Problem 1

matrix A = $\begin{bmatrix} a\_11 & a\_12 & a\_13 \\ a\_21 & a\_22 & a\_23 \\ a\_31 & a\_32 & a\_33 \end{bmatrix} = \begin{bmatrix} 2 & 5 & -8 \\ 5 & 0 & 7 \\ -3 & 5 & -1 \end{bmatrix}$

matrix E = $\begin{bmatrix} e\_11 & e\_12 & e\_13 \end{bmatrix} = \begin{bmatrix} 6 & -7 & -2 \end{bmatrix}$

matrix B = $\begin{bmatrix} b\_11 & b\_12 & b\_13 \\ b\_21 & b\_22 & b\_23 \end{bmatrix} = \begin{bmatrix} 9 & 2 & -4 \\ -6 & 8 & 0 \end{bmatrix}$

matrix F = $\begin{bmatrix} f\_11 & f\_12 & f\_13 \\ f\_21 & f\_22 & f\_23 \\ f\_31 & f\_32 & f\_33 \end{bmatrix} = \begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix}$

matrix C = $\begin{bmatrix} c\_11 \\ c\_21 \\ c\_31 \end{bmatrix} = \begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix}$

matrix D = $\begin{bmatrix} d\_11 & d\_12 & d\_13 \\ d\_21 & d\_22 & d\_23 \end{bmatrix} = \begin{bmatrix} 6 & -10 & -7 \\ -6 & 8 & -10 \end{bmatrix}$

Problem 1

−4B

$$-4\begin{bmatrix} 9 & 2 & ^-4 \\ ^-6 & 8 & 0 \end{bmatrix} = \begin{bmatrix} ^-36 & ^-8 & 16 \\ 24 & ^-32 & 0 \end{bmatrix}$$

−4 is called the scalar

This is called scalar multiplication

Problem 2

$B-2D = B + -2D$

$$\begin{bmatrix} 9 & 2 & {}^{-}4 \\ {}^{-}6 & 8 & 0 \end{bmatrix} + -2\begin{bmatrix} 6 & {}^{-}10 & {}^{-}7 \\ {}^{-}6 & 8 & {}^{-}10 \end{bmatrix}$$

We TRY to AVOID subtraction to prevent sign errors

$$\begin{bmatrix} 9 & 2 & {}^{-}4 \\ {}^{-}6 & 8 & 0 \end{bmatrix} + \begin{bmatrix} {}^{-}12 & 20 & 14 \\ 12 & {}^{-}16 & 20 \end{bmatrix}$$ Perform scalar multiplication FIRST

$$\begin{bmatrix} {}^{-}3 & 22 & 10 \\ 6 & {}^{-}8 & 20 \end{bmatrix}$$ Then add results

-2 is called the scalar

This has scalar multiplication and matrix addition

Problem 3

Matrix Multiplication

THIS IS ONLY POSSIBLE IF INNER DIMENSIONS ARE EQUAL

Matrix B $\begin{bmatrix} 9 & 2 & ^-4 \\ ^-6 & 8 & 0 \end{bmatrix}$ has dimension 2 x 3

Matrix A $\begin{bmatrix} 2 & 5 & ^-8 \\ 5 & 0 & 7 \\ ^-3 & 5 & ^-1 \end{bmatrix}$ has dimension 3 x 3

CHECK INNER DIMENSION

(2x3) (3x3) the inner dimensions are a match!

IF matrix multiplication is possible,

then the resultant matrix has the OUTER DIMENSION as its size

(2x3) (3x3) the inner dimensions are a match!

So matrix product BA has dimension 2 x 3 $\begin{bmatrix} \mathbf{ba\_11} & ba\_12 & ba\_13 \\ ba\_21 & ba\_22 & ba\_23 \end{bmatrix}$

Problem 3

Matrix Product BA

matrix A = $\begin{bmatrix} a\_11 & a\_12 & a\_13 \\ a\_21 & a\_22 & a\_23 \\ a\_31 & a\_32 & a\_33 \end{bmatrix} = \begin{bmatrix} 2 & 5 & ^-8 \\ 5 & 0 & 7 \\ ^-3 & 5 & ^-1 \end{bmatrix}$ matrix B = $\begin{bmatrix} b\_11 & b\_12 & b\_13 \\ b\_21 & b\_22 & b\_23 \end{bmatrix} = \begin{bmatrix} 9 & 2 & ^-4 \\ ^-6 & 8 & 0 \end{bmatrix}$

