## Problem 1

$$\frac{2\cdot x+7}{5\cdot x}+\frac{x-9}{7}=\frac{2\cdot x+7}{5\cdot x}\cdot\frac{7}{7}+\frac{x-9}{7}\cdot\frac{5x}{5x}$$

$$=\frac{14\cdot x+49}{5\cdot x\cdot 7}+\frac{5\cdot x^2-45\cdot x}{5\cdot x\cdot 7}$$

$$=\frac{14\cdot x+49+5\cdot x^2-45\cdot x}{5\cdot x\cdot 7}$$

$$=\frac{5\cdot x^2-31\cdot x+49}{35\cdot x}$$

16/18 completely simplified correctly

## Related work

$(2\cdot x+7)\cdot 7=14\cdot x+49$

$(5x)\cdot(x-9)=5\cdot x^2-45\cdot x$

$14\cdot x+49+5\cdot x^2-45\cdot x=5\cdot x^2-31\cdot x+49$

$5\cdot x^2-31\cdot x+49$

Quick checks of completely factored

D$=(^-31)^2-4\cdot 5\cdot 49$

This means that $5\cdot x^2-31\cdot x+49$ is NOT factorable

(this also means then function $f(x)=\frac{5\cdot x^2-31\cdot x+49}{35\cdot x}$

does not have any x intercepts because $D<0$)

since no single factor of 35 $=5\cdot 7$ is a common factor of each term and since all terms are not divisible by x, this is completely simplified

Problem 2 $\frac{3\cdot x^2+7\cdot x}{9\cdot x}+\frac{x^2-2\cdot x}{x-10}$

This problem has a fraction that can be simplified first

$$=\frac{3\cdot x+7}{9}+\frac{x^2-2\cdot x}{x-10}$$

$$=\frac{3\cdot x+7}{9}\cdot\frac{x-10}{x-10}+\frac{x^2-2\cdot x}{x-10}\cdot\frac{9}{9}$$

$$=\frac{3\cdot x^2-23\cdot x-70}{9\cdot (x-10)}+\frac{9\cdot x^2-18\cdot x}{9\cdot (x-10)}$$

$$=\frac{12\cdot x^2-41\cdot x-70}{9\cdot (x-10)}=\frac{(3\cdot x-14)\cdot (4\cdot x+5)}{9\cdot (x-10)}$$

5/18 completely simplified correctly

Related work

$(3\cdot x+7)\cdot (x-10)=3\cdot x^2-23\cdot x-70$

$(x^2-2\cdot x)\cdot 9=9\cdot x^2-18\cdot x$

$3\cdot x^2-23\cdot x-70+9\cdot x^2-18\cdot x = 12\cdot x^2-41\cdot x-70$

$12\cdot x^2-41\cdot x-70$

Quick checks of completely factored

D = $(-41)^2-4\cdot 12\cdot -70 \blacktriangleright 5041$ , $\sqrt{5041} \blacktriangleright 71$ means $12\cdot x^2-41\cdot x-70$ is factorable but $12\cdot x^2-41\cdot x-70=$ $(3\cdot x-14)\cdot (4\cdot x+5)$

I would build factors from roots because in this case,

AC = $840$

x=$\frac{41-\sqrt{5041}}{24} \blacktriangleright \frac{-5}{4}$ x=$\frac{41+\sqrt{5041}}{24} \blacktriangleright \frac{14}{3}$

$12\cdot 10^2-41\cdot 10-70 \blacktriangleright 720$ also means that x−10 is NOT a factor of $12\cdot x^2-41\cdot x-70$

since all terms are not divisible by any of the factors of 9 this is completely simplified

