Name_____

Separable Differential Equations

Date_____ Period____

Find the general solution of each differential equation.

1) $\frac{dy}{dx} = e^{x-y}$

2) $\frac{dy}{dx} = \frac{1}{\sec^2 y}$

3) $\frac{dy}{dx} = xe^{y}$

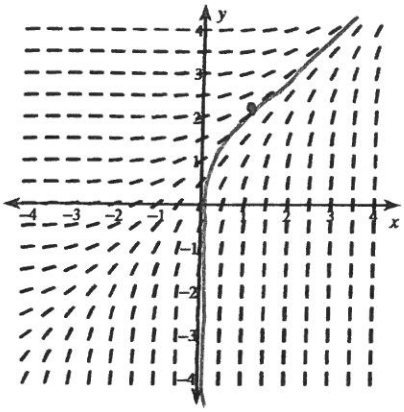
4) $\frac{dy}{dx} = \frac{2x}{e^{2y}}$

5) $\frac{dy}{dx} = 2y - 1$

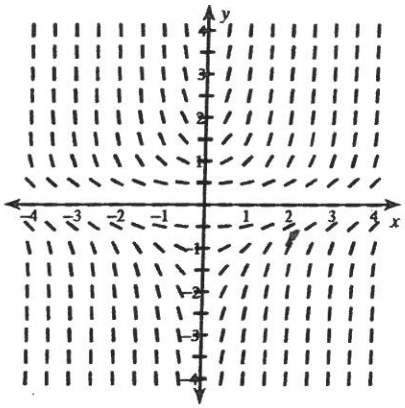
6) $\frac{dy}{dx} = 2yx + yx^2$

For each problem, find the particular solution of the differential equation that satisfies the initial condition. You may use a graphing calculator to sketch the solution on the provided graph.

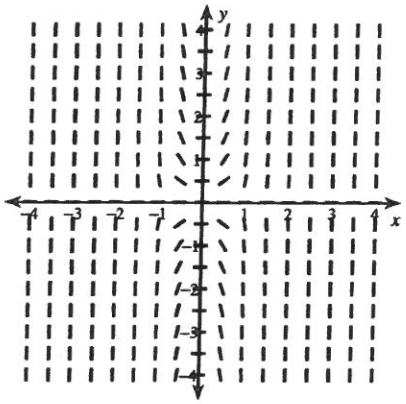
7) $\dfrac{dy}{dx} = 2e^{x-y}$, $y(1) = \ln(2e+1)$



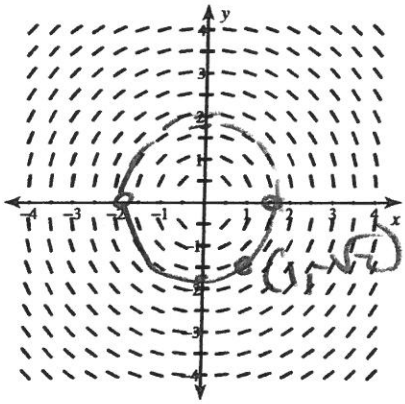
8) $\dfrac{dy}{dx} = xy^2$, $y(2) = -\dfrac{2}{5}$



9) $\dfrac{dy}{dx} = 12x^3y$, $y(0) = 2$



10) $\dfrac{dy}{dx} = -\dfrac{x}{y}$, $y(1) = -\sqrt{2}$



Solutions KUTA Separable Differential Equations

① $\frac{dy}{dx} = e^{x-y} \rightarrow \frac{dy}{dx} = \frac{e^x}{e^y} \rightarrow e^y dy = e^x dx$

$\int e^y dy = \int e^x dx$

$\boxed{e^y = e^x + C}$ ← general solution

② $\frac{dy}{dx} = \frac{1}{\sec^2 y} \rightarrow \sec^2 y\, dy = 1 dx$

$\int \sec^2 y\, dy = \int 1 dx$

$\boxed{\tan y = x + C}$ ← general solution

③ $\frac{dy}{dx} = xe^y \rightarrow \frac{1}{e^y} dy = x dx$

$\rightarrow e^{-y} dy = x dx$

$u = -y$

$du = -1 dx$

$-du = dx$

$\int e^{-y} dy = \int x dx$

$-1 \int e^{-y}(-1 dy) = \frac{1}{2}x^2 + C$

$-1 \int e^u du$

$-1e^u = \frac{1}{2}x^2 + C \rightarrow \boxed{-1e^{-y} = \frac{1}{2}x^2 + C}$

(4) $\frac{1}{2}e^{2y} = x^2 + C$

$1e^{2y} = 2x^2 + 2C$

$\ln e^{2y} = \ln(2x^2 + 2C)$

$2y = \ln(2x^2 + 2C)$

$$\boxed{y = \frac{\ln(2x^2 + 2C)}{2}}$$

Note 2C still unknown constant!

