Name____AP Calc PR 2 Review 4-23-19

Question 1: Derivative at a point

1. If $g(x) = \frac{1}{32}x^4 - 5x^2$ find $g'(4)$.

(A) –72
(B) –32
(C) –24
(D) 24
(E) 32

Question 3: Limit of a Rational Function

3. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is

(A) 0
(B) 10
(C) –10
(D) 5
(E) The limit does not exist.

Question 2: Domain of a Function

2. The domain of the function $f(x) = \sqrt{4 - x^2}$ is

(A) $x < -2$ or $x > 2$
(B) $x \leq -2$ or $x \geq 2$
(C) $-2 < x < 2$
(D) $-2 \leq x \leq 2$
(E) $x \leq 2$

Question 4: Derivative of a Rational Function

4. If $f(x) = \frac{x^5 - x + 2}{x^3 + 7}$, find $f'(x)$.

(A) $\frac{(5x^4 - 1)}{(3x^2)}$

(B) $\frac{(5x^4 - 1) - (3x^2)}{(x^3 + 7)}$

(C) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)}$

(D) $\frac{(x^5 - x + 2)(3x^2) - (x^3 + 7)(5x^4 - 1)}{(x^3 + 7)^2}$

(E) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)^2}$

Question 5: Limit Definition

5. Evaluate $\lim_{h \to 0} \frac{5\left(\frac{1}{2}+h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$.

(A) $\frac{5}{2}$

(B) $\frac{5}{16}$

(C) 40

(D) 160

(E) The limit does not exist.

Free Response part 1

1. Consider the curve defined by $y = x^4 + 4x^3$.

   (a) Find the equation of the tangent line to the curve at $x = -1$.

   (b) Find the coordinates of the absolute minimum.

   (c) Find the coordinates of the point(s) of inflection.

Show work for part a)

Show work for part b)

Show work for part c)

Question 1: Derivative at a point

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(A) –72

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(C) –24

(D) 24

(E) 32

Problem 1. If $g(x) = \frac{1}{32}x^4 - 5x^2$, find $g'(4)$.

First, take the derivative:

$$g'(x) = \frac{1}{32}(4x^3) - 5(2x) = \frac{x^3}{8} - 10x$$

Now, plug in 4 for x:

$$\frac{(4)^3}{8} - 10(4) = 8 - 40 = -32$$

The answer is (B).

Question 2: Domain of a Function

2. The domain of the function $f(x) = \sqrt{4 - x^2}$ is

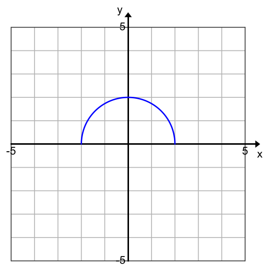
(A) $x < -2$ or $x > 2$

(B) $x \leq -2$ or $x \geq 2$

(C) $-2 < x < 2$

(D) $-2 \leq x \leq 2$

(E) $x \leq 2$



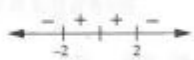
Problem 2. The domain of the function $f(x) = \sqrt{4 - x^2}$ is

When you have a square root in a function, the domain will require that the expression under the radical (the "radicand") not be negative. Thus, the domain will be those values where $4 - x^2$ is not negative.

In other words, $4 - x^2 \geq 0$.

We solve this by, first, factoring the expression on the left: $(2 + x)(2 - x) \geq 0$.

Next, we take the roots of the left side, which are –2 and 2, and put them on a number line:



Now, we pick a value in each of the three regions on the number line $x < -2$, $-2 < x < 2$, and $x > 2$. We plug the value into the expression $4 - x^2$ to see if we get a positive or negative value. If it's positive, then we include that region in the domain. If it's negative, then we exclude that region from the domain.

Let's try –3 for a value in the region $x < -2$.

We get: $4 - (-3)^2 = -5$, so we exclude the region $x < -2$ from the domain.

Now, we try 0 for a value in the region $-2 < x < 2$.

We get: $4 - (0)^2 = 4$, so we include the region $-2 < x < 2$ in the domain.

Finally, we try 3 for a value in the region $x > 2$.

We get: $4 - (3)^2 = -5$, so we exclude the region $x > 2$ from the domain.

Because the radicand is allowed to be zero, we include the endpoints in the domain. Therefore, the domain is $-2 \leq x \leq 2$.

Question 3: Limit of a Rational Function

3. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is

(A) 0

(B) 10

(C) −10

(D) 5

(E) The limit does not exist.

Problem 3. $\lim_{x \to 5} \frac{x^2 - 25}{x - 5}$ is

Notice that if we plug 5 into the expressions in the numerator and the denominator, we get: $\frac{0}{0}$, which is undefined. Before we give up, we need to see if we can simplify the limit so that it can be evaluated. If we factor the expression in the numerator, we get: $\frac{(x+5)(x-5)}{(x-5)}$, which can be simplified to $x + 5$.

Now, if we take the limit (by plugging in 5 for x), we get 10.

The answer is (B).

Question 4: Derivative of a Rational Function

4. If $f(x) = \frac{x^5 - x + 2}{x^3 + 7}$, find $f'(x)$.

(A) $\frac{(5x^4 - 1)}{(3x^2)}$

(B) $\frac{(5x^4 - 1) - (3x^2)}{(x^3 + 7)}$

(C) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)}$

(D) $\frac{(x^5 - x + 2)(3x^2) - (x^3 + 7)(5x^4 - 1)}{(x^3 + 7)^2}$

(E) $\frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)^2}$

Problem 4. If $f(x) = \frac{x^5 - x + 2}{x^3 + 7}$, find $f'(x)$.

