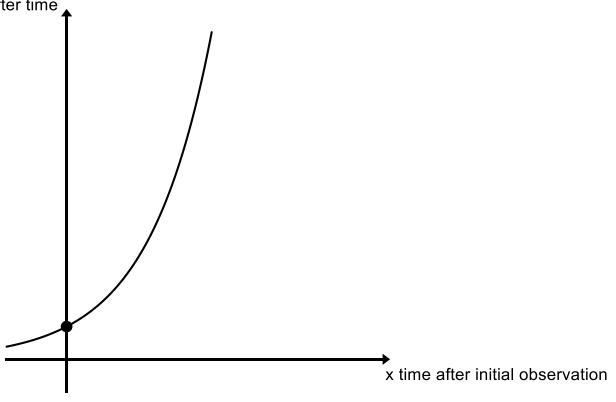
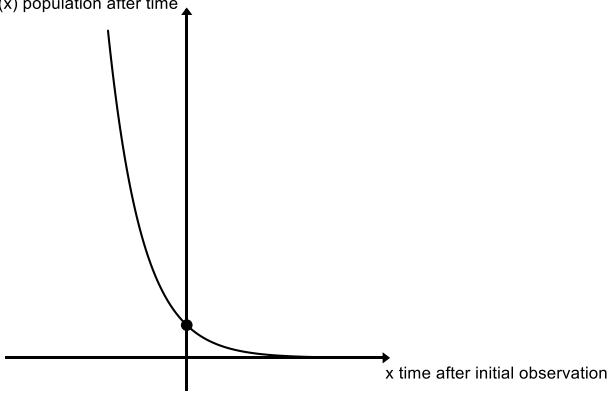


Name _____

Guided Notes and Practice Exponential Growth and Decay

A population is said to follow an exponential change if for each unit of time, current population changes by a particular percentage. That particular percentage is called the rate of change. This change is consistent, but the population that is impacted by the change will be new each time that the percentage change is applied.

Basic formula for exponential change model $A(t) = P(1+r)^t$ to make this easier for the use a calculator we might want to say this as $A(x) = P(1+r)^x$

<p>A = Amount left after time t</p> <p>P = initial amount or population</p> <p>r = rate of change (THIS MUST BE CONVERTED FROM %)</p> <p>t = amount of time that has passed since initial observation</p>	<p>This is an exponential population growth model $A(t) = P(1+r)^t$ or $A(x) = P(1+r)^x$</p> <p>The point on the y axis is ALWAYS $(0,P)$</p> <p>The point on the y axis is ALWAYS $(0,P)$</p>  <p>A graph showing an exponential growth curve starting from the y-axis at point (0, P). The curve is increasing and concave up, passing through a point marked with a dot on the curve.</p>
<p>P and r are the parameters of this model and $A(t)$ or $A(x)$ and t or x are called the variables of these model</p> <p>Parameter is a necessary numerical component that help to build a model</p> <p>Variable is the numerical input and output that build the coordinates of the model (x,y) or ($x, A(x)$) or ($t, A(t)$)</p> <p>x (or t) is said to be the independent variable and y (or $A(x)$ or $A(t)$) is said to be the dependent variable</p>	<p>This is an exponential population decay model $A(t) = P(1-r)^t$</p> <p>The point on the y axis is ALWAYS $(0,P)$</p>  <p>A graph showing an exponential decay curve starting from the y-axis at point (0, P). The curve is decreasing and concave down, passing through a point marked with a dot on the curve.</p>

