Practice Problems

Try some of the problems below. If you get stuck, don't worry! There are hints on the next page! But do try without looking at them first, chances are you won't get hints on your exam.

1. $\int_{-1}^{1} 3x^2\sqrt{x^3+5}\,dx$
2. $\int x^3(2+x^4)^5\,dx$
3. $\int_0^7 \sqrt{4+3x}\,dx$
4. $\int \frac{1}{(1-6t)^4}\,dt$
5. $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$
6. $\int \frac{\sec(1/x)}{x^2}\,dx$
7. $\int_{1/6}^{1/2} \csc(\pi t)\cot(\pi t)\,dt$
8. $\int x^2(x^3+5)^9\,dx$
9. $\int_0^1 xe^{-x^2}\,dx$
10. $\int (3t+2)^{2.4}\,dt$
11. $\int_0^{\pi/2} \cos(x)\sin(\sin(x))\,dx$
12. $\int \frac{x}{(x^2+1)^2}\,dx$
13. $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$
14. $\int e^x\sin(e^x)\,dx$
15. $\int_{-1}^{0} \frac{8x}{(4x^2+1)^2}\,dx$
16. $\int \frac{x}{x^2+1}\,dx$
17. $\int_0^1 -12x^2(4x^3-1)^3\,dx$
18. $\int \sec(2\theta)\tan(2\theta)\,d\theta$
19. $\int_{-1}^{2} 6x(x^2-1)^2\,dx$
20. $\int \sqrt{x}\sin(1+x^{3/2})\,dx$
21. $\int_0^1 \frac{24x}{(4x^2+4)^2}\,dx$
22. $\int (1+\tan(\theta))^5\sec^2(\theta)\,d\theta$
23. $\int_{-3}^{0} -\frac{8x}{(2x^2+3)^2}\,dx$
24. $\int e^{\cos(t)}\sin(t)\,dt$
25. $\int_0^1 \frac{16x}{(4x^2+4)^2}\,dx$
26. $\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$
27. $\int_{-1}^{0} 18x^2(3x^3+3)^2\,dx$
28. $\int \frac{\sin(\ln(x))}{x}\,dx$
29. $\int_0^1 -\frac{8x}{(4x^2+2)^2}\,dx$
30. $\int \frac{e^x}{e^x+1}\,dx$
31. $\int \frac{\cos(\pi/x)}{x^2}\,dx$
32. $\int \frac{\sin(x)}{1+\cos^2(x)}\,dx$
33. $\int \frac{1}{\cos^2(t)\sqrt{1+\tan(t)}}\,dt$

Challenge Problems

Below are some harder problems that require a little more thinking/algebraic manipulation to make the substitutions work.

1. $\int_0^1 \frac{x}{\sqrt{x+1}}\,dx$
2. $\int \frac{1}{2x^2-12x+26}\,dx$
3. $\int \frac{x}{1+x^4}\,dx$
4. $\int (x+3)\sqrt{x-1}\,dx$
5. $\int \frac{x^2}{\sqrt{1-x}}\,dx$
6. $\int x^3\sqrt{x^2+1}\,dx$
7. $\int \frac{1}{\sqrt{21-4x-x^2}}\,dx$
8. $\int_{-\pi/2}^{\pi/2} \frac{x^2\sin(x)}{1+x^6}\,dx$
9. $\int \frac{3x-1}{x^2+10x+28}\,dx$
10. $\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$
11. $\int_{-1}^{1} \frac{\sin(x)}{1+x^2}\,dx.$
12. $\int \frac{1}{e^x+1}\,dx$

