

Problem 4

Problem 1

$$\begin{aligned}\frac{2 \cdot x + 4}{5 \cdot x} + \frac{x - 5}{3} &= \frac{2 \cdot x + 4}{5 \cdot x} \cdot \frac{3}{3} + \frac{x - 5}{3} \cdot \frac{5x}{5x} \\ &= \frac{6 \cdot x + 12}{5 \cdot x \cdot 3} + \frac{5 \cdot x^2 - 25 \cdot x}{5 \cdot x \cdot 3} \\ &= \frac{6 \cdot x + 12 + 5 \cdot x^2 - 25 \cdot x}{5 \cdot x \cdot 3} \\ &= \frac{5 \cdot x^2 - 19 \cdot x + 12}{15 \cdot x} \\ &= \frac{(x - 3) \cdot (5 \cdot x - 4)}{15 \cdot x}\end{aligned}$$

14/21 completely simplified correctly

Related work

$$(2 \cdot x + 4) \cdot 3 = 6 \cdot x + 12$$

$$(5x) \cdot (x - 5) = 5 \cdot x^2 - 25 \cdot x$$

$$6 \cdot x + 12 + 5 \cdot x^2 - 25 \cdot x = 5 \cdot x^2 - 19 \cdot x + 12$$

$$5 \cdot x^2 - 19 \cdot x + 12$$

Quick checks of completely factored

$$D = (-19)^2 - 4 \cdot 5 \cdot 12 \rightarrow 121 \quad \sqrt{121} \rightarrow 11$$

This means that $5 \cdot x^2 - 19 \cdot x + 12$ IS factorable

$$5 \cdot x^2 - 19 \cdot x + 12 = (x - 3) \cdot (5 \cdot x - 4)$$

since no single factor of $15 = 3 \cdot 5$ is a common factor of each term and since all terms are not divisible by x , this is completely simplified

Problem 2

$$\frac{3 \cdot x^2 + 4 \cdot x}{2 \cdot x} + \frac{x^2 - 5 \cdot x}{x - 7}$$

This problem has a fraction that can be simplified first

$$\begin{aligned} &= \frac{3 \cdot x + 4}{2} + \frac{x^2 - 5 \cdot x}{x - 7} \\ &= \frac{3 \cdot x + 4}{2} \cdot \frac{x - 7}{x - 7} + \frac{x^2 - 5 \cdot x}{x - 7} \cdot \frac{2}{2} \\ &= \frac{3 \cdot x^2 - 17 \cdot x - 28}{2 \cdot (x - 7)} + \frac{2 \cdot x^2 - 10 \cdot x}{2 \cdot (x - 7)} \\ &= \frac{5 \cdot x^2 - 27 \cdot x - 28}{2 \cdot (x - 7)} \end{aligned}$$

6/21 completely simplified correctly

Related work

$$(3 \cdot x + 4) \cdot (x - 7) = \text{expand}((3 \cdot x + 4) \cdot (x - 7)) \rightarrow 3 \cdot x^2 - 17 \cdot x - 28$$

$$(x^2 - 5 \cdot x) \cdot 2 = \text{expand}((x^2 - 5 \cdot x) \cdot 2) \rightarrow 2 \cdot x^2 - 10 \cdot x$$

$$3 \cdot x^2 - 17 \cdot x - 28 + 2 \cdot x^2 - 10 \cdot x = 5 \cdot x^2 - 27 \cdot x - 28$$

$$5 \cdot x^2 - 27 \cdot x - 28$$

Quick checks of completely factored

$D = (-27)^2 - 4 \cdot 5 \cdot -28 \rightarrow 1289$ this means that $5 \cdot x^2 - 27 \cdot x - 28$ is NOT factorable

$5 \cdot 7^2 - 27 \cdot 7 - 28 \rightarrow 28$ also means that $x - 7$ is NOT a factor of $5 \cdot x^2 - 27 \cdot x - 28$

since all terms are not divisible by any of the factors of 2 this is completely simplified