(5) $\frac{1}{2}\ln|2y-1| = x + C$

$\ln|2y-1| = 2x + 2C$

$e^{\ln|2y-1|} = e^{2x+2C}$

$|2y-1| = e^{2x+2C}$

$2y - 1 = e^{2x+2C}$

$2y = 1 + e^{2x+2C}$

$$\boxed{y = \frac{1}{2} + \frac{1}{2}e^{2x+2C} = \frac{1}{2} + \frac{1}{2}e^{2C}e^{2x}}$$

note $e^{2C} = C$ still constant (unknown)

⑥ $\frac{dy}{dx} = 2yx + yx^2$

$\frac{dy}{dx} = y(2x + x^2)$

$\frac{dy}{y} = 2x + x^2 \, dx$

$\frac{1}{y} dy = 2x + x^2 \, dx$

$\int \frac{1}{y} dy = \int 2x + x^2 \, dx$

$\ln|y| = \frac{2x^2}{2} + \frac{x^3}{3} + C$

$\boxed{\ln|y| = x^2 + \frac{x^3}{3} + C}$

This is the end of GENERAL Solutions Problem

Check link to see Slope Fields

Now if you desire to graph these functions by hand you need y by itself

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① $e^y = e^x + C$

$\ln e^y = \ln(e^x + C)$

$\boxed{y = \ln(e^x + C)}$

② $\tan y = x + C$

$\tan^{-1}(\tan y) = \tan^{-1}(x + C)$

$\boxed{y = \tan^{-1}(x + C)}$

(3) $-1e^{-1y} = \frac{1}{2}x^2 + C$

$-1(-1e^{-1y}) = -\left(\frac{1}{2}x^2 + C\right)$

$e^{-1y} = -\frac{1}{2}x^2 + C \rightarrow$ note C always has + (but C can be negative)

$\ln(e^{-1y}) = \ln\left(-\frac{1}{2}x^2 + C\right)$

$-1y = \ln\left(-\frac{1}{2}x^2 + C\right)$

$\boxed{y = -\ln\left(-\frac{1}{2}x^2 + C\right)}$

(4) $\frac{dy}{dx} = \frac{2x}{e^{2y}}$

$e^{2y}\,dy = 2x\,dx$

$\int e^{2y}\,dy = \int 2x\,dx$

$\frac{1}{2}\int e^{2y}\,2\,dy = \frac{2}{2}x^2 + C$

$\frac{1}{2}\int e^{u}\,du = x^2 + C$

$\frac{1}{2}e^{u} = x^2 + C$

$\boxed{\frac{1}{2}e^{2y} = x^2 + C}$

(5) $\frac{dy}{dx} = 2y - 1 \rightarrow \frac{1}{2y-1}\,dy = 1\,dx$

$\int \frac{1}{2y-1}\,dy = \int dx$

$u = 2y - 1$

$du = 2\,dy$

$\frac{1}{2}du = dy$

$\frac{1}{2}\int \frac{1}{2y-1}\,2\,dy = x + C$

$\frac{1}{2}\int \frac{1}{u}\,du = x + C$

$\frac{1}{2}\ln|u| = x + C$

$\boxed{\frac{1}{2}\ln|2y-1| = x + C}$

⑥ $\ln|y| = x^2 + \frac{1}{3}x^3 + C$

$e^{\ln|y|} = e^{x^2 + \frac{1}{3}x^3 + C}$

$y = e^{x^2 + \frac{1}{3}x^3 + C}$

$y = e^C e^{x^2 + \frac{1}{3}x^3}$

$\boxed{y = C_1 e^{x^2 + \frac{1}{3}x^3}}$

again
$e^C = C_1$
a
unknown
constant!