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Problem 3

$$\frac{4\cdot x+3}{2\cdot x^3}-\frac{3x-5}{7}=\frac{4\cdot x+3}{2\cdot x^3}+\frac{-(3x-5)}{7}$$

$$=\frac{4\cdot x+3}{2\cdot x^3}+\frac{-3x+5}{7}$$

$$=\frac{4\cdot x+3}{2\cdot x^3}\cdot\frac{7}{7}+\frac{-3x+5}{7}\cdot\frac{2x^3}{2x^3}$$

$$=\frac{28\cdot x+21}{7\cdot 2\cdot x^3}+\frac{10\cdot x^3-6\cdot x^4}{7\cdot 2\cdot x^3}$$

$$=\frac{28\cdot x+21+10\cdot x^3-6\cdot x^4}{7\cdot 2\cdot x^3}$$

$$=\frac{-6\cdot x^4+10\cdot x^3+28\cdot x+21}{14\cdot x^3}$$

6/18 completely simplified correctly

Related Work

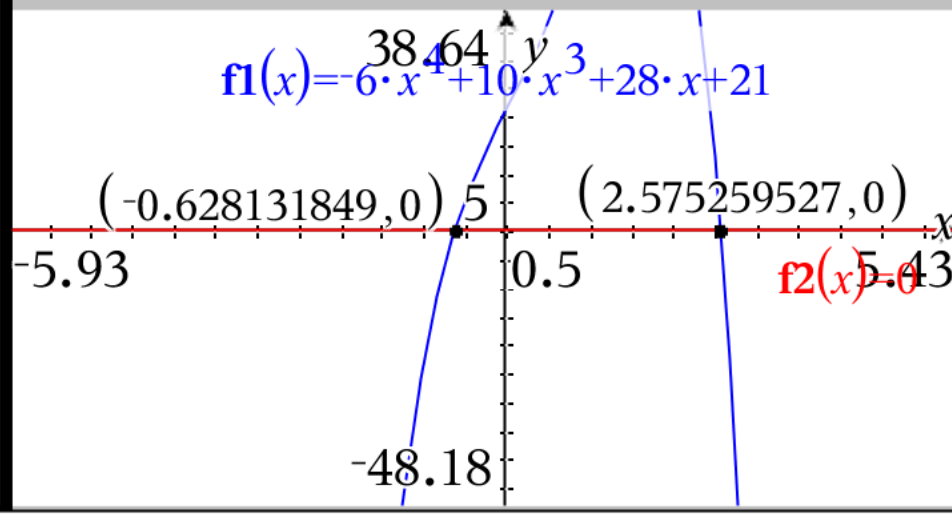
$(4\cdot x+3)\cdot 7=28\cdot x+21$

$(-3x+5)\cdot\left(2x^3\right)=10\cdot x^3-6\cdot x^4$

$28\cdot x+21+10\cdot x^3-6\cdot x^4=-6\cdot x^4+10\cdot x^3+28\cdot x+21$

since no single factor of $21=3\cdot 7$ is a factor of all terms and since all terms do not contain an x, this is completely simplified

roots of $-12\cdot x^4+8\cdot x^3+10\cdot x+25$ are IRRATIONAL or IMAGINARY also means that this is simplified (see below)



Problem 4 $\dfrac{x^2+3\cdot x}{2\cdot x+5}-\dfrac{4x^2-3\cdot x}{3x}$

This problem has a fraction that can be simplified first

$$\frac{x^2+3\cdot x}{2\cdot x+5}-\frac{4\cdot x-3}{3}$$

$$=\frac{x^2+3\cdot x}{2\cdot x+5}+\frac{-1(4\cdot x-3)}{3}=\frac{x^2+3\cdot x}{2\cdot x+5}+\frac{-4\cdot x+3}{3}$$

$$=\frac{x^2+3\cdot x}{2\cdot x+5}\cdot\frac{3}{3}+\frac{-4x+3}{3}\cdot\frac{2x+5}{2x+5}$$

$$=\frac{3\cdot x^2+9\cdot x}{3\cdot(2x+5)}+\frac{-8\cdot x^2-14\cdot x+15}{3\cdot(2x+5)}$$

$$=\frac{3\cdot x^2+9\cdot x+-8\cdot x^2-14\cdot x+15}{3\cdot(2\cdot x+5)}$$

$$=\frac{-5\cdot x^2-5\cdot x+15}{3\cdot(2x+5)}=\frac{-5\cdot(x^2+x-3)}{3\cdot(2\cdot x+5)}$$