There are some famous exponential change models in mathematics

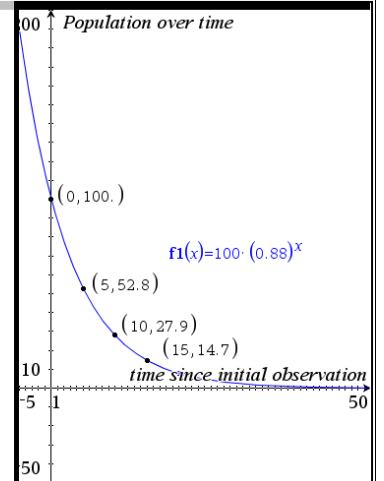
- The doubling model $A(x) = P(1+1)^x = P(2)^x$ in this case the rate of change is 100%
- The tripling model $A(x) = P(1+2)^x = P(3)^x$ in this case the rate of change is 200%
- The half model $A(x) = P(1-0.5)^x = P(0.5)^x$ in this case the rate of change is 50%

When we ADD the decimal version of the rate of change, we are building an exponential growth model.

When we SUBTRACT the decimal version of the rate of change, we are building an exponential growth model.

- Goal 1: Do recognize the type of model that is present?
 Goal 2: Can you determine the rate of change as a decimal?
 Goal 3: Can you determine the rate of change as a percent?
 Goal 4: Can you complete a table related to the population over time?
 Goal 5: Do you know the initial population from the model?
 Goal 6: Can you connect the graph of the model to its function?
 Goal 7: Can you label points from a table on the graph of a model? (including its initial point)

Model 1
 $A(x) = 100 \cdot (1 - 0.12)^x = 100 \cdot (0.88)^x$
 $A(0) = \text{population at time } 0 = 100.$
 Related Point (0, 100.)
 $A(5) = \text{population at time } 5 = 52.7732$
 Related Point (0, 52.7732)
 $A(10) = \text{population at time } 10 = 27.8501$
 Related Point (0, 27.8501)
 $A(15) = \text{population at time } 15 = 14.6974$
 Related Point (0, 14.6974)



For model 1

What type of model is present? _____ growth or decay

What is the rate of change as a decimal? _____ What is the rate of change as a percentage? _____ %

What was the initial population? _____

What does the point (15, 14.7) mean? The 15 tells us _____ while the 14.7 tells us _____

(the unit of time was not specifically mentioned but it can be seconds, minutes, hours, decades, etc. it just depends on scenario and population we are observing)

The domain of this model is $[0, \infty)$ The range of population in this model is $(0, 100]$ (we technically can never reach 0)

“Other” Questions that you might be asked to do when given the model, like this scenario

1. Complete the table related to the given model (Remember CTRL t makes a table with TI NSPIRE)

X	0	1	2	3	4	5	6
$A(x)$							

2. When does this population reach $\frac{1}{2}$ of its initial population?

- a. This simply asking you to:
 - i. know that $\frac{1}{2}$ of the initial population is 50
 - ii. solve the equation $50=100(0.88)^x$

3. When does this population fall below 75?

- a. This simply asking you to:
 - i. solve the equation $75=100(0.88)^x$
 - ii. state the INFINITELY many solutions using a set notation or inequality

4. When does this population exceed 16?

- a. This simply asking you to:
 - i. solve the equation $16=100(0.88)^x$
 - ii. state the INFINITELY many solutions using a set notation or inequality

