Name____AP Calc PR 2 Review DATE____________ Time Limit 10 minutes

Free Response part 2 DUE Tomorrow Suggested Time Limit 15 Minutes

2. The temperature on New Year's Day in Hinterland was given by $T(H) = -A - B\cos\left(\frac{\pi H}{12}\right)$, where T is the temperature in degrees Fahrenheit and H is the number of hours from midnight $(0 \le T < 24)$.

(a) The initial temperature at midnight was $-15°\ F$, and at noon of New Year's Day was $5°\ F$. Find A and B.

(b) Find the average temperature for the first 10 hours.

(c) Use the Trapezoid Rule with 4 equal subdivisions to estimate $\int_6^8 T(H)\, dH$.

(d) Find an expression for the rate that the temperature is changing with respect to H.

Show work for part a)

Show work for part b)

Show work for part c)

Show work for part d)

Problem 2. The temperature on New Year's Day in Hinterland was given by $T(H) = -A - B\cos\left(\frac{\pi H}{12}\right)$, where T is the temperature in degrees Fahrenheit and H is the number of hours from midnight ($0 \le T < 24$).

(a) Simply plug in the temperature, –15, for T and the time, midnight ($T = 0$), for H into the equation. We get: $-15 = -A - B\cos 0$, which simplifies to $-15 = -A - B$.

Now plug the temperature, 5, for T and the time, noon ($H = 12$), for H into the equation. We get: $5 = -A + B\cos(\pi)$, which simplifies to $5 = -A + B$.

Now we can solve the pair of simultaneous equations for A and B, and we get $A = 5°\,F$ and $B = 10°\,F$.

(b) In order to find the average value, we use the Mean Value Theorem for integrals, which says that the average value of $f(x)$ on the interval $[a, b]$ is:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Here, we have: $\frac{1}{10-0}\int_0^{10}\left(-5-10\cos\left(\frac{\pi H}{12}\right)\right)dH$

Evaluating the integral, we get:

$$\frac{1}{10}\left[\left(-5H-\frac{120}{\pi}\sin\left(\frac{\pi H}{12}\right)\right)\right]_0^{10} = \frac{1}{10}\left[\left(-50-\frac{120}{\pi}\sin\left(\frac{5\pi}{6}\right)\right)\right] = \frac{1}{10}\left[\left(-50-\frac{60}{\pi}\right)\right] \approx -6.910°F$$

(c) The Trapezoid Rule enables us to approximate the area under a curve with a fair degree of accuracy. The rule says that the area between the x-axis and the curve $y = f(x)$, on the interval $[a, b]$, with n trapezoids, is:

$$\frac{1}{2}\frac{b-a}{n}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + \dots + 2y_{n-1} + y_n\right]$$

Using the rule here, with $n = 4$, $a = 6$, and $b = 8$, we get:

$$\frac{1}{2}\cdot\frac{1}{2}\left[\left(-5-10\cos\frac{6\pi}{12}\right)+2\left(-5-10\cos\frac{13\pi}{24}\right)+2\left(-5-10\cos\frac{7\pi}{12}\right)+2\left(-5-10\cos\frac{15\pi}{24}\right)+\left(-5-10\cos\frac{8\pi}{12}\right)\right]$$

This is approximately $-4.890°\,F$.

(d) We simply take the derivative with respect to H:

$$\frac{dT}{dH} = -10\left(\frac{\pi}{12}\right)\left(-\sin\frac{\pi H}{12}\right) = \frac{5\pi}{6}\sin\frac{\pi H}{12}$$