(remember the address tells us the way to the entries)

ba_11 needs row 1 of matrix B to be multiplied by column 1 of matrix A

ba_12 needs row 1 of matrix B to be multiplied by column 2 of matrix A

ba_13 needs row 1 of matrix B to be multiplied by column 3 of matrix A

ba_21 needs row 2 of matrix B to be multiplied by column 1 of matrix A

ba_22 needs row 2 of matrix B to be multiplied by column 2 of matrix A

ba_23 needs row 2 of matrix B to be multiplied by column 3 of matrix A

Matrix Product BA

ROW 1 WORK

(remember the address tells us the way to the entries)

ba_11 needs row 1 of matrix B to be multiplied by column 1 of matrix A

$$\begin{bmatrix} 9 & 2 & {}^-4 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ {}^-3 \end{bmatrix} = (9)(2)+(2)(5)+({}^-4)({}^-3) = 18+10+12 = \begin{bmatrix} 40 \end{bmatrix}$$ This is the entry at ba_11

ba_12 needs row 1 of matrix B to be multiplied by column 2 of matrix A

$$\begin{bmatrix} 9 & 2 & {}^-4 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix} = (9)(5)+(2)(0)+({}^-4)(5) = 45+0+{}^-20 = \begin{bmatrix} 25 \end{bmatrix}$$ This is the entry at ba_12

ba_13 needs row 1 of matrix B to be multiplied by column 3 of matrix A

$$\begin{bmatrix} 9 & 2 & {}^-4 \end{bmatrix} \begin{bmatrix} {}^-8 \\ 7 \\ {}^-1 \end{bmatrix} = (9)({}^-8)+(2)(7)+({}^-4)({}^-1) = {}^-72+14+4 = \begin{bmatrix} {}^-54 \end{bmatrix}$$ This is the entry at ba_13

Matrix Product BA

ROW 2 WORK

(remember the address tells us the way to the entries)

ba_21 needs row 2 of matrix B to be multiplied by column 1 of matrix A

$$\begin{bmatrix} ^-6 & 8 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ ^-3 \end{bmatrix} = (^-6)(2)+(8)(5)+(0)(^-3) = {}^-12+40+0 = \begin{bmatrix} 28 \end{bmatrix}$$ This is the entry at ba_21

ba_22 needs row 2 of matrix B to be multiplied by column 2 of matrix A

$$\begin{bmatrix} ^-6 & 8 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix} = (^-6)(5)+(8)(0)+(0)(5) = {}^-30+0+0 = \begin{bmatrix} ^-30 \end{bmatrix}$$ This is the entry at ba_22

ba_23 needs row 2 of matrix B to be multiplied by column 3 of matrix A

$$\begin{bmatrix} ^-6 & 8 & 0 \end{bmatrix} \begin{bmatrix} ^-8 \\ 7 \\ ^-1 \end{bmatrix} = (^-6)(^-8)+(8)(7)+(0)(^-1) = 48+56+0 = \begin{bmatrix} 104 \end{bmatrix}$$ This is the entry at ba_23

Problem 3

NOW TECHNOLOGY certainly can be used to accomplish the multiplication of two matrices, but we do expect that you will be able to do both methods (by hand and with the assistance of a graphing calculator)

matrix A = $\begin{bmatrix} \mathbf{a\_11} & \mathbf{a\_12} & \mathbf{a\_13} \\ \mathbf{a\_21} & \mathbf{a\_22} & \mathbf{a\_23} \\ \mathbf{a\_31} & \mathbf{a\_32} & \mathbf{a\_33} \end{bmatrix} = \begin{bmatrix} 2 & 5 & -8 \\ 5 & 0 & 7 \\ -3 & 5 & -1 \end{bmatrix}$

matrix B = $\begin{bmatrix} \mathbf{b\_11} & \mathbf{b\_12} & \mathbf{b\_13} \\ \mathbf{b\_21} & \mathbf{b\_22} & \mathbf{b\_23} \end{bmatrix} = \begin{bmatrix} 9 & 2 & -4 \\ -6 & 8 & 0 \end{bmatrix}$

BA = $\begin{bmatrix} 9 & 2 & -4 \\ -6 & 8 & 0 \end{bmatrix} \begin{bmatrix} 2 & 5 & -8 \\ 5 & 0 & 7 \\ -3 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 40 & 25 & -54 \\ 28 & -30 & 104 \end{bmatrix}$

MATRIX MULTIPLICATION IS NOT COMMUTATIVE!

