Kuta Software - Infinite Calculus

Name____________________________________

Separable Differential Equations

Date__________________ Period____

Find the general solution of each differential equation.

1) $\frac{dy}{dx} = e^{x-y}$

2) $\frac{dy}{dx} = \frac{1}{\sec^2 y}$

3) $\frac{dy}{dx} = xe^{y}$

4) $\frac{dy}{dx} = \frac{2x}{e^{2y}}$

5) $\frac{dy}{dx} = 2y - 1$

6) $\frac{dy}{dx} = 2yx + yx^2$

① $\frac{dy}{dx} = e^{x-y} = \frac{e^x}{e^y}$

$\frac{dy}{\cancel{dx}} = \frac{e^x}{e^y} \rightarrow eydy = e^x dx$

$\int e^y dy = \int e^x dx$

$\boxed{e^y = e^x + C}$

$\ln(e^y) = \ln(e^x + C)$

$\boxed{y = \ln(e^x + C)}$

(2) $\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$\sec^2 y \, dy = 1 \, dx$

$\int \sec^2 y \, dy = \int 1 \, dx$

$\boxed{\tan y = x + C}$

$\boxed{y = \tan^{-1}(x+C)}$

(3) $\dfrac{dy}{dx} = \dfrac{x e^{y}}{1}$

$$\frac{dy}{e^{y}} = \frac{x\,dx}{1}$$

$$\frac{1}{e^{y}}\,dy = x\,dx$$

$$e^{-y}\,dy = x\,dx$$

$$\int e^{-y}\,dy = \int x\,dx$$

$u = -y$

$du = -dy$ $\qquad = \frac{1}{2}x^{2} + C$

$$\int e^{-y}\,\frac{dy}{-du}$$

u

$$\int e^{u} - du$$

$$-\int e^{u}\,du$$

$$\boxed{-e^{-y} = \tfrac{1}{2}x^{2} + C}$$

$C \neq C_1$

$$e^{-y} = -\tfrac{1}{2}x^{2} + C_1$$

$$\ln e^{-y} = \ln\left(-\tfrac{1}{2}x^{2} + C_1\right)$$

$$-y = \ln\left(-\tfrac{1}{2}x^{2} + C_1\right) \longrightarrow \boxed{y = -\ln\left(-\tfrac{1}{2}x^{2} + C_1\right)}$$

(4) $\frac{dy}{dx} = \frac{2x}{e^{2y}}$

$e^{2y}\,dy = 2x\,dx$

$\int e^{2y}\,dy = \int 2x\,dx$

$u = 2y$

$du = 2\,dy$

$\int e^{\overset{u}{2y}}\,\underbrace{dy}_{\frac{1}{2}du} = \frac{2}{2}x^2 + C$

$\int e^u \frac{1}{2}\,du$

$\frac{1}{2}\int e^u\,du$

$\frac{1}{2}e^u =$

$\boxed{\frac{1}{2}e^{2y} = x^2 + C}$

$\boxed{C_1 \neq C}$

$e^{2y} = 2x^2 + C_1$

$\ln e^{2y} = \ln(2x^2 + C_1)$

$2y = \ln(2x^2 + C_1) \rightarrow \quad y = \frac{1}{2}\ln(2x^2 + C_1)$

⑤ $\frac{dy}{dx} = 2y-1$

$dy = (2y-1)dx$

$\frac{1}{2y-1}\,dy = 1\,dx$

$\int \frac{1}{2y-1}\,dy = \int 1\,dx$

$u = 2y-1$

$du = 2\,dy$ $\qquad = x + C$

$\int \frac{1}{2y-1}\,\frac{dy}{\frac{1}{2}dy}$

u

$\int \frac{1}{u}\,\frac{1}{2}\,du$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u|$

$\boxed{\frac{1}{2}\ln|2y-1| = x + C}$

$\ln|2y-1| = 2x + C_1$

$e^{\ln|2y-1|} = e^{2x+C_1}$

$2y-1 = e^{2x+C_1}$

$2y-1 = e^{C_1}e^{2x}$

$2y-1 = e^{C_1}e^{2x}$

$+1 \qquad +1$

$2y = e^{C_1}e^{2x}+1$

$\frac{2y}{2} = \frac{e^{C_1}e^{2x}+1}{2}$

$\boxed{y = \frac{C_2e^{2x}+1}{2}}$

$C \neq C_1 \neq C_2$

(6) $\frac{dy}{dx} = 2yx + yx^2$

$$\frac{dy}{dx} = y(2x + x^2)$$

$$dy = y(2x+x^2)\,dx$$

$$\frac{dy}{y} = (2x+x^2)\,dx$$

$$\int \frac{1}{y}\,dy = \int (2x+x^2)\,dx$$

$$\ln|y| = \frac{2}{2}x^2 + \frac{1}{3}x^3 + c$$

$$\boxed{\ln|y| = 1x^2 + \frac{1}{3}x^3 + C}$$

$$e^{\ln|y|} = e^{1x^2 + \frac{1}{3}x^3 + C}$$

$$y = e^{1x^2 + \frac{1}{3}x^3 + c}$$

$$y = e^{1x^2 + \frac{1}{3}x^3}\, e^{c}$$

$$\boxed{y = C_1 e^{1x^2 + \frac{1}{3}x^3}}$$

$c \neq C_1$