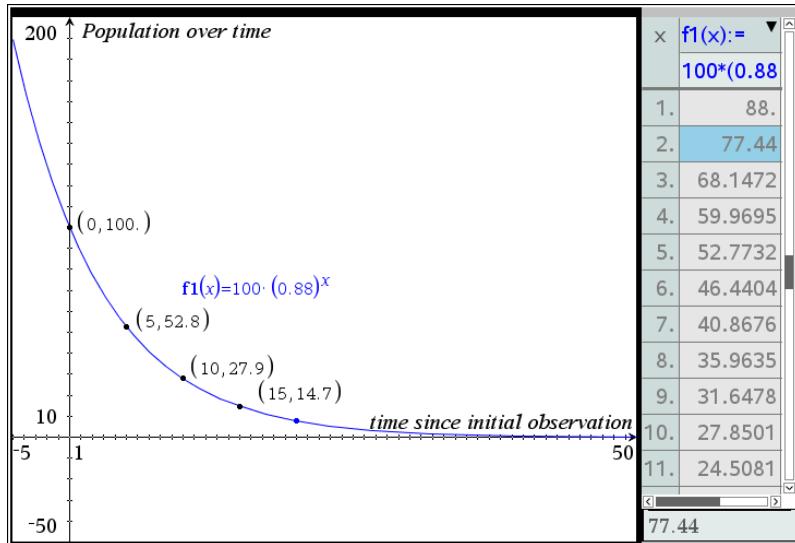
Model 1

1. Complete the table related to the given model (Remember CTRL t makes a table with TI NSPIRE)

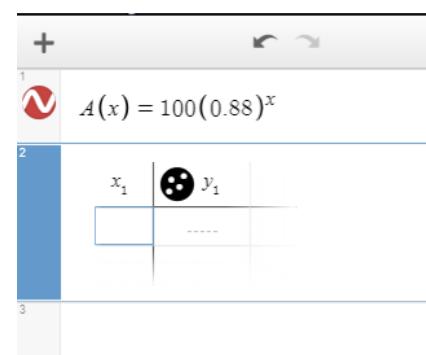
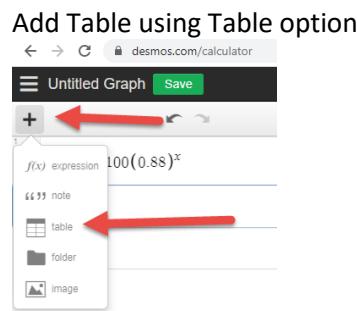
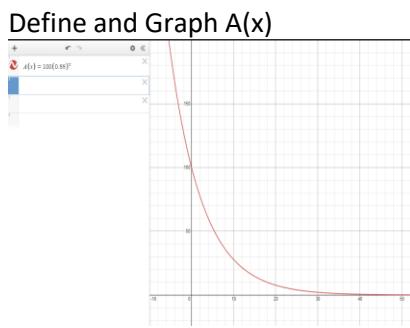
X	0	1	2	3	4	5	6
A(x)							

Option 1: TYPE EACH OF THOSE GIVEN X VALUES in $A(x) = 100(0.88)^x$

Option 2: Graph Function then hit CTRL T



Option 3: (Desmos option)



Define x_1 to be given x values

x_1	y_1
0
1
2
3
4
5
6

Change y_1 to $A(x_1)$

x_1	$A(x_1)$
0	100
1	88
2	77.44
3	68.1472
4	59.9695
5	52.7732
6	46.4404

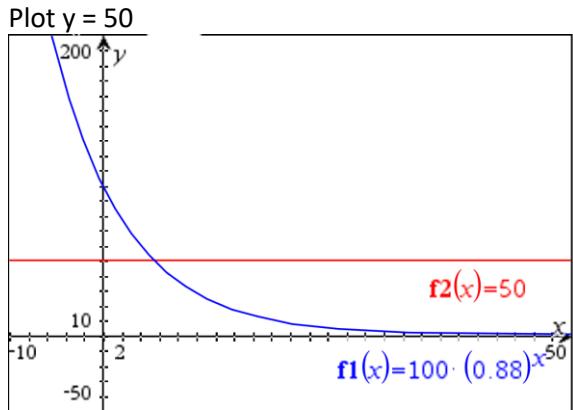
2. When does this population reach $\frac{1}{2}$ of its initial population?

a. This simply asking you to:

- i. know that $\frac{1}{2}$ of the initial population is 50
- ii. solve the equation $50=100(0.88)^x$

Option 1: Solve the equation $50=100(0.88)^x$ leads to $0.5 = 0.88^x$ which leads to $x = \log_{0.88} 0.5 \approx 5.422$