① $\int 3x^2\sqrt{x^3+5}\,dx$ $\qquad u = x^3$

$\int 3x^2(x^3+5)^{1/2}\,dx$ $\qquad du = 3x^2\,dx$

$\int (x^3+5)^{1/2}\,\underline{3x^2\,dx}$

$$\int u^{\frac{1}{2}}\,du = \frac{1}{\frac{3}{2}}u^{\frac{3}{2}} + C = \frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \boxed{\frac{2}{3}(x^3+5)^{\frac{3}{2}} + C}$$

$$\int_{-1}^{1} 3x^2\sqrt{x^3+5}\,dx = F(+1) - F(1)$$

$\underline{F(x) = \frac{2}{3}(x^3+5)^{\frac{3}{2}}}$

antiderivative

$$= \frac{2}{3}(6)^{\frac{3}{2}} - \frac{2}{3}(4)^{\frac{3}{2}}$$

$$= \frac{2}{3}\,6^1\cdot 6^{\frac{1}{2}} - \frac{2}{3}(2)^3$$

$$= 4\sqrt{6} - \frac{16}{3}$$

(2) $\int x^3(2+x^4)^5\,dx$ $\quad u = 2+x^4$

$du = 4x^3\,du$

$\int (2+x^4)^5 \underbrace{x^3dx}_{\frac{1}{4}du}$

$\int u^5 \frac{1}{4}du$

$\frac{1}{4}\int u^5\,du = \frac{1}{4}\frac{1}{6}(2+x^4)^6+C$

$= \boxed{\frac{1}{24}(2+x^4)^6+C}$

(3) $\int_0^7 \sqrt{4+3x}\,dx$ $\quad u = 4+3x$

$du = 3dx$

$\int \underbrace{(4+3x)}_{u}{}^{1/2} \underbrace{dx}_{\frac{1}{3}du} = \int u^{1/2}\frac{1}{3}du$

$\frac{1}{3}\int u^{\frac{1}{2}}du = \frac{1}{3}\frac{1}{\frac{3}{2}}u^{\frac{3}{2}}+C = \frac{1}{3}\cdot\frac{2}{3}u^{\frac{3}{2}}+C$

$= \boxed{\frac{2}{9}(4+3x)^{\frac{3}{2}}+C}$

$F(x)=\frac{2}{9}(4+3x)^{\frac{3}{2}} \quad = \int_0^7 f(x)\,dx$

$F(7)-F(0)$

$F(7)=\frac{2}{9}(25)^{\frac{3}{2}}=\frac{250}{9}$

$F(0)=\frac{2}{9}(4)^{\frac{3}{2}} = \frac{16}{9}$

$\frac{234}{9}$ $\pm$(?

④ $\int \frac{1}{(1-6t)^4}\,dt =$ $\qquad u = 1-6t$
$du = -6\,dt$

$$\int \frac{1}{\underbrace{(1-6t)^4}} \underbrace{dt}_{-\frac{1}{6}du} = \int \frac{1}{u^4}\,\frac{-1}{6}\,dt$$

$$= \frac{-1}{6}\int u^{-4}\,dt = \frac{-1}{6}\left(\frac{1}{-3}\right)u^{-3} + C$$

$$= \boxed{\frac{1}{18}\left(\frac{1}{(1-6t)^3}\right) + C}$$

*

⑤ $\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx$ $\qquad u = x^2$
$du = 2x\,dx$

$$\int \underbrace{\cos(x^2)}_{u}\ \underbrace{x\,dx}_{\frac{1}{2}du} = \int \cos u\ \frac{1}{2}du$$

$$= \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C = \boxed{\frac{1}{2}\sin(x^2)+C}$$

$$F(x) = \frac{1}{2}\sin(x^2)' = \int x\cos x^2\,dx$$

$$\int_0^{\sqrt{\pi}} f(x)\,dx = F(\sqrt{\pi}) - F(0)$$

$$= \frac{1}{2}\sin\pi - \frac{1}{2}\sin 0 = \frac{1}{2}(0) - \frac{1}{2}(0)$$

$$= 0$$

*⑥ $\int \frac{\sec(1/x)}{x^2}\,dx$ $\quad u = \frac{1}{x} = x^{-1}$

$du = -\frac{1}{x^2}\,dx$

$$\int \underbrace{\sec\left(\tfrac{1}{x}\right)}_{u}\ \underbrace{\tfrac{1}{x^2}\,dx}_{du} = \int \sec u\,du$$

$$= \ln|\sec u + \tan u| + C$$

$$= \boxed{\ln\left|\sec\left(\tfrac{1}{x}\right) + \tan\left(\tfrac{1}{x}\right)\right| + C}$$