(7) $\frac{dy}{dx} = 2e^{x-y}$

$\frac{dy}{dx} = 2\left(\frac{e^x}{e^y}\right)$

$\frac{dy}{dx} = \frac{2e^x}{e^y}$

$e^y dy = 2e^x dx$

$\int e^y dy = \int 2e^x dx$

$e^y = 2\int e^x dx$

$\boxed{e^y = 2e^x + C}$

$e^{\ln(2e+1)} = 2e^1 + C$

$2e+1 = 2e^1 + C$

$\boxed{C = 1}$

$\boxed{e^y = 2e^x + 1}$

$\ln e^y = \ln(2e^x + C)$

$\boxed{y = \ln(2e^x + C)}$

We know

$y(1) = \ln(2e+1)$

$x \qquad y$

OR

$e^{\ln(2e+1)} = e^{\ln(2e^1 + C)}$

$2e+1 = 2e + C$

$\boxed{C = 1}$

$\boxed{y = \ln(2e^x + 1)}$

Both OK

⑧ $\frac{dy}{dx} = xy^2$

$\frac{dy}{y^2} = x\,dx$

$\frac{1}{y^2}\partial y = x\,dx$

$y^{-2}dy = x\,dx$

$\int y^{-2}dy = \int x\,dx$

$\frac{y^{-1}}{-1} = \frac{x^2}{2} + C$

$\boxed{\frac{-1}{y} = \frac{1}{2}x^2 + C}$

$\frac{-1}{\frac{-2}{5}} = \frac{1}{2}(2)^2 + C$

$\frac{5}{2} = 2 + C$

$\frac{5}{2} - 2 = C$

$\boxed{C = \frac{1}{2}}$

$\frac{-2}{y} = x^2 + C$

$\boxed{\frac{-2}{x^2+C} = y}$

we know

$y(2) = \frac{-2}{5}$

$\downarrow \quad \downarrow$

$x \quad y$

$\frac{-2}{2^2+C} = \frac{-2}{5}$

$\frac{-2}{4+C} = \frac{-2}{5}$

$C = 1$

$\boxed{y = \frac{-2}{x^2+1}}$

$\boxed{\frac{-1}{y} = \frac{1}{2}x^2 + \frac{1}{2}}$

(9) $\frac{dy}{dx} = 12x^3 y$ $\quad y(0) = 2$

$x=0 \quad y=2$

$\frac{dy}{y} = 12x^3 dx$

$\frac{1}{y} dy = 12x^3 dx$

$\int \frac{1}{y} dy = \int 12x^3 dx$

$\ln|y| = 12\int x^3 dx$

$\ln|y| = \frac{12}{4} x^4 + C$

$\boxed{\ln|y| = 3x^4 + C}$

$\ln|2| = 3(0)^4 + C$

$\ln|2| = C$

$\boxed{\ln 2 = C}$

$\boxed{\ln|y| = 3x^4 + \ln 2}$

Also

$e^{\ln|y|} = e^{3x^4 + C}$

$|y| = e^{3x^4 + C}$

$= e^{3x^4} \cdot e^C$

$\boxed{y = Ce^{3x^4}}$

$y > 0$ if $C > 0$

$2 = Ce^{3(0)^4}$

$2 = C \cdot e^0$

$2 = 1C$

$\boxed{y = 2e^{3x^4}}$

(10) $\frac{dy}{dx} = -\frac{x}{y}$

$y\,dy = -x\,dx$

$\int y\,dy = \int -x\,dx$

$\boxed{\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C}$

$y^2 = -1x^2 + C$

$\sqrt{y^2} = \pm\sqrt{-1x^2 + C}$

$\boxed{y = \pm\sqrt{-1x^2 + C}}$

$-\sqrt{2} = -\sqrt{-1(1)^2 + C}$

$-\sqrt{2} = -\sqrt{-1 + C}$

$2 = -1 + C$

$\boxed{C = 3}$

$\boxed{y = -\sqrt{-1x^2 + 3}}$

$y(1) = -\sqrt{2}$

$x = 1 \qquad y = -\sqrt{2}$

$\frac{1}{2}(-\sqrt{2})^2 = -\frac{1}{2}(1)^2 + C$

$\frac{1}{2}(2) = -\frac{1}{2}(1) + C$

$1 = -\frac{1}{2} + C$

$+\frac{1}{2} \quad +\frac{1}{2}$

$\boxed{\frac{3}{2} = C}$

$\boxed{\frac{1}{2}y^2 = -\frac{1}{2}x^2 + \frac{3}{2}}$

$y^2 = -x^2 + 3$

$\boxed{x^2 + y^2 = 3}$

$(\sqrt{2})^2 = -1(1)^2 + C$

$2 = -1 + C$

$3 = C$

$y = -\sqrt{-1x^2 + C}$

because $y < 0$

$\sqrt{3}$