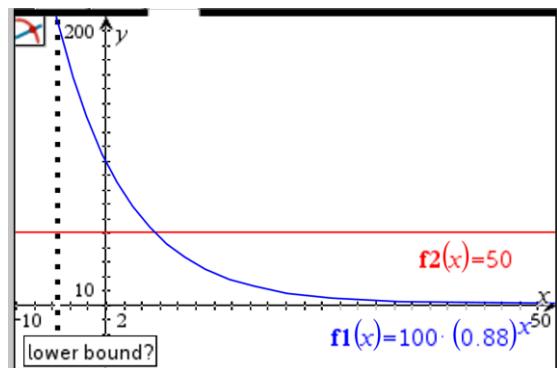
Option 2: Plot function and Use Analyze Graph Feature Intersection



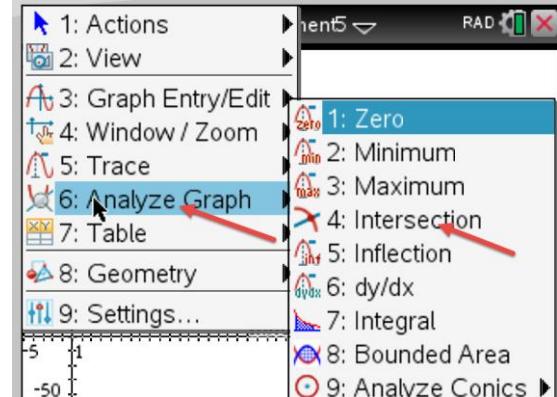
Select lower bound (point LEFT of intersection)

Select upper bound (point RIGHT of intersection)

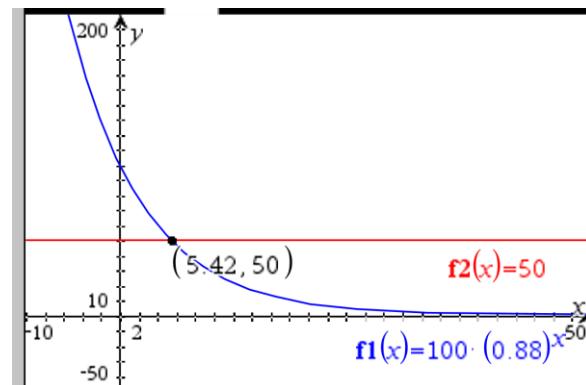
Hit enter



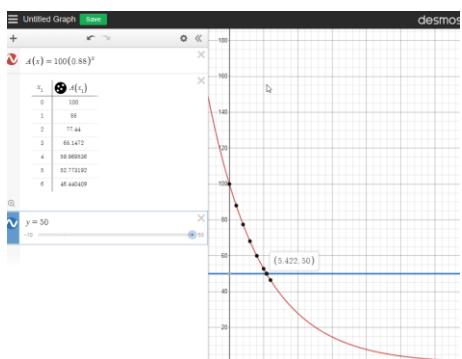
Press MENU Button, then Intersection



This should display answer



Option 3: Desmos Option Plot $y = 50$ and click on point of intersection (in person you will use TI NSPIRE not Desmos)



3. When does this population fall below 75?

a. This simply asking you to:

i. solve the equation $75=100(0.88)^x$

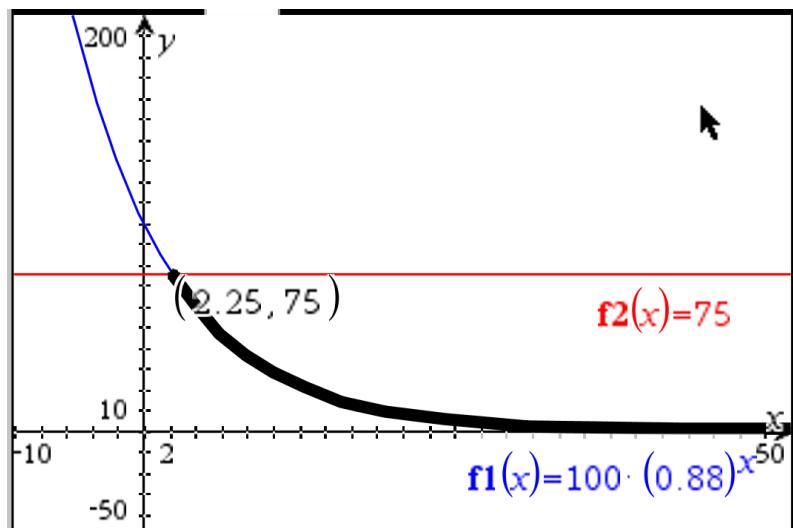
ii. state the INFINITELY many solutions using a set notation or inequality