2/18 completely simplified correctly

Related work

$(x^2+3\cdot x)\cdot 3=3\cdot x^2+9\cdot x$

$(-4x+3)\cdot(2x+5)=-8\cdot x^2-14\cdot x+15$

$3\cdot x^2+9\cdot x+-8\cdot x^2-14\cdot x+15=-5\cdot x^2-5\cdot x+15$

$-5\cdot x^2-5\cdot x+15=-5\cdot(x^2+x-3)$

D $=1^2-4\cdot 1\cdot -3 \blacktriangleright 13$

This means that $-5\cdot x^2-5\cdot x+15=-5\cdot(x^2+x-3)$ cannot be factored any further

OR

$-5\cdot\left(\frac{-5}{2}\right)^2+-5\cdot\frac{-5}{2}+15 \blacktriangleright \frac{-15}{4}$ this means that x $=\frac{-5}{2}$ is not a root and (2x+5) is not a factor of $-5x^2-5\cdot x+15$

and since all terms are not divisible by 3, this is completely simplified

## Problem 5

$$\frac{9\cdot x+4}{2\cdot x-7}+\frac{x-5}{x+2}$$

$$=\frac{9\cdot x+4}{2\cdot x-7}\cdot\frac{x+2}{x+2}+\frac{x-5}{x+2}\cdot\frac{2x-7}{2x-7}$$

$$=\frac{9\cdot x^2+22\cdot x+8}{(2x-7)(x+2)}+\frac{2\cdot x^2-17\cdot x+35}{(2x-7)(x+2)}$$

$$=\frac{9\cdot x^2+22\cdot x+8+2\cdot x^2-17\cdot x+35}{(2\cdot x-7)\cdot(x+2)}$$

$$=\frac{11\cdot x^2+5\cdot x+43}{(2\cdot x-7)\cdot(x+2)}$$

11/18 completely simplified correctly

Related work

$(9\cdot x+4)\cdot(x+2)=9\cdot x^2+22\cdot x+8$

$(2x-7)\cdot(x-5)=2\cdot x^2-17\cdot x+35$

$9\cdot x^2+22\cdot x+8+2\cdot x^2-17\cdot x+35=11\cdot x^2+5\cdot x+43$

$11\cdot x^2+5\cdot x+43$

$D=5^2-4\cdot 11\cdot 43 \blacktriangleright {}^-1867$

This means that $11\cdot x^2+5\cdot x+43$ is not factorable and has imaginary roots

Another check x = −2

$11\cdot(^-2)^2+5\cdot{}^-2+43 \blacktriangleright 77$ x = −2 is not a root of $11\cdot x^2+5\cdot x+43$ so x+2 is not a factor of $11\cdot x^2+5\cdot x+43$

Another check x = 7/2

$11\cdot\left(\frac{7}{2}\right)^2+\frac{5\cdot 7}{2}+43 \blacktriangleright \frac{781}{4}$ x = $\frac{7}{2}$ is not a root of $11\cdot x^2+5\cdot x+43$ so 2x−7 is not a factor of $11\cdot x^2+5\cdot x+43$

This is completely simplified

Problem 6

$$\frac{x^2+3x}{x+8}+\frac{x^2-5x}{x-2}$$

$$=\frac{x^2+3x}{x+8}\cdot\frac{\mathrm{x}-2}{\mathrm{x}-2}+\frac{x^2-5\mathrm{x}}{\mathrm{x}-2}\cdot\frac{x+8}{x+8}$$

$$=\frac{x^3+x^2-6\cdot x}{(x+8)(x-2)}+\frac{x^3+3\cdot x^2-40\cdot x}{(x+8)(x-2)}$$

$$=\frac{x^3+x^2-6\cdot x+x^3+3\cdot x^2-40\cdot x}{(x+8)(x-2)}$$

$$=\frac{2\cdot x^3+4\cdot x^2-46\cdot x}{(x+8)(x-2)}=\frac{2\cdot x\cdot (x^2+2\cdot x-23)}{(x-2)\cdot (x+8)}$$

