

Solve the following differential equations:

1. There are 120 grams of a radioactive substance whose half-life is 74 years. Determine how much of the substance will remain after 50 years.

2. A bacteria culture containing 2,400 cells 3 hours ago has now grown to 5,200 cells. Assuming the rate of growth is proportional to the number present, determine the time (from now) at which the population will reach 10,000.

3. A colony of bacteria with originally 100 cells has now grown to 400 in 2 hours. Assuming the rate of growth is proportional to the present number, find the number of bacteria in 5 hours from now.

4. There were 100 grams of a radioactive substance ten years ago, now there are only 32 grams. Find the substance's half-life.

5. A bank account contains \$1,200 and pays an annual interest rate of 5.25% compounded continuously. Determine the time at which the money doubles.

6. An object is 3400 years old, find the percentage of its original Carbon-14 content it should have now (Carbon-14 has a half-life of 5730 years).

7. Intensity of light beam passing through an absorbing medium decreases at a rate proportional to the intensity at any given depth. Suppose at the surface of the water, the intensity of a light bulb is 20 candelas and 14 candelas under a yard of water. Find the light intensity under 20 feet of water.

8. The population of a country is growing at a rate proportional to its population. If the growth rate per year is 5% of the current population, in how many years will the population double?

9. Suppose the prices for real estate grow exponentially. If a house was worth \$100,000 five years ago, and is now worth \$250,000, find the year (from now) in which the price will exceed \$500,000.

11. When a capacitor is being discharged, the equation describing the charge on one plate of the capacitor is

$$\frac{dq}{dt} + \frac{q}{C} = 0$$

where q is the charge (C), R is the resistance of the circuit (Ω), C is the capacitance of the capacitor (F), and t is time in seconds. If a 5 mF capacitor with an initial charge of $3\mu\text{C}$ is discharged through a 100 resistor, find the time when 90% of the charge has been drained.

12. A RL circuit has a resistance of $2\ \Omega$ an inductance of 5 H, and an initial current of 8 A. Find (a) the current in the circuit at any time t and (b) its current after 10 seconds. The equation describing the current in the circuit is

$$L \frac{dI}{dt} + RI = 0$$

where L is the inductance (H), I is the current (A), R is the resistance (Ω), and t is time in seconds.

Name _____ Date _____ Period _____

Worksheet 7.3—Separable Differential Equations

Show all work. No Calculator unless specified.

Multiple Choice

1. (OK, so you can use your calculator right away on a non-calculator worksheet. Use it on this one.) A sample of Kk-1234 (an isotope of Kulmakorpium) loses 99% of its radioactive matter in 199 hours. What is the half-life of Kk-1234?
(A) 4 hours (B) 6 hours (C) 30 hours (D) 100.5 hours (E) 143 hours

2. In which of the following models is $\frac{dy}{dt}$ directly proportional to y ?

I. $y = e^{kt} + C$

II. $y = Ce^{kt}$

III. $y = 28^{kt}$

IV. $y = 3\left(\frac{1}{2}\right)^{3t+1}$

- (A) I only (B) II only (C) I and II only (D) II and III only (E) II, III, and IV (F) all of them

3. (Use your calculator on this one, too, but get the exact answer first.) The rate at which acreage is being consumed by a plot of kudzu is proportional to the number of acres already consumed at time t . If there are 2 acres consumed when $t = 1$ and 3 acres consumed when $t = 5$, how many acres will be consumed when $t = 8$?
- (A) 3.750 (B) 4.000 (C) 4.066 (D) 4.132 (E) 4.600

Free Response

For problems 4 – 13, find the general solution to the following differential equations, then find the particular solution using the initial condition.