(unless dealing with VERY special matrices!)

Problem 4

Matrix Multiplication

THIS IS ONLY POSSIBLE IF INNER DIMENSIONS ARE EQUAL

Matrix B $\begin{bmatrix} 9 & 2 & -4 \\ -6 & 8 & 0 \end{bmatrix}$ has dimension 2 x 3

Matrix A $\begin{bmatrix} 2 & 5 & -8 \\ 5 & 0 & 7 \\ -3 & 5 & -1 \end{bmatrix}$ has dimension 3 x 3

CHECK INNER DIMENSION

(3x3) (2x3) the inner dimensions are NOT a match!

SO matrix multiplication is IMPOSSIBLE, *WHY?*

lets try to multiply row 1 of matrix A by column 1 of matrix B

$$\begin{bmatrix} 2 & 5 & -8 \end{bmatrix} \begin{bmatrix} 9 \\ -6 \end{bmatrix} = \begin{bmatrix} \mathbf{a\_11} & \mathbf{a\_12} & \mathbf{a\_13} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{b\_11} \\ \mathbf{b\_21} \end{bmatrix} = \text{Error: Dimension error}$$

each number (entry) in the row of matrix A must correspond to a number (entry) in the column of matrix B. THIS CANNOT happen UNLESS inner dimension is EQUAL!

Problem 5

$$\text{matrix } B = \begin{bmatrix} b\_11 & b\_12 & b\_13 \\ b\_21 & b\_22 & b\_23 \end{bmatrix} = \begin{bmatrix} 9 & 2 & ^-4 \\ ^-6 & 8 & 0 \end{bmatrix}$$

$$\text{matrix } D = \begin{bmatrix} d\_11 & d\_12 & d\_13 \\ d\_21 & d\_22 & d\_23 \end{bmatrix} = \begin{bmatrix} 6 & ^-10 & ^-7 \\ ^-6 & 8 & ^-10 \end{bmatrix}$$

$$\text{matrix } C = \begin{bmatrix} c\_11 \\ c\_21 \\ c\_31 \end{bmatrix} = \begin{bmatrix} 11 \\ ^-7 \\ ^-4 \end{bmatrix}$$

Note this is Matrix Subtraction (Addition of opposite) followed by Matrix Multiplication

(B−D)C = (B +−D)C

Step 1) Change signs of matrix D

Step 2) Add B to −D

Step 3) Check inner dimension of matrix B−D (2x3) and matrix C (3x1)

(2x3) (3x1)→ We can multiply the matrices and the resultant matrix has dimension 2x1

(matrix addition and subtraction does NOT change dimensions)

Step 4 Multiply matrix B −D and matrix C

Problem 5 cont

$$\begin{bmatrix} 9 & 2 & ^-4 \\ ^-6 & 8 & 0 \end{bmatrix} -1 \begin{bmatrix} 6 & ^-10 & ^-7 \\ ^-6 & 8 & ^-10 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 2 & ^-4 \\ ^-6 & 8 & 0 \end{bmatrix} + -1 \begin{bmatrix} 6 & ^-10 & ^-7 \\ ^-6 & 8 & ^-10 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 2 & ^-4 \\ ^-6 & 8 & 0 \end{bmatrix} + \begin{bmatrix} ^-6 & 10 & 7 \\ 6 & ^-8 & 10 \end{bmatrix}$$

$B - D = \begin{bmatrix} 3 & 12 & 3 \\ 0 & 0 & 10 \end{bmatrix}$ Notice the SIZE of the matrix did not change!

$$(B-D)C = \begin{bmatrix} 3 & 12 & 3 \\ 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 11 \\ ^-7 \\ ^-4 \end{bmatrix} = \begin{bmatrix} \square \\ \square \end{bmatrix}$$

this is what the resultant matrix looks like

Problem 5

ROW 1 WORK

(remember the address tells us the way to the entries)