10/18 completely simplified correctly

Related work

$(x^2+3\cdot x)\cdot (x-2)=x^3+x^2-6\cdot x$

$(x^2-5x)\cdot (x+8)=x^3+3\cdot x^2-40\cdot x$

$x^3+x^2-6\cdot x+x^3+3\cdot x^2-40\cdot x$

$=2\cdot x^3+4\cdot x^2-46\cdot x$

since none of the polynomials are equal or multiples of each other this is completely simplified

$x^2+2x-23$

$D=2^2-4\cdot 1\cdot {}^{-}23 \;\blacktriangleright\; 96$

This means that $x^2+2x-23$ is not factorable and therefore neither $(x+8)$ nor $(x-2)$ are factors of the numerator

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Problem 7

$$\frac{2x+5}{2\cdot x^3+7x}-\frac{3x-2}{x-9}=\frac{2x+5}{2\cdot x^3+7x}+\frac{-(3x-2)}{x-9}$$

$$=\frac{2\cdot x+5}{2\cdot x^3+7x}+\frac{-3x+2}{x-9}$$

$$=\frac{2\cdot x+5}{2\cdot x^3+7x}\cdot\frac{\text{x}-9}{\text{x}-9}+\frac{-3x+2}{\text{x}-9}\cdot\frac{2x^3+7x}{2x^3+7x}$$

$=$

$$\frac{2\cdot x^2-13\cdot x-45}{\left(2\cdot x^3+7x\right)(x-9)}+\frac{{}^{-}6\cdot x^4+4\cdot x^3-21\cdot x^2+14\cdot x}{\left(2\cdot x^3+7x\right)(x-9)}$$

$$=\frac{2\cdot x^2-13\cdot x-45+{}^{-}6\cdot x^4+4\cdot x^3-21\cdot x^2+14\cdot x}{\left(2\cdot x^3+7\cdot x\right)\cdot(x-9)}$$

$$=\frac{{}^{-}6\cdot x^4+4\cdot x^3-19\cdot x^2+x-45}{\left(2\cdot x^3+7\cdot x\right)\cdot(x-9)}$$

4/18 completely simplified correctly

Related Work

$(2\cdot x+5)\cdot(x-9)=2\cdot x^2-13\cdot x-45$

$(-3x+2)\cdot\left(2x^3+7x\right)={}^{-}6\cdot x^4+4\cdot x^3-21\cdot x^2+14\cdot x$

$2\cdot x^2-13\cdot x-45+{}^{-}6\cdot x^4+4\cdot x^3-21\cdot x^2+14\cdot x$

$={}^{-}6\cdot x^4+4\cdot x^3-19\cdot x^2+x-45$

$\left(2\cdot x^3+7\cdot x\right)\cdot(x-9)=2\cdot x^4-18\cdot x^3+7\cdot x^2-63\cdot x$

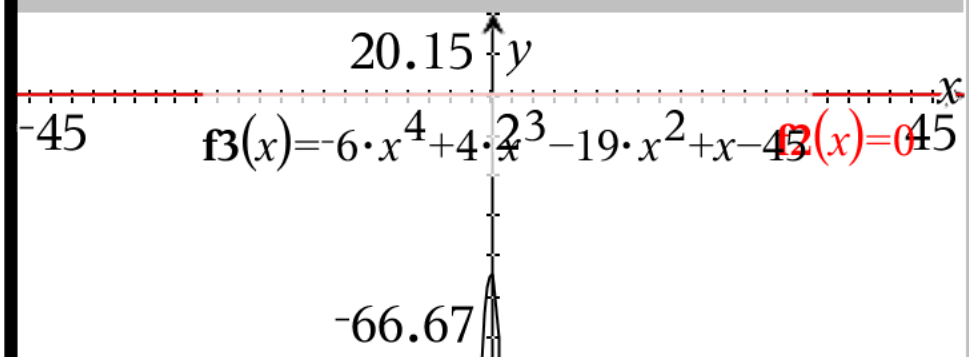
$\left(2\cdot x^3+7\cdot x\right)\cdot(x-9)=x\cdot(x-9)\cdot\left(2\cdot x^2+7\right)$