*⑦ $\int_{1/6}^{1/2} \csc(\pi t)\cot(\pi t)\,dt$ $\quad u = \pi t \quad du = \pi\,dt$

$$\int \csc(\underbrace{\pi t}_{u})\cot(\underbrace{\pi t}_{u})\underbrace{dt}_{\frac{1}{\pi}du} = \int \csc u \cot u \,\tfrac{1}{\pi}\,du$$

$$= \frac{1}{\pi}\int \csc u \cot u\,dt = \frac{1}{\pi}(-\csc u) + C$$

$$= -\frac{1}{\pi}\csc u + C = \boxed{-\frac{1}{\pi}\csc(\pi t) + C}$$

$$F(t) = -\frac{1}{\pi}\csc(\pi t) \to \int_{1/6}^{1/2} f(t)\,dt \quad F(t)\Big|_{1/6}^{1/2}$$

$$= F\left(\tfrac{1}{2}\right) - F\left(\tfrac{1}{6}\right) = -\frac{1}{\pi}\csc\left(\frac{\pi}{2}\right) - -\frac{1}{\pi}\csc\left(\frac{\pi}{6}\right)$$

$$= -\frac{1}{\pi}(1) + \frac{1}{\pi}(2) = -\frac{1}{\pi} + \frac{2}{\pi} = \frac{1}{\pi}$$

(8) $\int x^2(x^3+5)^9\,dx$ $\quad u = x^3+5$

$du = 3x^2\,dx$

$$\int \underbrace{(x^3+5)^9}_{u}\ \underbrace{x^2\,dx}_{\frac{1}{3}du} = \int u^9 \tfrac{1}{3}\,du$$

$$= \frac{1}{3}\int u^9\,du = \frac{1}{3}\cdot\frac{1}{10}u^{10}+C$$

$$= \boxed{\frac{1}{30}(x^3+5)^9+C}$$

(9) $\int_0^1 x e^{-x^2}\,dx$

$u = -x^2 \quad du = -2x\,dx$

$\int x e^{-x^2}\,dx$

$$\int e^{\overset{u}{-x^2}}\ \underbrace{x\,dx}_{-\frac{1}{2}du} = \int e^u\, -\tfrac{1}{2}\,du = -\frac{1}{2}\int e^u\,du$$

$$= -\frac{1}{2}e^u + C = \boxed{-\frac{1}{2}e^{-x^2}+C}$$

$F(x) = \int x e^{-x^2}\,dx = -\frac{1}{2}e^{-x^2}$

$\int_0^1 f(x)\,dx = F(1) - F(0) = -\frac{1}{2}e^{-(1)^2} - -\frac{1}{2}e^{-(0)^2}$

$$= -\frac{1}{2}e^{-1} + \frac{1}{2}e^0 = \boxed{-\frac{1}{2}e^{-1}+\frac{1}{2}} = \frac{-1}{2e}+\frac{e}{2e}$$

$$= \frac{e-1}{2e}$$

(10) $\int (3t+2)^{2.4}\,dt$ $\quad u = 3t+2$

$du = 3\,dt$

$$\int \underbrace{(3t+2)}_{u}^{2.4} \underbrace{dt}_{\frac{1}{3}du} = \int u^{2.4}\tfrac{1}{3}\,du$$