(b−d)c_11 needs row 1 of matrix B−D to be multiplied by column 1 of matrix C

$$\begin{bmatrix} 3 & 12 & 3 \end{bmatrix}\begin{bmatrix} 11 \\ ^{-}7 \\ ^{-}4 \end{bmatrix} = (3)(11)+(12)(^{-}7)+(3)(^{-}4) = 33+^{-}84+^{-}12 = \begin{bmatrix} ^{-}63 \end{bmatrix}$$

This is the entry at (b−d)c_11

ROW 2 WORK

(remember the address tells us the way to the entries)

(b−d)c_21 needs row 2 of matrix B−D to be multiplied by column 1 of matrix C

$$\begin{bmatrix} 0 & 0 & 10 \end{bmatrix}\begin{bmatrix} 11 \\ ^{-}7 \\ ^{-}4 \end{bmatrix} = (0)(11)+(0)(^{-}7)+(10)(^{-}4) = 0+0+^{-}40 = \begin{bmatrix} ^{-}40 \end{bmatrix}$$

This is the entry at (b−d)c_21

OR $\begin{bmatrix} 3 & 12 & 3 \\ 0 & 0 & 10 \end{bmatrix}\begin{bmatrix} 11 \\ ^{-}7 \\ ^{-}4 \end{bmatrix} = \begin{bmatrix} ^{-}63 \\ ^{-}40 \end{bmatrix}$

Problem 5

NOTE: IF given TECHNOLOGY, you can go directly to inputting the matrices all at same time

$$\left(\begin{bmatrix} 9 & 2 & ^-4 \\ ^-6 & 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & ^-10 & ^-7 \\ ^-6 & 8 & ^-10 \end{bmatrix}\right)\begin{bmatrix} 11 \\ ^-7 \\ ^-4 \end{bmatrix} = \begin{bmatrix} ^-63 \\ ^-40 \end{bmatrix}$$

JUST REMEMBER THE ( )'s and it should do it all at once!

Problem 6

matrix A = $\begin{bmatrix} a\_11 & a\_12 & a\_13 \\ a\_21 & a\_22 & a\_23 \\ a\_31 & a\_32 & a\_33 \end{bmatrix} = \begin{bmatrix} 2 & 5 & -8 \\ 5 & 0 & 7 \\ -3 & 5 & -1 \end{bmatrix}$ matrix F = $\begin{bmatrix} f\_11 & f\_12 & f\_13 \\ f\_21 & f\_22 & f\_23 \\ f\_31 & f\_32 & f\_33 \end{bmatrix} = \begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix}$

2A +3F

Step1) perform BOTH scalar products

Step 2) add 2A to 3F

$$2\begin{bmatrix} 2 & 5 & -8 \\ 5 & 0 & 7 \\ -3 & 5 & -1 \end{bmatrix} + 3\begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 4 & 10 & -16 \\ 10 & 0 & 14 \\ -6 & 10 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 24 & 12 \\ 6 & -18 & 0 \\ 0 & 6 & -18 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 34 & -4 \\ 16 & -18 & 14 \\ -6 & 16 & -20 \end{bmatrix}$$

Problem 7 This problem is a fun one

matrix C = $\begin{bmatrix} c\_11 \\ c\_21 \\ c\_31 \end{bmatrix} = \begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix}$ matrix E = $\begin{bmatrix} e\_11 & e\_12 & e\_13 \end{bmatrix} = \begin{bmatrix} 6 & -7 & -2 \end{bmatrix}$

matrix F = $\begin{bmatrix} f\_11 & f\_12 & f\_13 \\ f\_21 & f\_22 & f\_23 \\ f\_31 & f\_32 & f\_33 \end{bmatrix} = \begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix}$

FCE

Step 1 check the inner dimensions

F (3x3) C (3x1) E (1x 3)

FC (3x3) (3x1) the matrix FC will have dimension 3x1

(FC) (3x1) E (1x3)

FCE (3x1) (1x 3) the matrix FCE will have dimension 3x3

Step 2 Find FC

Step 3 Find (FC)(E)

Problem 7 FC

FC = $\begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix}\begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix} = \begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$ This is what FC will look like! and FCE will look like $\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix}$