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Problem 3

$$\begin{aligned}
 \frac{3 \cdot x + 4}{5 \cdot x^3} - \frac{2x - 5}{7} &= \frac{3 \cdot x + 4}{5 \cdot x^3} + \frac{-(2x - 5)}{7} \\
 &= \frac{3 \cdot x + 4}{5 \cdot x^3} + \frac{-2x + 5}{7} \\
 &= \frac{3 \cdot x + 4}{5 \cdot x^3} \cdot \frac{7}{7} + \frac{-2x + 5}{7} \cdot \frac{5x^3}{5x^3} \\
 &= \frac{21 \cdot x + 28}{7 \cdot 5 \cdot x^3} + \frac{25 \cdot x^3 - 10 \cdot x^4}{7 \cdot 5 \cdot x^3} \\
 &= \frac{21 \cdot x + 28 + 25 \cdot x^3 - 10 \cdot x^4}{7 \cdot 5 \cdot x^3} \\
 &= \frac{-10 \cdot x^4 + 25 \cdot x^3 + 21 \cdot x + 28}{35 \cdot x^3}
 \end{aligned}$$

14/21 completely simplified correctly

Related Work

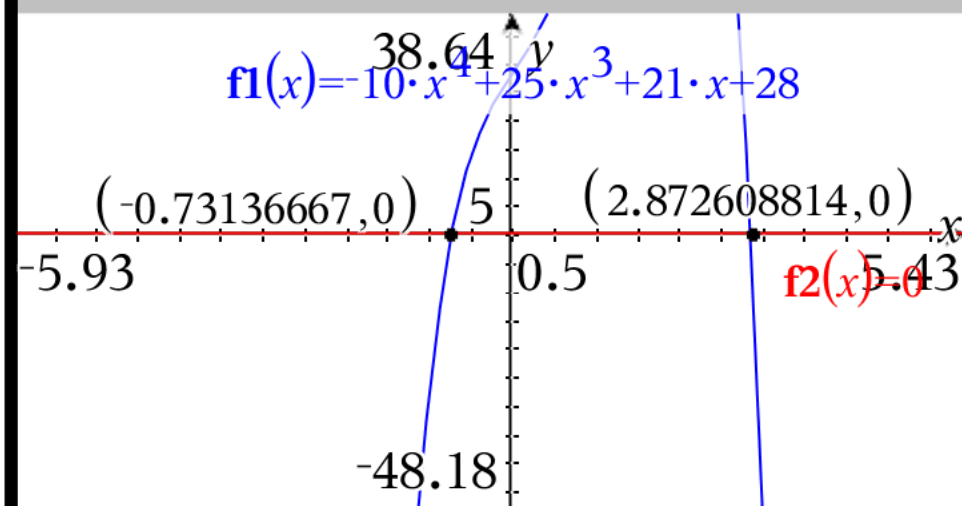
$$(3 \cdot x + 4) \cdot 7 = 21 \cdot x + 28$$

$$(-2x + 5) \cdot (5x^3) = 25 \cdot x^3 - 10 \cdot x^4$$

$$21 \cdot x + 28 + 25 \cdot x^3 - 10 \cdot x^4 = -10 \cdot x^4 + 25 \cdot x^3 + 21 \cdot x + 28$$

since no single factor of $35 = 5 \cdot 7$ is a factor of all terms and since all terms do not contain an x , this is completely simplified

roots of $-10 \cdot x^4 + 25 \cdot x^3 + 21 \cdot x + 28$ are IRRATIONAL or IMAGINARY also means that this is simplified (see below)



