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To assist in the comprehension of a common error area, I will employ some INFORMAL vocabulary.

EQUAL Degree $=$ the highest exponent is the SAME in both the numerator (TOP of fraction) and the denominator (BOTTOM of fraction)

BOTTOM HEAVY = when a rational function has an exponent that is LARGER than all of the exponents in the DENOMINATOR

TOP HEAVY = when a rational function has an exponent that is LARGER than all of the exponents in the NUMERATOR

- When a rational function has EQUAL degree, then a horizontal asymptote exists and must be determined by the fraction created by the lead coefficients of the numerator and denominator
- $y=\frac{\text { lead coefficient of NUMERATOR }}{\text { lead coefficient of DENOMINATOR }}$
- When a rational function is BOTTOM HEAVY, then a horizontal asymptote exists and is $y=0$ (the $x$ axis)
- $y=0$
- When a rational function is TOP HEAVY, then a horizontal asymptote does NOT exist.
- A slant asymptote exists when the difference in degree of the numerator and denominator is exactly 1
- An oblique asymptote exists when the difference in degree of the numerator and denominator exceeds 1


## ALL ANSWERS ON FUTURE ASSIGNMENTS AND ASSESSMENTS MUST BE IN PROPER FORMAT

Lines are stated as lines and points are stated as points

BAD example

1. $f(x)=\frac{x^{2}-4}{3 x-9}$

X intercept(s) 2 or - 2
Y intercept $\frac{4}{9}$
Horizontal asymptote ???? or blank
Vertical asymptote 3

## GOOD example

1. $f(x)=\frac{x^{2}-4}{3 x-9}=\frac{(x-2)(x+2)}{3(x-3)}$
$X$ intercept(s) (-2,0) or (2,0)
Y intercept ( $0, \frac{4}{9}$ )
Horizontal asymptote
NONE or this has a slant asymptote
Vertical asymptote $\mathrm{x}=3$

To find the $y$ intercept of any function

1) Evaluate the function at $x=0$

To find the x intercepts of a rational function

1) Factor the numerator
2) Factor the denominator
3) Check to see if a hole is present FIRST
4) Cancel off any common terms
5) Set remaining factors from NUMERATOR equal to 0 and solve for $x$ (these numbers are the x intercepts)
6) STATE as a POINT $(x, 0)$

To determine if a rational function has a hole

1) Factor both the numerator and denominator
2) Does the numerator and denominator have a factor that has a variable in common?
3) If YES on 2) then a hole is present on the graph of the rational function
4) If YES on 2) set canceled factor equal to zero and solve for $x$ (this is the $x$ of your hole and a domain restriction)
5) If YES on 2) replace $x$ in NEW version of rational function with the solution you just found in 4)

To find vertical asymptotes of a rational function

1) Factor the numerator
2) Factor the denominator
3) Check to see if a hole is present FIRST
4) Cancel off any common terms
5) Set remaining factors from DENMINATOR equal to 0 and solve for $x$ (these numbers are the $x$ values of the vertical asymptotes \& domain restrictions)
6) STATE as an EQUATION $x=$ $\qquad$

## Intercepts \& Asymptotes

Determine the intercepts and asymptotes of each of the rational functions (if NONE, then state so)

1. $f(x)=\frac{x^{2}-100}{3 x+15}$

X intercept(s) $\qquad$

Y intercept $\qquad$

Horizontal asymptote $\qquad$

Vertical asymptote $\qquad$

Does this rational function have a hole? $\qquad$

If this rational function has a hole, then state it $\qquad$
2. $g(x)=\frac{20 x+40}{x^{2}-16}$

X intercept(s) $\qquad$
Y intercept $\qquad$

Horizontal asymptote $\qquad$
Vertical asymptote $\qquad$

Does this rational function have a hole? $\qquad$

If this rational function has a hole, then state it $\qquad$
3. $h(x)=\frac{-4 x^{2}-24 x}{x^{2}-36}$

X intercept(s) $\qquad$

Y intercept $\qquad$
Horizontal asymptote $\qquad$

Vertical asymptote $\qquad$

Does this rational function have a hole? $\qquad$

If this rational function has a hole, then state it $\qquad$
4. $j(x)=\frac{x^{2}-6 x-7}{2 x+14}$

X intercept(s) $\qquad$

Y intercept $\qquad$

Horizontal asymptote $\qquad$

Vertical asymptote $\qquad$

Does this rational function have a hole? $\qquad$

If this rational function has a hole, then state it $\qquad$
5. $k(x)=\frac{x+1}{x^{2}-7 x-8}$

X intercept(s) $\qquad$
Y intercept $\qquad$

Horizontal asymptote $\qquad$
Vertical asymptote $\qquad$

Does this rational function have a hole? $\qquad$
If this rational function has a hole, then state it $\qquad$
6. $m(x)=\frac{x^{2}+2 x-8}{2 x^{2}-8 x}$

X intercept(s) $\qquad$

Y intercept $\qquad$
Horizontal asymptote $\qquad$

Vertical asymptote $\qquad$

Does this rational function have a hole? $\qquad$

If this rational function has a hole, then state it $\qquad$
7. $w(x)=\frac{x^{2}-2 x-8}{2 x^{2}-8 x}$

X intercept(s) $\qquad$

Y intercept $\qquad$
Horizontal asymptote $\qquad$

Vertical asymptote $\qquad$
Does this rational function have a hole? $\qquad$

If this rational function has a hole, then state it $\qquad$