roots of ${}^{-}6\cdot x^4+4\cdot x^3-19\cdot x^2+x-45$ are IMAGINARY also means that this is simplified (see below)

${}^{-}6\cdot 9^4+4\cdot 9^3-19\cdot 9^2+9-45 \blacktriangleright {}^{-}38025$

also means that x = 9 is NOT a root of numerator

and (x−9) is not a factor of ${}^{-}6\cdot x^4+4\cdot x^3-19\cdot x^2+x-45$



Problem 8 $\frac{x^2-2\cdot x}{3\cdot x+5}-\frac{12x^2-8\cdot x}{4x^2+6x}$

This problem has a fraction that can be simplified first

$$\frac{x^2-2\cdot x}{3\cdot x+5}-\frac{6x-4}{2x+3}$$

$$=\frac{x^2-2\cdot x}{3\cdot x+5}+\frac{-1(6\cdot x-4)}{2x+3}$$

$$=\frac{x^2-2\cdot x}{3\cdot x+5}+\frac{-6\cdot x+4}{2x+3}$$

$$=\frac{x^2-2\cdot x}{3\cdot x+5}\cdot\frac{2x+3}{2x+3}+\frac{-6x+4}{2x+3}\cdot\frac{3x+5}{3x+5}$$

$$=\frac{2\cdot x^3-x^2-6\cdot x}{(2x+3)(3x+5)}+\frac{-18\cdot x^2-18\cdot x+20}{(2x+3)(3x+5)}$$

$$=\frac{x^3-3\cdot x^2-18\cdot x+-6\cdot x^2-x+12}{(2x+3)(3x+5)}=\frac{2\cdot x^3-19\cdot x^2-24\cdot x+20}{(2\cdot x+3)\cdot(3\cdot x+5)}$$

3/18 completely simplified correctly

Related work

$(x^2-2\cdot x)\cdot(2x+3)=2\cdot x^3-x^2-6\cdot x$

$(-6x+4)\cdot(3x+5)=-18\cdot x^2-18\cdot x+20$

$2\cdot x^3-x^2-6\cdot x+-18\cdot x^2-18\cdot x+20=2\cdot x^3-19\cdot x^2-24\cdot x+20$

$(2\cdot x+3)\cdot(3x+5)=6\cdot x^2+19\cdot x+15$

$2\cdot x^3-19\cdot x^2-24\cdot x+20$ cannot be factored

OR

$$2\cdot\left(\frac{-3}{2}\right)^3-19\cdot\left(\frac{-3}{2}\right)^2-\frac{24\cdot -3}{2}+20 \blacktriangleright \frac{13}{2}$$

this means that x $=\frac{-3}{2}$ is not a root of $2\cdot x^3-19\cdot x^2-24\cdot x+20$