We need to use the Quotient Rule, which is:

Given $f(x) = \frac{g(x)}{h(x)}$, then $f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$

Here, we have:

$$f'(x) = \frac{(x^3 + 7)(5x^4 - 1) - (x^5 - x + 2)(3x^2)}{(x^3 + 7)^2}$$

The answer is (E).

Question 5: Limit Definition

5. Evaluate $\lim_{h \to 0} \frac{5\left(\frac{1}{2} + h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$.

(A) $\frac{5}{2}$

(B) $\frac{5}{16}$

(C) 40

(D) 160

(E) The limit does not exist.

Problem 5. Evaluate $\lim_{h \to 0} \frac{5\left(\frac{1}{2} + h\right)^4 - 5\left(\frac{1}{2}\right)^4}{h}$.

Notice how this limit takes the form of the definition of the derivative, which is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Here, if we think of $f(x)$ as $5x^4$, then this expression gives the derivative of $5x^4$ at the point $x = \frac{1}{2}$.

The derivative of $5x^4$ is $f'(x) = 20x^3$.

At $x = \frac{1}{2}$, we get $f'\left(\frac{1}{2}\right) = 20\left(\frac{1}{2}\right)^3 = \frac{5}{2}$

The answer is (A).

Free Response part 1

1. Consider the curve defined by $y = x^4 + 4x^3$.

   (a) Find the equation of the tangent line to the curve at $x = -1$.

   (b) Find the coordinates of the absolute minimum.

   (c) Find the coordinates of the point(s) of inflection.

PROBLEM 1. Consider the curve defined by $y = x^4 + 4x^3$.

(a) If we want to find the equation of the tangent line, first we need to find the y-coordinate that corresponds to $x = -1$.

It is:

$$y = (-1)^4 + 4(-1)^3 = 1 - 4 = -3$$

Next, we need to find the derivative of the curve at $x = -1$.

It is $\frac{dy}{dx} = 4x^3 + 12x^2$ and, at $x = -1$, $\left.\frac{dy}{dx}\right|_{x=-1} = 4(-1)^3 + 12(-1)^2 = 8$.

Now we have the slope of the tangent line and a point that it goes through. We can use the point-slope formula for the equation of a line, $(y - y_1) = m(x - x_1)$, and plug in what we have just found.

We get:

$$(y + 3) = 8(x + 1), \text{ which can be rewritten as } y = 8x + 5$$

Show work for part b)

(b) First, we set the derivative equal to zero and solve for x:

$$\frac{dy}{dx} = 4x^3 + 12x^2 = 0$$

$$4x^2(x + 3) = 0$$

$$x = 0 \text{ or } x = -3$$

Now, we can use the second derivative test to determine whether a critical value is the x-coordinate of a minimum or a maximum. The second derivative test is the following:

If c is a critical point, then:

c is the x-coordinate of a maximum if $f''(c) < 0$, and

c is the x-coordinate of a minimum if $f''(c) > 0$.

By the way, c is the x-coordinate of a point of inflection if $f''(c) = 0$, and the second derivative changes sign at that point.

So now we need to find the second derivative:

$$\frac{d^2y}{dx^2} = 12x^2 + 24x$$

If we plug in $x = -3$, we get:

$$\frac{d^2y}{dx^2}\, 12(-3)^2 + 24(-3) = 36$$

So, the curve has a minimum at $x = -3$. Finally, to get the y-coordinate of the minimum, we plug $x = -3$ into the original equation and we get:

$$y = (-3)^4 + 4(-3)^3 = 81 - 108 = -27$$

Thus, the curve has an absolute minimum at $(-3, -27)$.

Show work for part c)

(c) In order to find points of inflection, we need to set the second derivative equal to zero. We have the second derivative from part (b) above.

$$12x^2 + 24x = 0$$
$$12x(x + 2) = 0$$
$$x = 0 \text{ or } x = -2$$

Next, we need to check if the second derivative changes sign at both of these points. We can do this by trying points on the number line in the different intervals created by these points. If we try a point to the left of $x = -2$, for example $x = -3$, and plug it into the second derivative, we get $\frac{d^2y}{dx^2} = 36$. If we then try a point between $x = 0$ and $x = -2$, for example $x = -1$, we get $\frac{d^2y}{dx^2} = -12$. Finally, if we try a point to the right of $x = 0$, for example $x = 1$, we get $\frac{d^2y}{dx^2} = 36$. The second derivative changes sign at both $x = 0$ and $x = -2$, so these are both the x-coordinates of points of inflection. To get the y-coordinates, simply plug $x = 0$ and $x = -2$ into the original equation: $y = x^4 + 4x^3$. We find that the points of inflection are (0, 0) and (−2, −16).

Name____________________________________

Date __________________________

Question 1

A	B	C	D	E

Question 2

A	B	C	D	E

Question 3

A	B	C	D	E

Question 4

A	B	C	D	E

Question 5

A	B	C	D	E

Grade Scale:
5=105 4= 102 3=100 2 =85 1 = 70 0 = 60

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