Pre-Calculus/Trig 3
UNIT 1: Algebra II Review – SECTION 7 WORKSHEET #1

Name: KEY
Date:

GRAPHING RATIONAL FUNCTIONS

To Identify Types of Discontinuity:

Step 1: **HOLES (Removable Discontinuities)**
- ✓ Factor numerator & denominator
- ✓ Simplify
- ✓ If anything cancels, then there is a hole (More than one factor cancels → More than one hole)
- ✓ Find the ordered pair, (x, y), substitute *x* into the ***SIMPLIFIED EQUATION*** to get *y*

Step 2: **VERTICAL ASYMPTOTES *(USE SIMPLIFIED EQUATION)***
- ✓ Set simplified equation denominator = 0, solve for *x*

Step 3: **HORIZONTAL ASYMPTOTES – *Two Cases (USE SIMPLIFIED EQUATION)***
- ✓ Degree of Denominator = Degree of Numerator → $y =$ ratio of leading coefficients
- ✓ Degree of Denominator > Degree of Numerator → $y = 0$

Step 4: **SLANT ASYMPTOTES *(Exists only if Horizontal Asymptote is not present) (USE SIMPLIFIED EQUATION)***
- ✓ Degree of Numerator is ***ONE*** degree larger than the Degree of Denominator
- ✓ Use Long Division
- ✓ Ignore the remainder
- ✓ Answer in the form $y = mx + b$

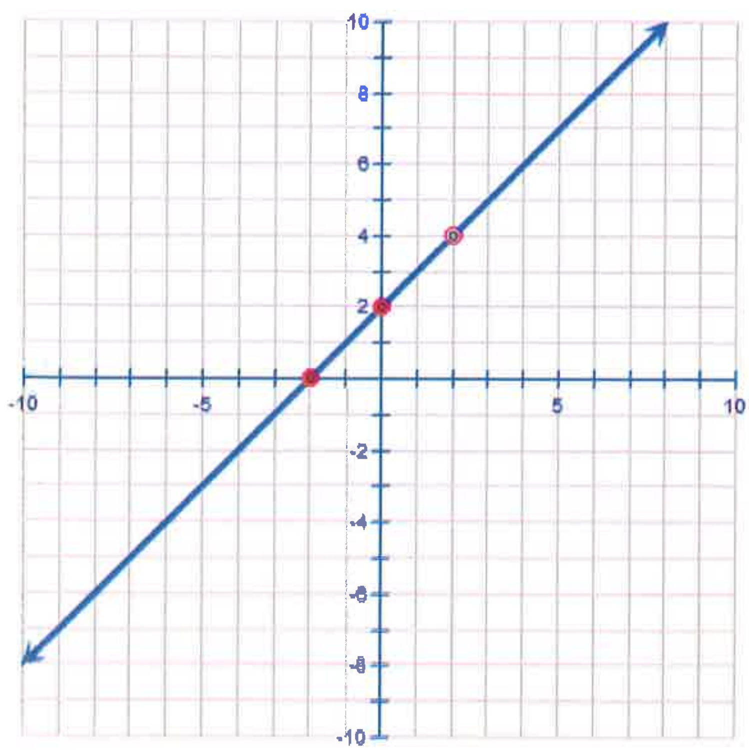
Directions: State each discontinuity, x-intercept, and y-intercept. Then sketch a graph.

1.) $f(x) = \frac{x^2-4}{x-2}$: $\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = x+2$