Repeat same steps as previous problem, just with $y = 75$

Then state the answer using set notation

$75 = 100(0.88)^x$ leads to $0.75 = (0.88)^x$ which leads to $x = \log_{0.88} 0.75 \approx 2.25$

So when $x \geq 2.25$ then $A(x) \leq 75$ or if $x \in [2.25, \infty)$ then $y \in [75, 0)$



4. When does this population exceed 16?

a. This simply asking you to:

i. solve the equation $16=100(0.88)^x$

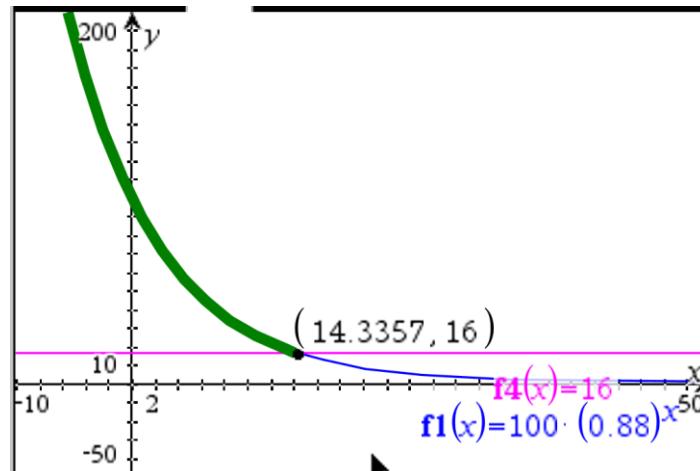
ii. state the INFINITELY many solutions using a set notation or inequality

Repeat same steps as previous problem, just with $y = 16$

Then state the answer using set notation

$16 = 100(0.88)^x$ leads to $0.16 = (0.88)^x$ which leads to $x = \log_{0.88} 0.16 \approx 14.3357$

So when $0 \leq x \leq 14.3357$ then $A(x) \geq 16$ or if $x \in [0, 14.3357)$ then $y \in [16, \infty)$



Practice for you

1. Given model $A(x) = 200(1.25)^x$

What type of model is present? _____ growth or decay

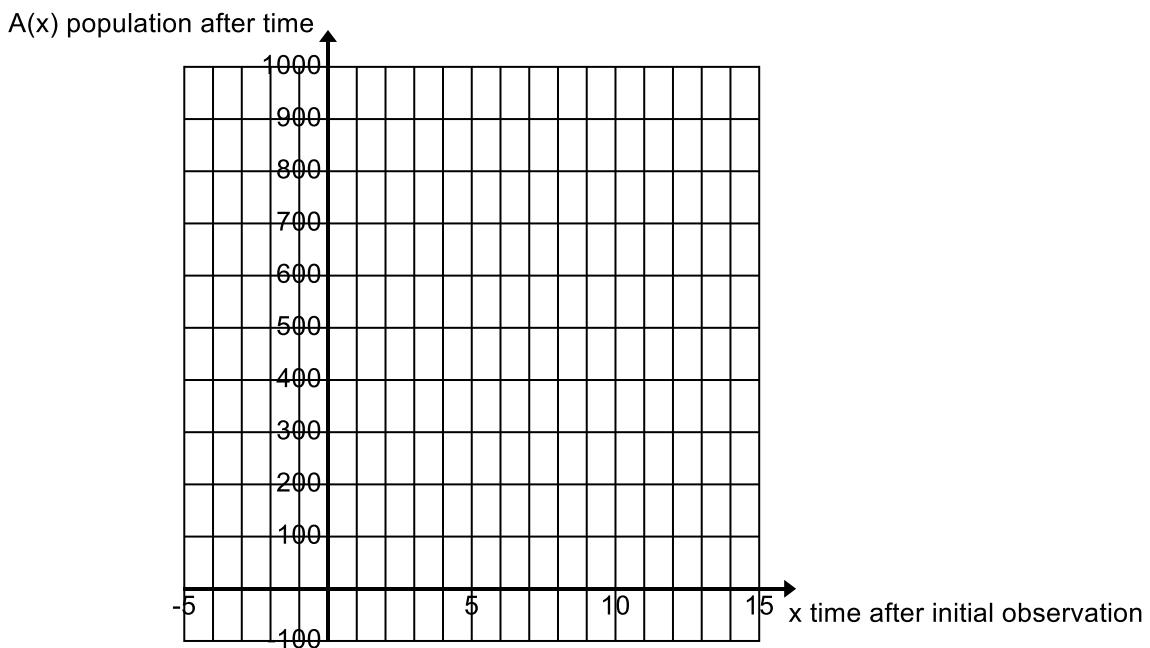
What is the rate of change as a decimal? _____ What is the rate of change as a percentage? _____ %

