Name_____

Adding/Subtracting Rational Expressions

Date_____ Period____

Simplify each expression.

1) $\frac{u-v}{8v}+\frac{6u-3v}{8v}$

2) $\frac{m-3n}{6m^3n}-\frac{m+3n}{6m^3n}$

3) $\frac{5}{a^2+3a+2}+\frac{5a+1}{a^2+3a+2}$

4) $\frac{5}{10n^2+16n+6}+\frac{n-6}{10n^2+16n+6}$

5) $\frac{r+6}{3r-6}+\frac{r+1}{3r-6}$

6) $\frac{x+2}{2x^2+13x+20}-\frac{x+3}{2x^2+13x+20}$

7) $\frac{6}{x-1}-\frac{5x}{4}$

8) $6-\frac{x+5}{(7x-5)(x+4)}$

9) $\frac{3}{x+7}+\frac{4}{x-8}$

10) $\frac{3}{4v^2+4v}-\frac{7}{2}$

11) $\frac{7}{3}-\frac{8}{12x-8}$

12) $\frac{5}{n+5}+\frac{4n}{2n+6}$

13) $\frac{2x}{5x+4}+\frac{6x}{2x+3}$

14) $\frac{2}{3x^2+12x}+\frac{8}{2x}$

15) $\dfrac{7n}{n+1} + \dfrac{8}{n-7}$

16) $\dfrac{2}{n+8} + \dfrac{4}{n+1}$

17) $\dfrac{3}{8} - \dfrac{3}{3x+4}$

18) $\dfrac{3}{b-8} + \dfrac{7}{b+3}$

19) $\dfrac{3}{x+6} + \dfrac{7}{x-2}$

20) $\dfrac{4}{x+1} - \dfrac{2}{x+2}$

21) $\dfrac{5n+5}{5n^2+35n-40} + \dfrac{7n}{3n}$

22) $\dfrac{3}{n-5} + \dfrac{6}{3n-8}$

23) $\dfrac{\frac{25}{4}}{\frac{1}{5} - \frac{4}{25}}$

24) $\dfrac{8}{\frac{4}{9} + \frac{16}{9}}$

25) $\dfrac{\frac{a}{25} - \frac{a}{5}}{a}$

26) $\dfrac{\frac{5}{4}}{\frac{5}{m} - \frac{4}{m}}$

Critical thinking questions:

27) Simplify: $\dfrac{a}{b} + \dfrac{c}{d}$

28) Split into a sum of two rational expressions with unlike denominators:

$$\frac{2x+3}{x^2+3x+2}$$

EVENS WORKED OUT SOLUTIONS

(2) $$\frac{m-3n}{6m^3n} - \frac{m+3n}{6m^3n} = \frac{m-3n}{6m^3n} + \frac{-(m+3n)}{6m^3n}$$

$$= \frac{m-3n-m-3n}{6m^3n} = \frac{-6n}{6m^3n} = \boxed{\frac{6n \cdot -1}{6n \cdot m^3}}$$

$$= \boxed{\frac{-1}{m^3}}$$

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(4) $$\frac{5}{10n^2+16n+6} + \frac{n-6}{10n^2+16n+6}$$

$$\boxed{\frac{n-1}{10n^2+16n+6}} = \frac{n-1}{2(5n^2+8n+3)}$$

$n-1$ not a factor of de-ominator

(6) $\dfrac{x+2}{2x^2+13x+20} - \dfrac{x+3}{2x^2+13x+20}$ LCD ✓

$$\frac{x+2}{2x^2+13x+20} + \frac{-(x+3)}{2x^2+13x+20}$$

$$\frac{x+2}{LCD} + \frac{-x-3}{LCD}$$

$$\frac{x-x+2-3}{LCD}$$

$$\frac{-1}{LCD} = \boxed{\frac{-1}{2x^2+13x+20}}$$

(8) $\frac{6}{1} - \frac{x+5}{(7x-5)(x+4)}$ LCD $(7x-5)(x+4)$

$$\frac{6(7x-5)(x+4)}{1(7x-5)(x+4)} + \frac{-1(x+5)}{LCD}$$

$$\frac{6(7x^2-5x+28x-20)}{LCD} + \frac{-x-5}{LCD}$$

$$\frac{6(7x^2+23x-20) - x-5}{LCD}$$

$$\frac{42x^2+138x-120-x-5}{LCD}$$

$$\boxed{\frac{42x^2+137x-125}{(7x-5)(x+4)}}$$

$$\boxed{\frac{42x^2+137x-125}{7x^2+23x-20}}$$

(10) $\dfrac{3}{4v^2+4v} - \dfrac{7}{2}$ LCD $(2, 4v^2+4v)$

$= 4v^2+4v$

$$\frac{3}{4v(v+1)} + \frac{-7}{2}$$

$$\frac{3}{2\cdot 2v(v+1)} + \frac{-7}{2}$$

$$\frac{3}{4v(v+1)} + \frac{-7(2v)(v+1)}{2(2v)(v+1)}$$

$$\frac{3}{LCD} + \frac{-14v(v+1)}{LCD}$$

$$\frac{3 + -14v^2 - 14v}{LCD}$$

$$\boxed{\frac{-14v^2 - 14v + 3}{4v^2+4v}} \quad \text{or} \quad \boxed{\frac{-14v^2 - 14v + 3}{4v(v+1)}}$$