HOLE: $x-2=0 \rightarrow y = (2)+2$
$x = 2$, $y = 4$

X-INT: $0 = x+2$
$x = -2$

Y-INT: $y = 0+2$
$y = 2$



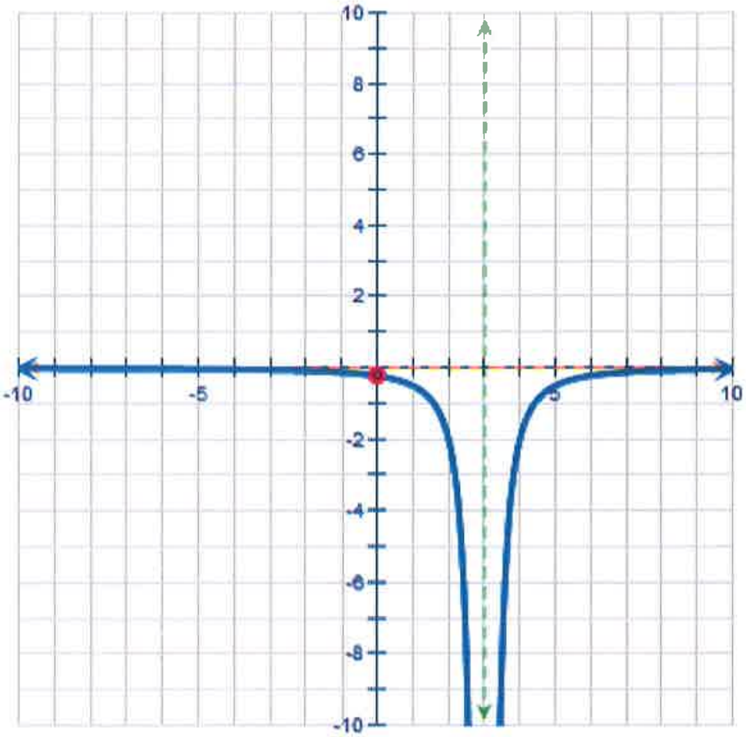
HOLE(S)	VERTICAL ASYMPTOTE(S)	HORIZONTAL ASYMPTOTE	SLANT ASYMPTOTE	x-intercept(s)	y-intercept
(2, 4)	NONE	NONE	NONE	(-2, 0)	(0, 2)

ENTIRE DENOMINATOR CANCELED → NO ASYMPTOTES!

2.) $f(x) = \frac{-2}{(x-3)^2}$

VA $x-3=0$
$x=3$ $\rightarrow$ $y=$

Y-INT: $\frac{-2}{(0-3)^2} = \frac{-2}{(-3)^2} = \frac{-2}{9}$

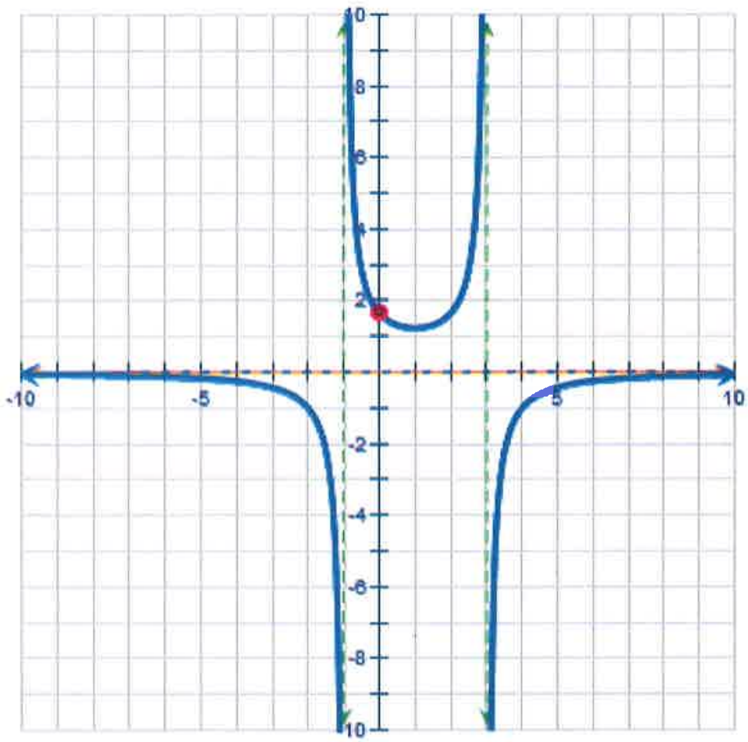


HOLE(S)	VERTICAL ASYMPTOTE(S)	HORIZONTAL ASYMPTOTE	SLANT ASYMPTOTE	x-intercept(s)	y-intercept
NONE	$X=3$	$Y=0$	NONE	NONE	$(0, -\frac{2}{9})$