$$= \frac{1}{3}\int u^{2.4}\,du = \frac{1}{3}\,\frac{1}{3.4}\,u^{3.4} + C$$

$$= \frac{1}{10.2}\,u^{3.4} + C = \frac{10}{102}(3t+2)^{3.4} + C$$

$$= \boxed{\frac{5}{51}(3t+2)^{3.4} + C}$$

$$= \frac{5}{51}(3t+2)^{\frac{34}{10}} + C$$

$$= \frac{5}{51}(3t+2)^{\frac{17}{5}} + C$$

$$= \frac{5}{51}(3t+2)^{3}(3t+2)^{\frac{2}{5}} + C$$

$$= \frac{5}{51}(3t+2)^{3}\sqrt[5]{(3t+2)^{2}} + C$$

all equivalen

(11) $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\, dx$

$u = \sin x$

$du = \cos x\, dx$

$$\int \sin(\underbrace{\sin x}_{u}) \underbrace{\cos x\, dx}_{du}$$

$$\int \sin u\, du = -\cos u + C$$

$$= \boxed{-\cos(\sin x) + C}$$

$F(x) = \int \cos(x) \sin(\sin x)\, dx = -\cos(\sin x)$

$$\int_0^{\pi/2} f(x)\, dx = F\left(\tfrac{\pi}{2}\right) - F(0)$$

$$= -\cos\left(\sin \tfrac{\pi}{2}\right) + \cos(\sin(0))$$

$$= -\cos(1) + \cos(0)$$

$$= -\cos(1) + 1$$

$$= \boxed{1 - \cos(1)}$$

(12) $\int \frac{x}{(x^2+1)^2}\,dx$ $\qquad u = x^2+1$

$\qquad du = 2x\,dx$

$$\int \underbrace{\frac{1}{(x^2+1)^2}}_{u} \underbrace{x\,dx}_{\frac{1}{2}du} = \int \frac{1}{u^2}\,\frac{1}{2}\,du$$

$$\frac{1}{2}\int \frac{1}{u^2}\,du = \frac{1}{2}\int u^{-2}\,du$$

$$= \frac{1}{2}(-1)\,u^{-1}+C = \boxed{-\frac{1}{2}(x^2+1)^{-1}+C}$$

$$= \boxed{\frac{-1}{2(x^2+1)}+C}$$

(13) $\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$ $\qquad u = \sin^{-1}(x)$

$\qquad du = \frac{1}{\sqrt{1-x^2}}\,dx$

$$\int \underbrace{\sin^{-1}(x)}_{u}\,\underbrace{\frac{1}{\sqrt{1-x^2}}\,dx}_{du} = \int u\,du$$

$$= \frac{1}{2}u^2+C = \boxed{\frac{1}{2}\left[\sin^{-1}(x)\right]^2+C}$$

(14) $\int e^x \sin(e^x)\,dx$ $\quad u = e^x$

$du = e^x dx$

$$\int \sin(\underset{u}{e^x})\,\underbrace{e^x dx}_{du}$$

$$\int \sin u \, du = -\cos u + C$$

$$= \boxed{-\cos(e^x) + C}$$

(15) $\int_{-1}^{0} \frac{8x}{(4x^2+1)^2}\,dx$ $\quad u = 4x^2+1$

$du = 8x\,dx$

$$\int \frac{1}{\underset{u}{(4x^2+1)^2}} \underbrace{8x\,dx}_{du} = \int \frac{1}{u^2}\,du$$