What was the initial population? _____

Complete the table related to the given model (Remember CTRL t makes a table with TI NSPIRE)

X	0	1	2	3	4	5	6
A(x)							

Sketch a graph of $A(x)$ label at least three points ONE POINT MUST be the y intercept



2. When does this population reach DOUBLE its initial population?

3. When does this population fall below 400?

4. When does this population exceed 525?

Practice for you

5. Given model $A(x) = 50(4)^x$

What type of model is present? _____ growth or decay

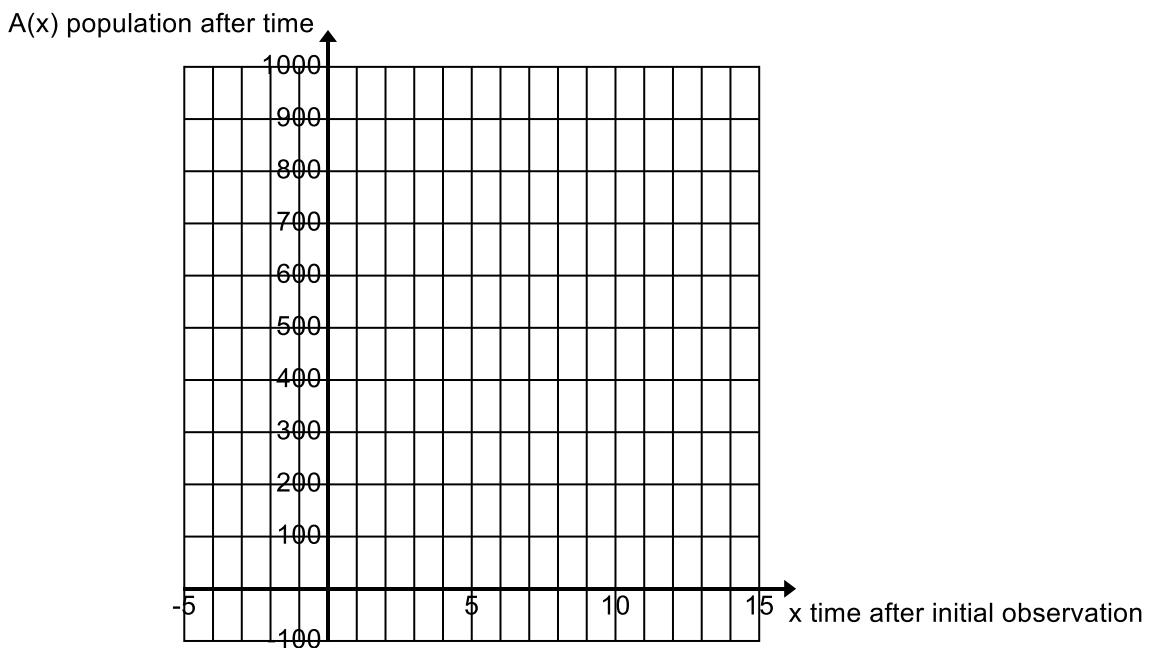
What is the rate of change as a decimal? _____ What is the rate of change as a percentage? _____ %

