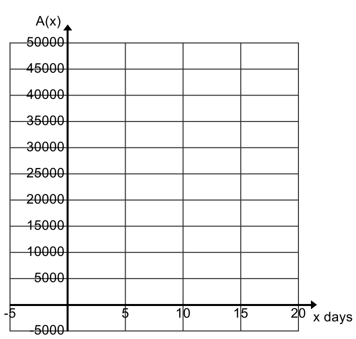
__________________________________Guided Notes Applications of Exponential Models and Natural change models

Growth Models	Decay Models	Compound Interest Models
Exponential Growth Model $A(x) = P(1+r)^x$ $= P(b)^x$ A(x) = population after time x P = initial population r = rate of change as a decimal x = time passed since initial observation b= change factor (growth factor)>1 Natural Growth Model $A(x) = Pe^{rx}$ A(x) = population after time x P = initial population r = rate of change as a decimal x = time passed since initial e^r = change factor > 1 THIS MODEL needs to be given or the phrases “follows the natural growth model” or CONTINUOUSLY grows $r > 0$	Exponential Decay Model $A(x) = P(1-r)^x$ $= P(b)^x$ A(x) = population after time x P = initial population r = rate of change as a decimal x = time passed since initial observation b=change factor (decay factor) < 1 Natural Decay Model $A(x) = Pe^{rx}$ A(x) = population after time x P = initial population r = rate of change as a decimal x = time passed since initial e^r = change factor < 1 THIS MODEL needs to be given or the phrases “follows the natural growth model” or CONTINUOUSLY grows $r < 0$	$A(x) = P(1+\frac{r}{n})^{nx}$ A(x) = account balance after time x P = initial deposit into account r = rate of change as a decimal n = number of times interest is compound over a year x = time passed since initial observation when $n \to \infty$ then $(1+\frac{1}{n})^n \to e$ annually → n = 1 quarterly→ n = 4 monthly → n = 12 weekly → n = 52 daily → n = 365 continuously → $A(x)= Pe^{rx}$

1. A population of 12000 bugs is said to grow exponentially at a rate of 1.2% per day
   a. State the model ___________________________________

X (number of days since initial observation)	0	5	10	15	20
A(x) (Number of bugs since initial observation)					

   b. Label at least three points on the graph



   c. Determine when this bug population doubles

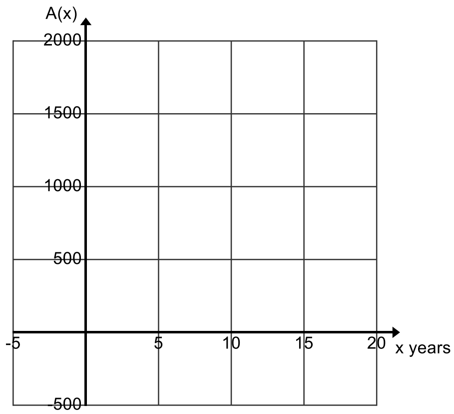
   d. Determine when this bug population is 1.6 times the original population

2. A population of 100 slugs reaches 175 in 90 days , if this population of slugs is said to grow exponentially, then determine the rate of change as a decimal, as a percent, and write the population model.

3. A population of 1500 salmon is said to decay exponentially at a rate of 3.56% per year
   a. State the model ___________________________________

X (number of years since initial observation)	0	5	10	15	20
A(x) (Number of fish since initial observation)					

   b. Label at least three points on the graph



   c. Determine when this salmon population is half of its initial size

   d. Determine when this salmon population is 1000

4. A population of 150 coyotes reaches 120 in 2 years, if this population of wolves is said to decay exponentially, then determine the rate of change as a decimal, as a percent, and write the population model.

5. Complete the table for the following expression b = $\left(1+\frac{1}{n}\right)^{n}$

n	2	12	52	365
b				
n	1000	2000	5000	10000
b				

6. Use your calculator to determine the number of seconds in a year

7. Determine b = $\left(1+\frac{1}{n}\right)^{n}$ with the number of seconds you found in #6 as n

8. Use your calculator to determine the value of e

Simple interest model account P=1000 n = 1 r = 0.025	Monthly compounded interest P=1000 n=12 r = 0.025	Weekly compounded interest P=1000 n=52 r = 0.025	Daily compounded interest P=1000 n=365 r = 0.025	Continuously compounded interest P=1000 n =∞ r =0.025
State the Account Balance model	State the Account Balance model	State the Account Balance model	State the Account Balance model	State the Account Balance model
Complete the table	Complete the table	Complete the table	Complete the table	Complete the table

x years	A(x)	x years	A(x)	x years	A(x)	x years	A(x)	x years	A(x)
0		0		0		0		0	
5		5		5		5		5	
10		10		10		10		10	
15		15		15		15		15	
20		20		20		20		20	

9. What was the balance in the account for the 1000 investment at 2.5% interest in a SIMPLE interest account after 20 years?