3.) $f(x) = \frac{-5}{x^2-2x-3} = \frac{-5}{(x-3)(x+1)}$

VA: $x-3=0$ $x+1=0$
$x=3$ $x=-1$

Y-INT: $y = \frac{-5}{(0-3)(0+1)} = \frac{-5}{(-3)(1)} = \frac{-5}{-3} = \frac{5}{3}$



HOLE(S)	VERTICAL ASYMPTOTE(S)	HORIZONTAL ASYMPTOTE	SLANT ASYMPTOTE	x-intercept(s)	y-intercept
NONE	$X=-1$ $X=3$	$Y=0$	NONE	NONE	$(0, \frac{5}{3})$

4.) $f(x) = \frac{x^3+4x^2-21x}{x^2+4x-21} = \frac{x(x^2+4x-21)}{(x+7)(x-3)}$

$= \frac{x\cancel{(x+7)}\cancel{(x-3)}}{\cancel{(x+7)}\cancel{(x-3)}}$

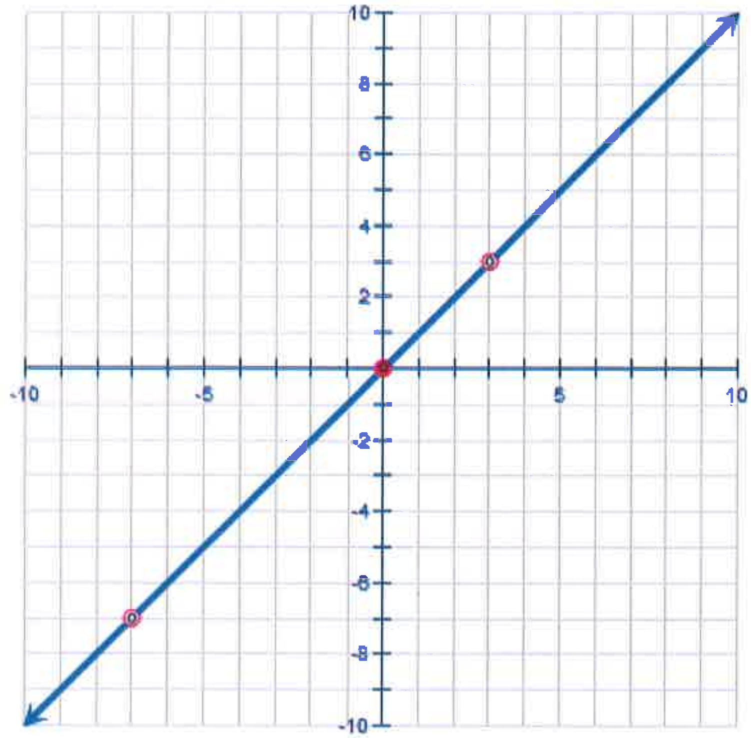
$= x$

HOLES: $x+7=0$, $x=-7$ → $y=(-7)$, $y=-7$

$x-3=0$, $x=3$ → $y=(3)$, $y=3$

X-INT: $0=x$

Y-INT: $y=0$



HOLE(S)	VERTICAL ASYMPTOTE(S)	HORIZONTAL ASYMPTOTE	SLANT ASYMPTOTE	*x*-intercept(s)	*y*-intercept
(-7, -7) (3, 3)	NONE	NONE	NONE	(0, 0)	(0, 0)