What was the initial population? _____

Complete the table related to the given model (Remember CTRL t makes a table with TI NSPIRE)

X	0	1	2	3	4	5	6
A(x)							

Sketch a graph of $A(x)$ label at least three points ONE POINT MUST be the y intercept



6. When does this population reach TRIPLE its initial population?

7. When does this population fall below 150?

8. When does this population exceed 25?

Practice for you

9. Given model $A(x) = 600(0.45)^x$

What type of model is present? _____ growth or decay

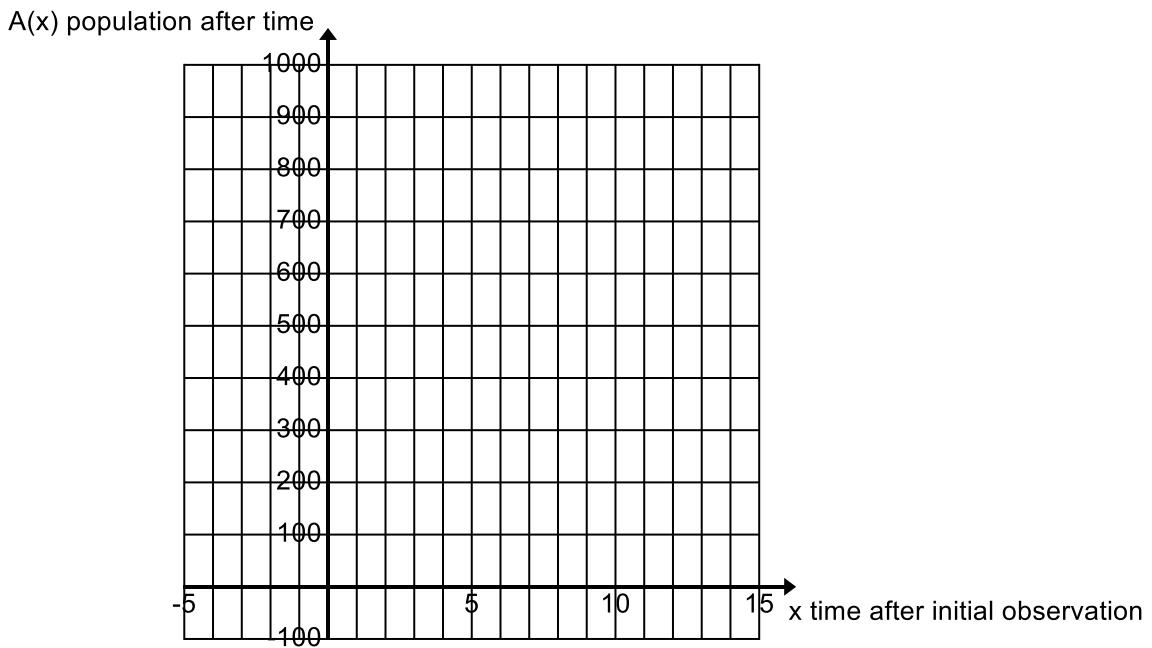
What is the rate of change as a decimal? _____ What is the rate of change as a percentage? _____ %

What was the initial population? _____

Complete the table related to the given model (Remember CTRL t makes a table with TI NSPIRE)

X	0	1	2	3	4	5	6
A(x)							

Sketch a graph of $A(x)$ label at least three points ONE POINT MUST be the y intercept



10. When does this population reach $\frac{1}{4}$ of its initial population?

11. When does this population fall below 200?

12. When does this population exceed 375?