(12) $\frac{5}{n+5} + \frac{4n}{2n+6}$

$\frac{5}{n+5} + \frac{2 \cdot 2n}{2(n+3)}$

Always Reduce 1st when possible

$\frac{5}{n+5} + \frac{2n}{n+3}$

LCD $(n+5)(n+3)$

$\frac{5(n+3)}{(n+5)(n+3)} + \frac{2n}{n+3}\frac{(n+5)}{(n+5)}$

$\frac{5n+15}{LCD} + \frac{2n^2+10n}{LCD}$

$\frac{2n^2+15n+15}{LCD}$

$\boxed{\frac{2n^2+15n+15}{(n+5)(n+3)}} = \boxed{\frac{2n^2+15n+15}{n^2+8n+15}}$

(12) Method (2)

$$\frac{5}{n+5} + \frac{4n}{2n+6} = \frac{5}{n+5} + \frac{4n}{2(n+3)}$$

$$LCD\ (n+5)(2)(n+3) = 2(n+5)(n+3)$$

$$\frac{5}{(n+5)} \cdot \frac{2(n+3)}{2(n+3)} + \frac{4n}{2(n+3)} \frac{(n+5)}{(n+5)}$$

$$\frac{10(n+3)}{LCD} + \frac{4n(n+5)}{LCD} = \frac{10n+30}{LCD} + \frac{4n^2+20n}{LCD}$$

$$\frac{4n^2+20n+10n+30}{LCD} = \frac{4n^2+30n+30}{2(n+3)(n+5)}$$

$$= \frac{\cancel{2}(2n^2+15n+15)}{\cancel{2}(n+3)(n+5)}$$

$$= \boxed{\frac{2n^2+15n+15}{(n+3)(n+5)}} = \boxed{\frac{2n^2+15n+15}{n^2+8n+15}}$$

(14) $\dfrac{2}{3x^2+12x} + \dfrac{8}{2x}$

Always Simplify 1st when possible

$\dfrac{2}{3x(x+4)} + \dfrac{2 \cdot 4}{2x}$

$\dfrac{2}{3x(x+4)} + \dfrac{4}{x}$

LCD $(3x(x+4), x)$
$= 3x(x+4)$

$\dfrac{2}{3x(x+4)} + \dfrac{4}{x} \dfrac{3(x+4)}{(3)(x+4)}$

$\dfrac{2}{LCD} + \dfrac{12(x+4)}{LCD}$

$\dfrac{2+12x+48}{LCD} = \dfrac{12x+50}{LCD} = \dfrac{2(6x+25)}{LCD}$

$= \dfrac{2(6x+25)}{3x(x+4)} = \boxed{\dfrac{12x+50}{3x^2+12x}}$

(14) Method (2)

$$\frac{2}{3x^2+12x} + \frac{8}{2x} = \frac{2}{3x(x+4)} + \frac{8}{2x}$$

$$\text{LCD}\left(3x(x+4),\ 2x\right) = 3\cdot 2x(x+4)$$
$$= 6x(x+4)$$

$$\frac{2}{2}\,\frac{2}{3x(x+4)} + \frac{8}{2x}\,\frac{3(x+4)}{(3)(x+4)}$$

$$\frac{4}{\text{LCD}} + \frac{24(x+4)}{\text{LCD}} = \frac{4}{\text{LCD}} + \frac{24x+96}{\text{LCD}}$$

$$\frac{24x+100}{\text{LCD}} = \frac{24x+100}{6x(x+4)} = \frac{4(6x+25)}{6x(x+4)}$$

$$= \frac{2\cdot 2(6x+25)}{2\cdot 3x(x+4)} = \boxed{\frac{2(6x+25)}{3x(x+4)}}$$

$$= \boxed{\frac{12x+50}{3x^2+12x}}$$

(16) $\frac{2}{n+8} + \frac{4}{n+1}$

LCD $(n+8)(n+1)$
$=(n+8)(n+1)$

$$\frac{2}{(n+8)}\frac{(n+1)}{(n+1)} + \frac{4}{(n+1)}\frac{(n+8)}{(n+8)}$$

$$\frac{2n+2}{LCD} + \frac{4n+32}{LCD}$$

$$\frac{6n+34}{LCD} = \boxed{\frac{2(3n+17)}{(n+8)(n+1)}}$$

$$= \boxed{\frac{6n+34}{n^2+9n+8}}$$

(18) $\frac{3}{b-8} + \frac{7}{b+3}$

LCD $(b-8)(b+3) = (b-8)(b+3)$

$$\frac{3}{(b-8)}\frac{(b+3)}{(b+3)} + \frac{7}{(b+3)}\frac{(b-8)}{(b-8)} = \frac{3b+9}{LCD} + \frac{7b-56}{LCD}$$