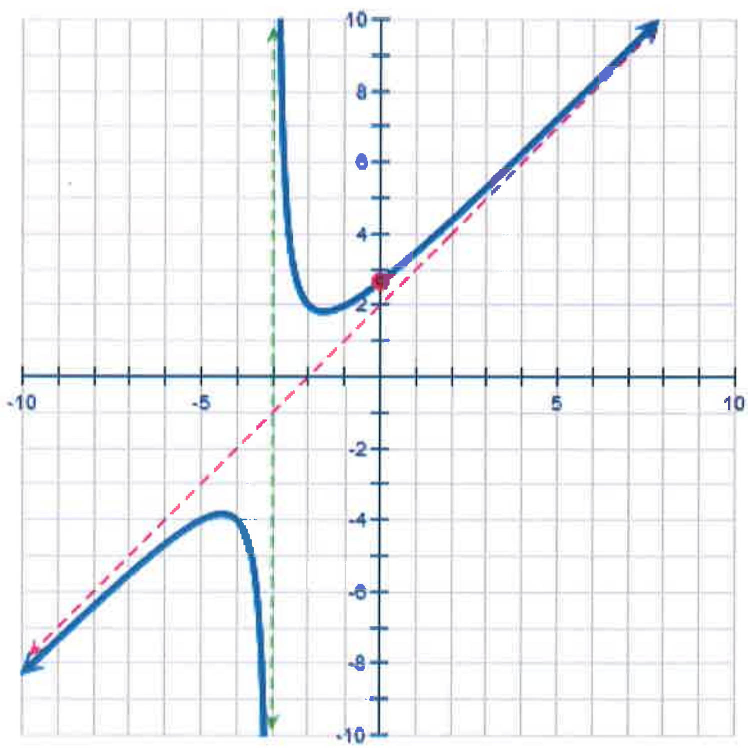
5.) $f(x) = \frac{x^2+5x+8}{x+3} =$

VA: $x+3=0$, $x=-3$

SA:
$$\begin{array}{r|rrr} -3 & 1 & 5 & 8 \\ & \downarrow & -3 & -6 \\ \hline & 1 & 2 & 2 \end{array}$$

$y = x+2$

Y-INT: $y = \frac{(0)^2+5(0)+8}{(0)+3} = \frac{8}{3}$



HOLE(S)	VERTICAL ASYMPTOTE(S)	HORIZONTAL ASYMPTOTE	SLANT ASYMPTOTE	*x*-intercept(s)	*y*-intercept
NONE	$x=-3$	NONE	$y=x+2$	NONE	$(0, \frac{8}{3})$

6.) $f(x) = \frac{x^2+x-2}{(x+2)(x^2-2x-15)} = \frac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}(x-5)(x+3)}$

$= \frac{x-1}{(x-5)(x+3)}$

HOLES: $x+2=0 \rightarrow$ $x=-2$ $y = \frac{-2-1}{(-2-5)(-2+3)}$

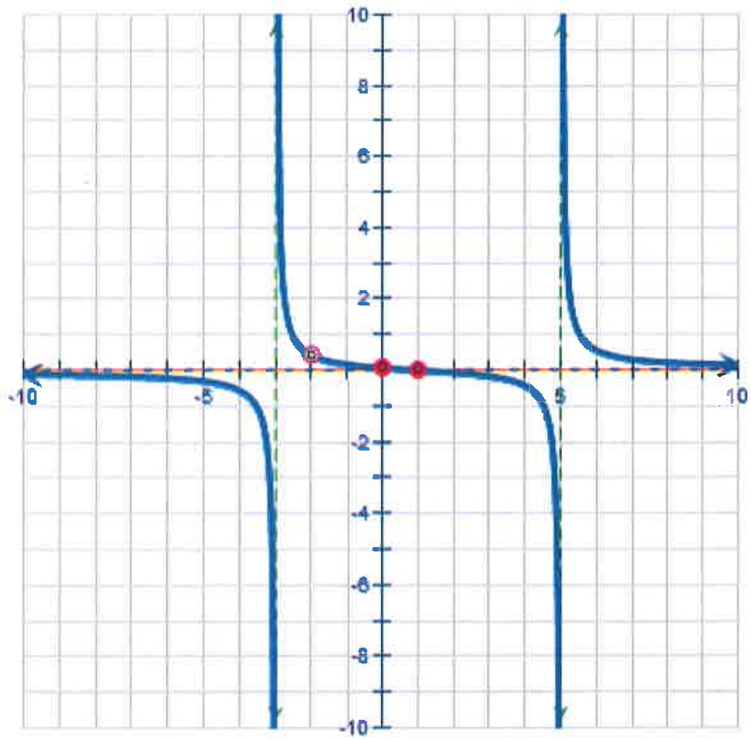
$y = \frac{-3}{-7(1)} = \frac{-3}{-7} = \frac{3}{7}$