10. How much interest was paid (Interest = A(x) -P) after 20 years in the simple interest account?

11. What did you notice about both the account balances (A(x)) and the Interest earned (A(x) -P) as n got larger?

12. The average cost of attending four years of college is said to be approximately 102,000.
    a. If you got a federal Stafford loan at a rate of 4.53% for 20 years, then how much would you eventually pay for college?

    b. How much of that was interest? (The federal loan program follows the continuously compounded interest model)

13. A certain radioactive element decays really slowly. If 99.5% of the material is still present in 100 years
    a. Use the natural decay model to determine the model for this radioactive element.

    b. State the rate of decay per year as a percent.

14. Another radioactive element is said to follow this model $A(x) = Pe^{-0.0021x}$.
    a. Determine the amount of time this model will take to reduce 100 grams of the material to 50 grams of the material. (this is called the radioactive half life of the material)

    b. What was the rate of decay as a percent per year?

15. A population of rabbits is said to follow the natural growth model at a rate of 2.4% per year. Given that your initial population is 500 rabbits:
    a. Complete the related table

Time in years	0	5	10	15	20
Population after time					

    b. Determine when this rabbit population reaches 15000 (round to three decimal places)

1. How long will it take for $2800 to grow to $3000 if invested at 4% interest compounded semiannually?
1.7 years

2. How long will it take for $2800 to grow to $3500 if invested at 5% interest compounded quarterly?
4.5 years

3. How long will it take for $1000 to grow to $1750 if invested at 5.75% compounded quarterly?
9.8 years

4. Three thousand dollars is left in a bank savings account drawing interest compounded quarterly at 5%. How long will it take for the balance to triple?
22.0 years

5. A thousand dollars is left in a credit union drawing 7%, compounded monthly, how many years will it need to be left to produce an ending balance of $2500?
13.1 years

6. $1750 is invested in an account earning 3.5% interest compounded continuously. How long will it need to be in an account to double?
19.8 years

7. What principal will grow to $3000 if invested at 3% interest compounded semiannually for 10 years?
$2227.41

8. What principal will amount to $2500 if invested at 5% interest compounded continuously for 7.5 years?
$1718.22

Another way to signify the "natural growth or natural decay model" is to say that the population change is proportional to the population

Solve each exponential growth/decay problem.

1) For a period of time, an island's population grows at a rate proportional to its population. If the growth rate is 3.8% per year and the current population is 1543, what will the population be 5.2 years from now?

2) During the exponential phase, E. coli bacteria in a culture increase in number at a rate proportional to the current population. If the growth rate is 1.9% per minute and the current population is 172.0 million, what will the population be 7.2 minutes from now?

3) Radioactive isotope Carbon-14 decays at a rate proportional to the amount present. If the decay rate is 12.10% per thousand years and the current mass is 135.2 mg, what will the mass be 2.2 thousand years from now?

4) A savings account balance is compounded continuously. If the interest rate is 3.1% per year and the current balance is $1077.00, in how many years will the balance reach $1486.73?

1. The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.26%.

   a. Find the growth factor for the world population.

   b. Suppose the rate of increase continues to be 1.26% . Write a function to model the world population

   c. Let x be the number of years past the year 2000. Find the world population in 2010.

2. A computer valued at $6500 depreciates at the rate of 14.3% per year.

   a. Write a function that models the value of the computer.

   b. Find the value of the computer after three years.

3. The population of a certain animal species decreases at a rate of 3.5% per year. You have counted 80 of the animals in the habitat you are studying.

   a. Write a function that models the change in the animal population.

   b. Graph the function. Estimate the number of years until the population fist drops below 15 animals

4. Write an exponential function to model each situation. Find the value of each function after five years.

   a. A $12,500 car depreciates 9% each year

   b. A baseball card bought for $50 increases 3% in value each year.

5. A new car that sells for $18,000 depreciates 25% each year. Write a function that models the value of the car. Find the value of the car after 4 yr.

6. A new truck that sells for $29,000 depreciates 12% each year. Write a function that models the value of the truck. Find the value of the truck after 7 yr.

7. The bear population increases at a rate of 2% per year. There are 1573 bear this year. Write a function that models the bear population. How many bears will there be in 10 yr?

8. An investment of $75,000 increases at a rate of 12.5% per year. Find the value of the investment after 30 yr.

9. The population of an endangered bird is decreasing at a rate of 0.75% per year. There are currently about 200,000 of these birds. Write a function that models the bird population. How many birds will there be in 100 yr?