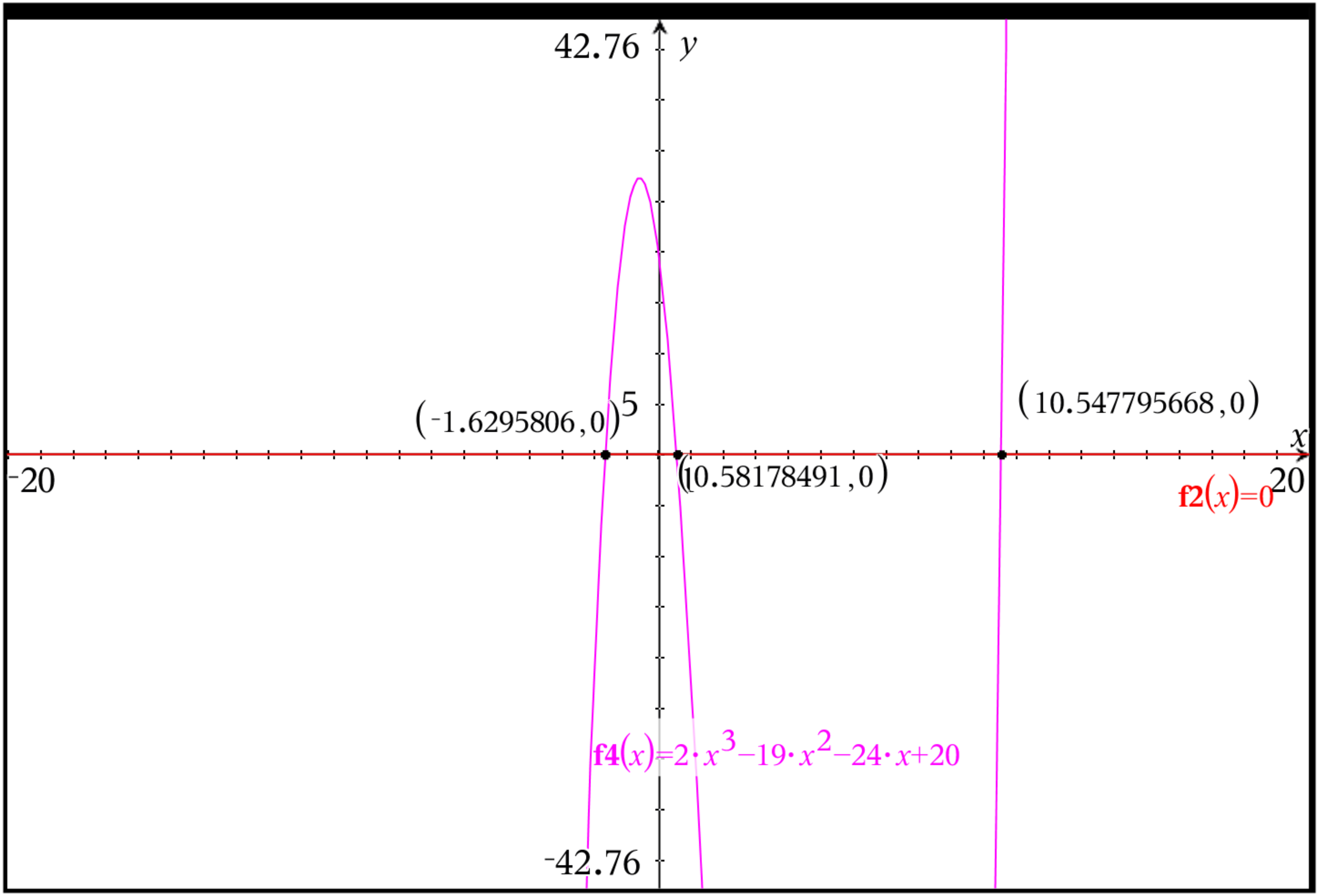
and (2x+3) is not a factor of $2\cdot x^3-19\cdot x^2-24\cdot x+20$

OR

$$2\cdot\left(\frac{-5}{3}\right)^3-19\cdot\left(\frac{-5}{3}\right)^2-\frac{24\cdot -5}{3}+20 \blacktriangleright \frac{-55}{27}$$

this means that x $=\frac{-5}{3}$ is not a root of $2\cdot x^3-19\cdot x^2-24\cdot x+20$

and $(3x+5)$ is not a factor of $2\cdot x^3-19\cdot x^2-24\cdot x+20$


42.76
y
(-1.6295806,0)
5
(10.547795668,0)
x
-20
(0.58178491,0)
20
f2(x)=0
f4(x)=2·x^3−19·x^2−24·x+20
-42.76