Problem 4 $\frac{x^2+4\cdot x}{5\cdot x+2} - \frac{3x^2-5\cdot x}{3x}$

This problem has a fraction that can be simplified first

$$\begin{aligned} & \frac{x^2+5\cdot x}{5\cdot x+2} - \frac{3\cdot x-5}{3} \\ &= \frac{x^2+4\cdot x}{5\cdot x+2} + \frac{-1(3\cdot x-5)}{3} = \frac{x^2+5\cdot x}{5\cdot x+2} + \frac{-3\cdot x+5}{3} \\ &= \frac{x^2+4\cdot x}{5\cdot x+2} \cdot \frac{3}{3} + \frac{-3x+5}{3} \cdot \frac{5x+2}{5x+2} \\ &= \frac{3\cdot x^2+12\cdot x}{3\cdot (5x+2)} + \frac{-15\cdot x^2+19\cdot x+10}{3\cdot (5x+2)} \\ &= \frac{3\cdot x^2+12\cdot x-15\cdot x^2+19\cdot x+10}{3\cdot (5\cdot x+2)} \\ &= \frac{-12\cdot x^2+31\cdot x+10}{3\cdot (5\cdot x+2)} \end{aligned}$$

5/21 completely simplified correctly

Related work

$$(x^2+4\cdot x) \cdot 3 = 3\cdot x^2+12\cdot x$$

$$(-3x+5) \cdot (5x+2) = -15\cdot x^2+19\cdot x+10$$

$$3\cdot x^2+12\cdot x-15\cdot x^2+19\cdot x+10 = -12\cdot x^2+31\cdot x+10$$

$$-12\cdot x^2+31\cdot x+10$$

$$D = 31^2 - 4 \cdot (-12) \cdot 10 = 1441$$

This means that $-12\cdot x^2+31\cdot x+10$

OR

$$-12 \cdot \left(\frac{-2}{5}\right)^2 + 31 \cdot \frac{-2}{5} + 10 = \frac{-108}{25} \text{ this means that } x = \frac{-2}{5} \text{ is}$$

not a root of $-12\cdot x^2+31\cdot x+10$

and since all terms are not divisible by 3, this is completely simplified

Problem 5 $\frac{2 \cdot x + 3}{5 \cdot x - 1} + \frac{x - 1}{x + 2}$

$$= \frac{2 \cdot x + 3}{5 \cdot x - 1} \cdot \frac{x + 2}{x + 2} + \frac{x - 1}{x + 2} \cdot \frac{5x - 1}{5x - 1}$$

$$= \frac{2 \cdot x^2 + 7 \cdot x + 6}{(5x - 1)(x + 2)} + \frac{5 \cdot x^2 - 6 \cdot x + 1}{(5x - 1)(x + 2)}$$

$$= \frac{2 \cdot x^2 + 7 \cdot x + 6 + 5 \cdot x^2 - 6 \cdot x + 1}{(5 \cdot x - 1) \cdot (x + 2)}$$

$$= \frac{7 \cdot x^2 + x + 7}{(5 \cdot x - 1) \cdot (x + 2)}$$

12/21 completely simplified correctly

Related work

$$(2 \cdot x + 3) \cdot (x + 2) = 2 \cdot x^2 + 7 \cdot x + 6$$

$$(x - 1) \cdot (5x - 1) = 5 \cdot x^2 - 6 \cdot x + 1$$

$$2 \cdot x^2 + 7 \cdot x + 6 + 5 \cdot x^2 - 6 \cdot x + 1 = 7 \cdot x^2 + x + 7$$

$$7 \cdot x^2 + x + 7$$

$$D = 1^2 - 4 \cdot 7 \cdot 7 \rightarrow -195$$

This means that $7 \cdot x^2 + x + 7$ is not factorable and has imaginary roots

Another check $x = -2$

$7 \cdot (-2)^2 + (-2) + 7 \rightarrow 33 \cdots x = -2$ is not a root of $7 \cdot x^2 + x + 7$
so $x + 2$ is not a factor of $7 \cdot x^2 + x + 7$