4. $\frac{dy}{dx} = \frac{x}{y}$, $y(1) = -2$

5. $\frac{dy}{dx} = -\frac{x}{y}$, $y(4) = 3$

6. $\frac{dy}{dx} = \frac{y}{x}$, $y(2) = 2$

7. $\frac{dy}{dx} = 2xy$, $y(0) = -3$

8. $\frac{dy}{dx} = (y+5)(x+2)$, $y(0) = -1$

9. $\frac{dy}{dx} = \cos^2 y$, $y(0) = 0$

$$10. \frac{dy}{dx} = (\cos x)e^{y+\sin x}, y(0) = 0 \quad 11. \frac{dy}{dx} = e^{x-y}, y(0) = 2 \quad 12. \frac{dy}{dx} = -2xy^2, y(1) = 0.25$$

$$13. \frac{dy}{dx} = \frac{4\sqrt{y} \ln x}{x}, y(e) = 1$$

For problems 14 – 17, find the solution of the differential equation $\frac{dy}{dt} = ky$ that satisfies the given conditions.

$$14. k = 1.5, y(0) = 100$$

$$15. k = -0.5, y(0) = 200$$

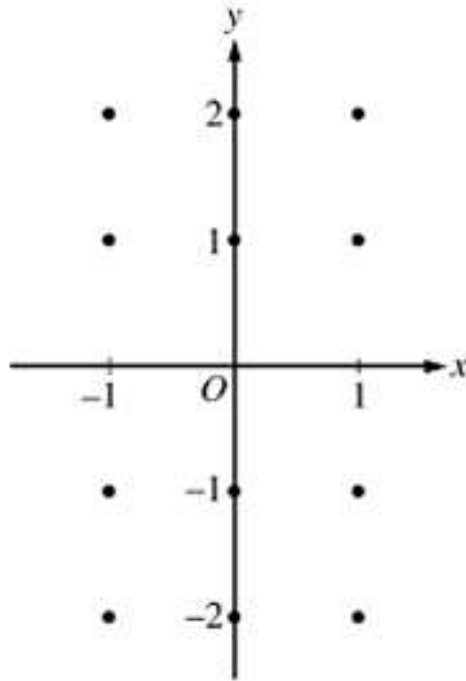
$$16. y(0) = 50, y(5) = 100$$

$$17. y(1) = 55, y(10) = 30 \text{ (divide one by the other)}$$

18. AP 2010B-5 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

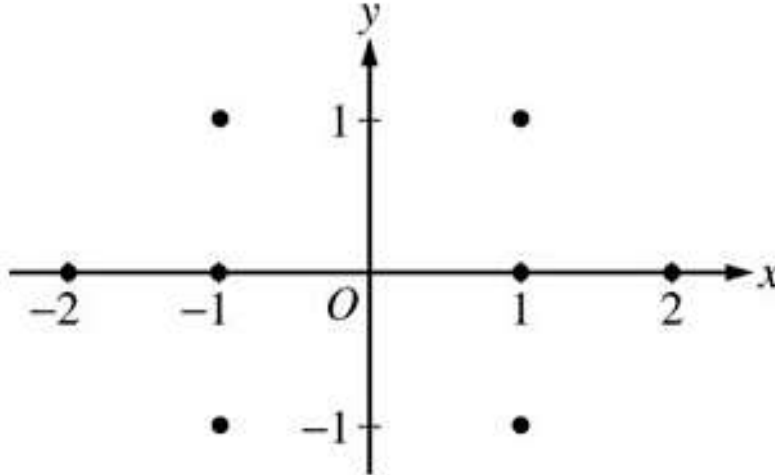


- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy -plane for which $y \neq 0$. Describe all points in the xy -plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

19. AP 2006-5

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

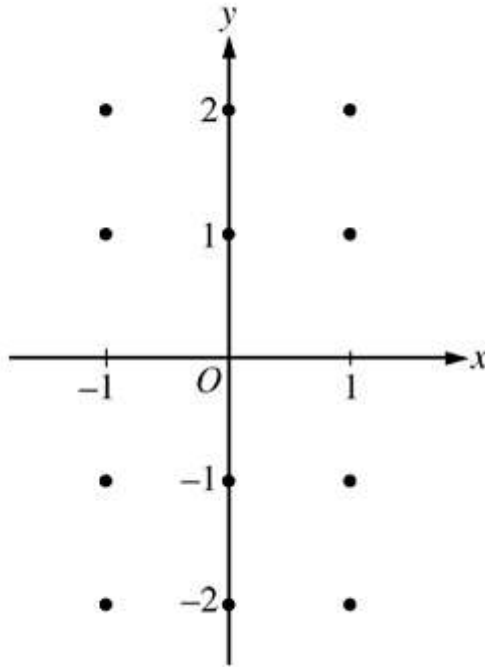


- (b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

20. AP 2005-6

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



- (b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.
- (c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.