$$\int u^{-2}\,du = -1\,u^{-1} + C = -1(4x^2+1)^{-1} + C$$

$F(x) = \frac{-1}{(4x^2+1)}$ $\quad \int_{-1}^{0} f(x)\,dx =$

$$F(0) - F(1) = \frac{-1}{1} - \frac{-1}{5}$$

$$= -1 + \frac{1}{5} = \frac{-4}{5}$$

(14) $\int e^x \sin(e^x)\,dx$

$u = e^x \quad du = e^x\,dx$

$\int \sin(\underbrace{e^x}_{u}) \underbrace{e^x dx}_{du} = \int \sin u\,du$

$-\cos u + C = \boxed{-\cos(e^x) + C}$

(15) $\int_{-1}^{0} -12x^2(4x^3-1)^3\,dx \qquad u = 4x^3-1$

$du = 12x^2\,dx$

$-\int \underbrace{(4x^3-1)^3}_{u} \underbrace{12x^2 dx}_{du} = -\int u^3\,du$

$= -\frac{1}{4}u^4 + C = \boxed{-\frac{1}{4}(4x^3-1)^4 + C}$

$F(x) = \int -12x^2(4x^3-1)^3\,dx = -\frac{1}{4}(4x^3-1)^4$

$\int_{-1}^{0} f(x)\,dx = F(0) - F(-1)$

$= -\frac{1}{4}(4(0)^3-1)^4 + \frac{1}{4}(4(-1)^3-1)^4$

$= -\frac{1}{4}(-1)^4 + \frac{1}{4}(-5)^4 = \frac{1}{4} + \frac{625}{4} = \frac{626}{4}$

$= 313$

(16) $\int \frac{x}{x^2+1}\,dx$ $\quad u = x^2+1$

$du = 2x\,dx$

$\int \frac{1}{\underbrace{x^2+1}_{u}} \underbrace{x\,dx}_{\frac{1}{2}du} = \int \frac{1}{u}\,\frac{1}{2}\,du$

$= \frac{1}{2}\int \frac{1}{u}\,du = \boxed{\frac{1}{2}\ln|x^2+1| + C}$

(17) $\int_0^1 -12x^2(4x^3-1)^3\,dx$ $\quad u = 4x^3$

$du = 12x^2\,du$

$-\int (4x^3-1)^3 \underbrace{12x^2\,dx}_{du}$

$-\int u^3\,du = -\frac{1}{4}u^4 + C = \boxed{-\frac{1}{4}(4x^3-1)^4 + C}$

$F(x) = \int f(x)\,dx = \frac{-1}{4}(4x^3-1)^4$

$\int_0^1 f(x)\,dx = F(1) - F(0) = \frac{-1}{4}(3)^4 - \frac{-1}{4}(-1)^4$

$= \frac{-81}{4} + \frac{1}{4} = \frac{-80}{4} = -20$

(18) $\int \sec(2\theta)\tan(2\theta)\,d\theta$

$u = 2\theta \quad du = 2\,d\theta$

$\int \sec\underbrace{(2\theta)}_{u} \tan\underbrace{(2\theta)}_{u} \underbrace{d\theta}_{\frac{1}{2}du}$

$\int \sec u \tan u \ \frac{1}{2} du$

$\frac{1}{2}\int \sec u \tan u \, du$

$\frac{1}{2}\sec u + C = \boxed{\frac{1}{2}\sec(2\theta) + C}$

(19) $\int_{-1}^{2} 6x(x^2-1)^2\,dx \qquad u = x^2-1$
$du = 2x\,dx$

$\int \underbrace{(x^2-1)^2}_{u} \underbrace{6x\,dx}_{3\,du} = \int u^2 \, 3\,du$

$= 3\int u^2\,du = 3\cdot\frac{1}{3}u^3 + C = u^3 + C$

$= \boxed{(x^2-1)^2 + C} \qquad F(x) = \int 6x(x^2-1)^2 dx$
$= (x^2-1)^3$

$\int_{-1}^{2} f(x)\,dx = F(2) - F(-1) = (2^2-1)^3 - ((-1)^2-1)^3$
$= 3^3 - 0^3$
$= 27$

(20) $\int \sqrt{x}\, \sin(1+x^{3/2})\, dx$

$\int x^{\frac{1}{2}} \sin(1+x^{3/2})\, dx$ $\quad u = 1+x^{3/2}$

$du = \frac{3}{2} x^{\frac{1}{2}} dx$

$* \int \sin(\underbrace{1+x^{3/2}}_{u})\, \underline{x^{\frac{1}{2}} dx}$

When dealing with needing a factor multiplier

$\frac{3}{2}\frac{2}{3}\int \sin(1+x^{\frac{3}{2}})\, x^{\frac{1}{2}} dx$

$\frac{1}{\frac{3}{2}} \int \sin(1+x^{\frac{3}{2}})\, \frac{3}{2} x^{\frac{1}{2}} dx$