Another check $x = 1/5$

$$7 \cdot \left(\frac{1}{5}\right)^2 + \frac{1}{5} + 7 \rightarrow \frac{187}{25} \quad x = \frac{1}{5} \text{ is not a root of } 7 \cdot x^2 + x + 7$$

so $5x - 1$ is not a factor of $7 \cdot x^2 + x + 7$

This is completely simplified

Problem 6 $\frac{x^2+4x}{x+2} + \frac{x^2-2x}{x-7}$

$$= \frac{x^2+4x}{x+2} \cdot \frac{x-7}{x-7} + \frac{x^2-2x}{x-7} \cdot \frac{x+2}{x+2}$$

$$= \frac{x^3-3 \cdot x^2-28 \cdot x}{(x+2) \cdot (x-7)} + \frac{x^3-4 \cdot x}{(x+2) \cdot (x-7)}$$

$$= \frac{x^3-3 \cdot x^2-28 \cdot x + x^3-4 \cdot x}{(x+2)(x-7)}$$

$$= \frac{2 \cdot x^3-3 \cdot x^2-32 \cdot x}{(x+2)(x-7)} = \frac{x \cdot (2 \cdot x^2-3 \cdot x-32)}{(x-7) \cdot (x+2)}$$

10/21 completely simplified correctly

Related work

$$(x^2+4 \cdot x) \cdot (x-7) = x^3-3 \cdot x^2-28 \cdot x$$

$$(x^2-2x) \cdot (x+2) = x^3-4 \cdot x$$

$$x^3-3 \cdot x^2-28 \cdot x + x^3-4 \cdot x = 2 \cdot x^3-3 \cdot x^2-32 \cdot x$$

since none of the polynomials are equal or multiples of each other this is completely simplified

$$2 \cdot x^3-3 \cdot x^2-32 \cdot x = x \cdot (2 \cdot x^2-3 \cdot x-32)$$

$$D = (-3)^2 - 4 \cdot 2 \cdot -32 \rightarrow 265$$

This means that $2x^2-3x-32$ is not factorable and therefore neither $(x+2)$ nor $(x-7)$ are factors of the numerator

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$$\begin{aligned}
 \text{Problem 7 } \frac{3x+4}{5 \cdot x^3+2x} - \frac{2x-5}{x-7} &= \frac{3x+4}{5 \cdot x^3+2x} + \frac{-(2x-5)}{x-7} \\
 &= \frac{3 \cdot x+4}{5 \cdot x^3+2x} + \frac{-2x+5}{x-7} \\
 &= \frac{3 \cdot x+4}{5 \cdot x^3+2x} \cdot \frac{x-7}{x-7} + \frac{-2x+5}{x-7} \cdot \frac{5x^3+2x}{5x^3+2x} \\
 &= \\
 &= \frac{3 \cdot x^2-17 \cdot x-28}{(5 \cdot x^3+2x)(x-7)} + \frac{-10 \cdot x^4+25 \cdot x^3-4 \cdot x^2+10 \cdot x}{(5 \cdot x^3+2x)(x-7)} \\
 &= \frac{3 \cdot x^2-17 \cdot x-28+-10 \cdot x^4+25 \cdot x^3-4 \cdot x^2+10 \cdot x}{(5 \cdot x^3+2x)(x-7)} \\
 &= \frac{-10 \cdot x^4+25 \cdot x^3-x^2-7 \cdot x-28}{(5 \cdot x^3+2x)(x-7)}
 \end{aligned}$$