VA: $x-5=0$ $x=5$ $x+3=0$ $x=-3$

X-INT: $x-1=0$ $x=1$

Y-INT: $y = \frac{0-1}{(0-5)(0+3)}$

$y = \frac{-1}{-5(3)} = \frac{1}{15}$



HOLE(S)	VERTICAL ASYMPTOTE(S)	HORIZONTAL ASYMPTOTE	SLANT ASYMPTOTE	x-intercept(s)	y-intercept
$\left(-2, \frac{3}{7}\right)$	$x=5$ $x=-3$	$y=0$	NONE	$(1,0)$	$\left(0, \frac{1}{15}\right)$

7.) $f(x) = \frac{x^2+3x-4}{x} = \frac{(x+4)(x-1)}{x}$

VA: $x=0$

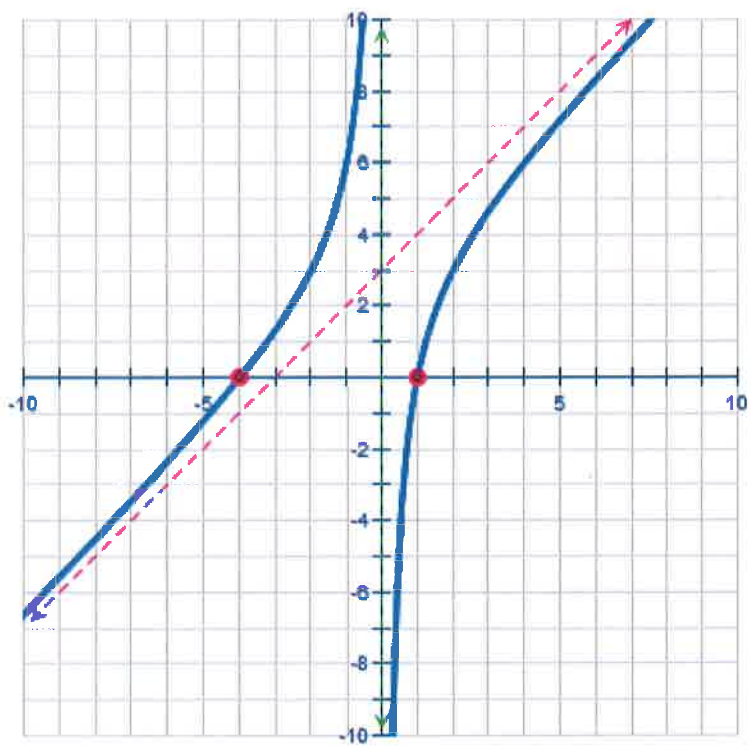
SA:
$$\begin{array}{r|rrr} 0 & 1 & 3 & -4 \\ & \downarrow & 0 & 0 \\ \hline & 1 & 3 & -4 \end{array}$$

$y = x+3$

X-INT: $x+4=0$ $x=-4$ $x-1=0$ $x=1$

Y-INT: $y = \frac{(0)^2+3(0)-4}{0} = \frac{-4}{0} \rightarrow$ UNDEFINED

$\Rightarrow$ NO Y-INT



HOLE(S)	VERTICAL ASYMPTOTE(S)	HORIZONTAL ASYMPTOTE	SLANT ASYMPTOTE	x-intercept(s)	y-intercept
NONE	$x=0$	NONE	$y=x+3$	$(-4,0)$ $(1,0)$	NONE