IF YOU HAVE TECHNOLOGY, you can do this in ONE STEP

FC = $\begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix}\begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix} = \begin{bmatrix} -72 \\ 64 \\ 10 \end{bmatrix}$

Then (FC)(E)

(FC)E= $\begin{bmatrix} -72 \\ 64 \\ 10 \end{bmatrix}\begin{bmatrix} 6 & -7 & -2 \end{bmatrix} = \begin{bmatrix} -432 & 504 & 144 \\ 384 & -448 & -128 \\ 60 & -70 & -20 \end{bmatrix}$ FCE = $\begin{bmatrix} -432 & 504 & 144 \\ 384 & -448 & -128 \\ 60 & -70 & -20 \end{bmatrix}$

OR YOU can do it directly in one step!

FCE = $\begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix}\begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix}\begin{bmatrix} 6 & -7 & -2 \end{bmatrix} = \begin{bmatrix} -432 & 504 & 144 \\ 384 & -448 & -128 \\ 60 & -70 & -20 \end{bmatrix}$

Problem 7 Matrix Product FC WITHOUT TECHNOLOGY

matrix F = $\begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix}$ matrix C = $\begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix}$ FC will look like $\begin{bmatrix} \square \\ \square \\ \square \end{bmatrix}$

COLUMN 1 WORK (remember the address tells us the way to the entries)

fc_11 needs row 1 of matrix F to be multiplied by column 1 of matrix C

$\begin{bmatrix} 0 & 8 & 4 \end{bmatrix} \begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix} = (0)(11)+(8)(-7)+(4)(-4) = 0 + -56 + -16 = \begin{bmatrix} -72 \end{bmatrix}$ This is the entry at fc_11

fc_21 needs row 2 of matrix F to be multiplied by column 1 of matrix C

$\begin{bmatrix} 2 & -6 & 0 \end{bmatrix} \begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix} = (2)(11)+(-6)(-7)+(0)(-4) = 22 + 42 + 0 = \begin{bmatrix} 64 \end{bmatrix}$ This is the entry at fc_21

fc_31 needs row 3 of matrix F to be multiplied by column 1 of matrix C

$\begin{bmatrix} 0 & 2 & -6 \end{bmatrix} \begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix} = (0)(11)+(2)(-7)+(-6)(-4) = 0 + -14 + 24 = \begin{bmatrix} 10 \end{bmatrix}$ This is the entry at fc_31

So we NOW know FC = $\begin{bmatrix} 0 & 8 & 4 \\ 2 & -6 & 0 \\ 0 & 2 & -6 \end{bmatrix} \begin{bmatrix} 11 \\ -7 \\ -4 \end{bmatrix} = \begin{bmatrix} -72 \\ 64 \\ 10 \end{bmatrix}$

Now we need to multiply FC by E

(FC)E= $\begin{bmatrix} -72 \\ 64 \\ 10 \end{bmatrix} \begin{bmatrix} 6 & -7 & -2 \end{bmatrix}$

fce_11 $= -72(6) = -432$ fce_12 $= -72(-7) = 504$ fce_13 $= -72(-2) = 144$

fce_21 $= 64(6) = 384$ fce_22 $= 64(-7) = -448$ fce_23 $= 64(-2) = -128$

fce_31 $= 10(6) = 60$ fce_32 $= 10(-7) = -70$ fce_33 $= 10(-2) = -20$

FCE = $\begin{bmatrix} -72 \\ 64 \\ 10 \end{bmatrix} \begin{bmatrix} 6 & -7 & -2 \end{bmatrix} = \begin{bmatrix} -432 & 504 & 144 \\ 384 & -448 & -128 \\ 60 & -70 & -20 \end{bmatrix}$

	A	B	C	D	E	F	G	H	I	J	K	L	M
=													
1		matrix_a				matrix_c		matrix_e					
2		2	5	-8		11		6	-7	-2			
3		5	0	7		-7				matrix_f			
4		-3	5	-1		-4				0	8	4	
5		matrix_b				matrix_d				2	-6	0	
6		9	2	-4		6	-10	-7		0	2	-6	
7		-6	8	0		-6	8	-10					
8													
9													
10													
11													
12													
13													
14													
15													
16													
17													
18													
19													
20													

A1