$\frac{2}{3} \int \sin u\, du = \boxed{-\frac{2}{3} \cos(1+x^{\frac{3}{2}}) + C}$

OR $\int \sin(\underbrace{1+x^{3/2}}_{u})\, \frac{x^{\frac{1}{2}} dx}{\frac{2}{3} du}$

$\int \sin u\, \frac{2}{3} du = \frac{2}{3} \int \sin u\, du$

$= \boxed{-\frac{2}{3} \sin(1+x^{\frac{3}{2}}) + C}$

(21) $\displaystyle\int_0^1 \frac{24x}{(4x^2+4)^2}\,dx$ $\quad u = 4x^2+4$

$du = 8x\,dx$

$$\int \frac{1}{\underbrace{(4x^2+4)^2}_{u}} \; \frac{24x\,dx}{3\,du} = \int \frac{1}{u^2}\,3\,du$$

$$3\int \frac{1}{u^2}\,du = 3\int u^{-2}\,du = 3(-1)u^{-1}+C$$

$$= \boxed{-3(4x^2+4)^{-1}+C = \frac{-3}{4x^2+4}+C}$$

So $F(x) = \int f(x)\,dx = \dfrac{-3}{4x^2+4}$

$$\int_0^1 f(x)\,dx = F(1)-F(0)$$

$$= \frac{-3}{4(1)^2+4} - \frac{-3}{4}$$

$$= \frac{-3}{8} + \frac{3}{4} = \frac{-3}{8} + \frac{6}{8}$$

$$= \frac{3}{8}$$

(22) $\int (1+\tan\theta)^5 \sec^2\theta \, d\theta$

$u = 1+\tan\theta \quad du = \sec^2\theta \, d\theta$

$\int \underbrace{(1+\tan\theta)^5}_{u} \underbrace{\sec^2\theta \, d\theta}_{du}$

$\int u^5 \, du = \frac{1}{6} u^6 + C$

$= \boxed{\frac{1}{6}(1+\tan\theta)^6 + C}$

(23) $\int_{-3}^{0} \frac{-8x}{(2x^2+3)^2} dx \qquad u = 2x^2+3$
$du = 4x\,dx$

$\int \frac{1}{\underbrace{(2x^2+3)^2}_{u}} \underbrace{-8x\,dx}_{-2du} = \int \frac{1}{u^2} -2du$