5/21 completely simplified correctly

Related Work

$$(3 \cdot x+4) \cdot (x-7) = 3 \cdot x^2-17 \cdot x-28$$

$$(-2x+5) \cdot (5x^3+2x) = -10 \cdot x^4+25 \cdot x^3-4 \cdot x^2+10 \cdot x$$

$$3 \cdot x^2-17 \cdot x-28+-10 \cdot x^4+25 \cdot x^3-4 \cdot x^2+10 \cdot x$$

$$= -10 \cdot x^4+25 \cdot x^3-x^2-7 \cdot x-28$$

$$(5 \cdot x^3+2x)(x-7) = 5 \cdot x^4-35 \cdot x^3+2 \cdot x^2-14 \cdot x$$

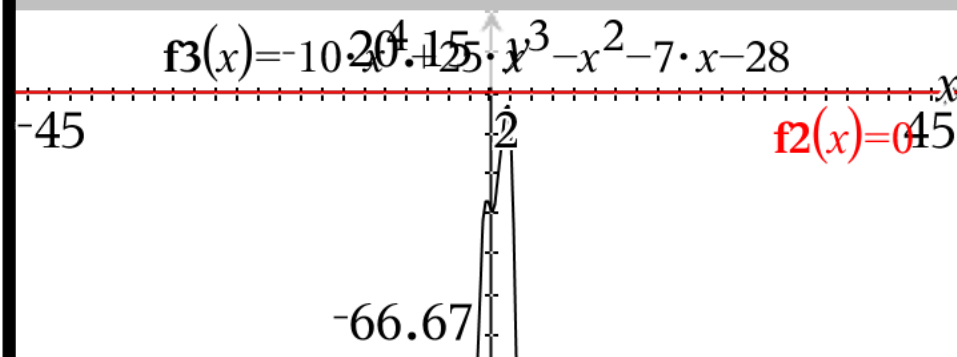
$$(5 \cdot x^3+2x)(x-7) = x \cdot (x-7) \cdot (5 \cdot x^2+2)$$

roots of $-10 \cdot x^4+25 \cdot x^3-x^2-7 \cdot x-28$ are IMAGINARY also means that this is simplified (see below)

$$-10 \cdot 7^4+25 \cdot 7^3-1 \cdot 7^2+-7 \cdot 7-45 \rightarrow -15578$$

also means that $x = 7$ is NOT a root of numerator

and $(x-7)$ is not a factor of $-10 \cdot x^4+25 \cdot x^3-x^2-7 \cdot x-28$



Problem 8 $\frac{x^2-4\cdot x}{5\cdot x+2} - \frac{4x^2-10\cdot x}{2x^2+6x}$

This problem has a fraction that can be simplified first

$$\begin{aligned} & \frac{x^2-4\cdot x}{5\cdot x+2} - \frac{2\cdot x-5}{x+3} \\ &= \frac{x^2-4\cdot x}{5\cdot x+2} + \frac{-1(2\cdot x-5)}{x+3} \\ &= \frac{x^2-4\cdot x}{5\cdot x+2} + \frac{-2\cdot x+5}{x+3} \\ &= \frac{x^2-4\cdot x}{5\cdot x+2} \cdot \frac{x+3}{x+3} + \frac{-2x+5}{x+3} \cdot \frac{5x+2}{5x+2} \\ &= \frac{x^3-x^2-12\cdot x}{(x+3)\cdot (5\cdot x+2)} + \frac{-10\cdot x^2+21\cdot x+10}{(x+3)(5x+2)} \\ &= \frac{x^3-x^2-12\cdot x-10\cdot x^2+21\cdot x+10}{(x+3)\cdot (5\cdot x+2)} = \frac{x^3-11\cdot x^2+9\cdot x+10}{(x+3)(5x+2)} \end{aligned}$$

2/21 completely simplified correctly

Related work

$$(x^2-4\cdot x) \cdot (x+3) = x^3-x^2-12\cdot x$$

$$(-2x+5) \cdot (5x+2) = -10\cdot x^2+21\cdot x+10$$

$$x^3-x^2-12\cdot x-10\cdot x^2+21\cdot x+10 = x^3-11\cdot x^2+9\cdot x+10$$

$$(x+3)(5x+2) = 5\cdot x^2+17\cdot x+6$$

$$x^3-11\cdot x^2+9\cdot x+10 \text{ cannot be factored}$$

OR

$$(-3)^3-11\cdot (-3)^2+9\cdot -3+10 \rightarrow -143$$

$$\text{this means that } x = -3 \text{ is not a root of } x^3-11\cdot x^2+9\cdot x+10$$

$$\text{and } (x+3) \text{ is not a factor of } x^3-11\cdot x^2+9\cdot x+10$$

OR

$$\left(\frac{-5}{2}\right)^3-11\cdot \left(\frac{-5}{2}\right)^2+\frac{9\cdot -5}{2}+10 \rightarrow \frac{-775}{8}$$

$$\text{this means that } x = \frac{-5}{2} \text{ is not a root of } x^3-11\cdot x^2+9\cdot x+10$$

$$\text{and } (5x+2) \text{ is not a factor of } x^3-11\cdot x^2+9\cdot x+10$$