$$= \boxed{\frac{10b-47}{(b+3)(b-8)}} = \boxed{\frac{10b-47}{b^2-5b-15}}$$

(20) $\frac{4}{x+1} - \frac{2}{x+2}$

$\frac{4}{x+1} + \frac{-2}{x+2}$

LCD $(x+1)(x+2)$
$= (x+1)(x+2)$

$\frac{4}{(x+1)} \cdot \frac{(x+2)}{(x+2)} + \frac{-2}{(x+2)} \frac{(x+1)}{(x+1)}$

$\frac{4x+8}{LCD} + \frac{-2x-2}{LCD}$

$$\boxed{\frac{2x+6}{(x+1)(x+2)} = \frac{2(x+3)}{(x+1)(x+2)}}$$

$$\boxed{\frac{2x+6}{x^2+3x+2}}$$

(22) $\frac{3}{n-5} + \frac{6}{3n-8}$

LCD $(n-5)(3n-8)$

$= (n-5)(3n-8)$

$$\frac{3}{(n-5)}\frac{(3n-8)}{(3n-8)} + \frac{6}{(3n-8)}\frac{(n-5)}{(n-5)}$$

$$\frac{9n-24}{LCD} + \frac{6n-30}{LCD} = \frac{15n-54}{LCD}$$

$$\boxed{\begin{aligned} \frac{15n-54}{(n-5)(3n-8)} &= \frac{3(5n-18)}{(n-5)(3n-8)} \\ \frac{15n-54}{3n^2-15n-8n+40} &= \frac{15n-54}{3n^2-23n+40} \end{aligned}}$$

(24) $\dfrac{8}{\frac{4}{9}+\frac{16}{9}}$

Method ①

$$\frac{8}{\frac{20}{9}} = \frac{8}{1} \div \frac{20}{9}$$

$$= \frac{8}{1} \cdot \frac{9}{20} = \frac{72}{20}$$

$$= \frac{4 \cdot 18}{4 \cdot 5} = \boxed{\frac{18}{5}}$$

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(24) method ②

$$\frac{8}{\frac{4}{9}+\frac{16}{9}} \quad \frac{9}{9} = \frac{72}{4+16}$$

$$= \frac{72}{20} = \frac{4}{4} \cdot \frac{18}{5} = \frac{18}{5}$$

(26) $\dfrac{\frac{5}{4}}{\frac{5}{m}-\frac{4}{m}}$

Method (1)

$$\frac{\frac{5}{4}}{\frac{1}{m}} = \frac{5}{4} \div \frac{1}{m} = \frac{5}{4} \cdot \frac{m}{1}$$

$$= \boxed{\frac{5m}{4}}$$

Method (2)

$$\frac{\frac{5}{4}}{\frac{5}{m}-\frac{4}{m}} \quad \frac{4m}{4m} = \frac{\frac{20m}{4}}{\frac{20m}{m}-\frac{16m}{m}}$$

$$= \frac{5m}{20-16}$$

$$= \frac{5m}{4}$$

(28) $\dfrac{2x+3}{x^2+3x+2} = \dfrac{2x+3}{(x+2)(x+1)}$

$$\frac{2x+3}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$$

$$\frac{(2x+3)}{\cancel{(x+2)(x+1)}} \cdot \cancel{(x+2)(x+1)} = (x+2)(x+1)\left[\frac{A}{x+2} + \frac{B}{x+1}\right]$$

$$2x+3 = \frac{A(x+1)\cancel{(x+2)}}{\cancel{(x+2)}} + \frac{B(x+2)\cancel{(x+1)}}{\cancel{(x+1)}}$$

$$2x+3 = A(x+1) + B(x+2)$$

Let $x = -1$

$$2(-1)+3 = A(-1+1) + B(-1+2)$$

$$-2+3 = A(0) + B(1)$$

$$1 = 0 + B \qquad B = 1$$

Let $x = -2$

$$2(-2)+3 \qquad A(-2+1) + B(-2+2)$$

$$-4+3 \qquad A(-1) + B(0)$$

$$-1 = -1A \qquad A = 1$$

(28) so $\boxed{\dfrac{2x+3}{x^2+3x+2} = \dfrac{1}{x+2} + \dfrac{1}{x+1}}$

this process is called partial fraction decomposition

it is <u>useful</u> skill in uplevel mathematics classes