$= -2\int u^{-2} du = -2(-1)u^{-1} + C$

$= 2(2x^2+3)^{-1} + C = \frac{2}{(2x^2+3)} + C$

$F(x) = \frac{2}{2x^2+3} \qquad \int_{-3}^{0} f(x)\,dx = F(0) - F(-3)$

$= \frac{2}{3} - \frac{2}{21} = \frac{14}{21} - \frac{2}{21}$

$= \frac{12}{21} = \frac{4}{7}$

(24) $\int e^{\cos t}\sin t\,dt$ $\quad u=\cos t$

$du=-\sin t\,dt$

$$\int e^{\cos t}\,\frac{\sin t\,dt}{-du} = \int e^{u}\,-1\,du$$

u

$$= -\int e^{u}\,du = -e^{u}+C = -e^{\cos t}+C$$

(25) $\int_0^1 \frac{16x}{(4x^2+4)^2}\,dx$ $\quad u=4x^2+4$

$du=8x\,dx$

$$\int \frac{1}{(4x^2+4)^2}\,\frac{16x\,dx}{2\,du} = \int \frac{1}{u^2}\,2\,du$$

u

$$= 2\int u^{-2}\,du = 2(-1)\,u^{-1}+C$$

$$= -2\,(4x^2+4)^{-1}+C = \frac{-2}{4x^2+4}+C$$

$F(x)=\frac{-2}{4x^2+4}$ $\qquad \int_0^1 f(x)\,dx = F(1)-F(0)$

$$= \frac{-2}{8} - \frac{-2}{4}$$

$$= \frac{-1}{4}+\frac{2}{4} = \frac{1}{4}$$

(26) $\int \frac{\tan^{-1}(x)}{1+x^2}\,dx$ $\quad u = \tan^{-1}(x)$

$du = \frac{1}{1+x^2}\,dx$

$\int \underbrace{\tan^{-1}(x)}_{u} \underbrace{\frac{1}{1+x^2}\,dx}_{du}$

$\int u\,du = \frac{1}{2}u^2 + C = \boxed{\frac{1}{2}\left[\tan^{-1}(x)\right]^2 + C}$

(27) $\int_{-1}^{0} 18x^2(3x^3+3)^2\,dx$ $\quad u = 3x^3+3$

$du = 9x^2\,dx$

$\int \underbrace{(3x^3+3)^2}_{u} \underbrace{18x^2\,dx}_{2du} = \int u^2\, 2\,du$

$= 2\int u^2\,du = 2\left(\frac{1}{3}\right)(u)^3 + C = \boxed{\frac{2}{3}(3x^3+3)^3 + C}$

$F(x) = \int f(x)\,dx = \frac{2}{3}(3x^3+3)^3$

$\int_{-1}^{0} f(x)\,dx = F(0) - F(1) = \frac{2}{3}(3(0)^3+3)^3 - \frac{2}{3}(3(-1)^3+3)^3$

$= \frac{2}{3}(3)^3 - \frac{2}{3}(-3+3)^3 = \frac{2}{3}(27) - \frac{2}{3}(0)^3$

$= \frac{54}{3} - 0 = 18$

30) $\int \frac{e^x}{e^x+1}\,dx$ $\qquad u = e^x+1$

$\qquad\qquad\qquad\qquad du = e^x\,dx$

$$\int \underbrace{\frac{1}{e^x+1}}_{u}\ \underbrace{e^x\,dx}_{du} = \int \frac{1}{u}\,du$$

$$= \ln|u| + C = \boxed{\ln|e^x+1| + C}$$

31) $\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\,dx$ $\qquad u = \frac{\pi}{x} \qquad du = \frac{-\pi}{x^2}\,dx$

$\qquad\qquad\qquad\qquad = \pi x^{-1} \qquad = \pi(-1)x^{-2}\,dx$

$$\int \underbrace{\cos\left(\frac{\pi}{x}\right)}_{u}\ \underbrace{\frac{1}{x^2}\,dx}_{-\frac{1}{\pi}dx} = \int \cos u \ \frac{-1}{\pi}\,du$$

$$= \frac{-1}{\pi}\int \cos u\,du = \frac{-1}{\pi}\sin u + C$$

$$= \boxed{-\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right) + C}$$

(32) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$$\int \frac{1}{\underbrace{1^2}_{a} + (\underbrace{\cos x}_{u})^2} \cdot \frac{\sin x\,dx}{-1\,du}$$

$$-1\int \frac{1}{a^2+u^2}\,du = -1\,\frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C$$

$$= \frac{-1}{1}\tan^{-1}\left(\frac{\cos x}{1}\right)+C$$

$$= \boxed{-\tan^{-1}(\cos x)+C}$$

33 $\displaystyle\int \frac{1}{\cos^2(t)\sqrt{1+\tan t}}\,dt$

$$\int \frac{1}{\cos^2(t)} \cdot \frac{1}{\sqrt{1+\tan t}}\,dt$$

$$= \int \frac{1}{(1+\tan t)^{\frac{1}{2}}} \sec^2 t\,dt$$

$$= \int (1+\tan t)^{-1/2} \sec^2(t)\,dt$$

$u = 1+\tan t \quad du = \sec^2 t\,dt$

$$\int u^{-1/2}\,du = \frac{1}{\frac{1}{2}} u^{\frac{1}{2}} + C$$

$$= \boxed{2(1+\tan t)^{1/2} + c}$$

$$= \boxed{2\sqrt{1+\tan t} + c}$$