Name____Properties of Logarithms Period ______

1. Find the value of y.

(1) $\log_5 25 = y$ (2) $\log_3 1 = y$ (3) $\log_{16} 4 = y$ (4) $\log_2 \frac{1}{8} = y$

(5) $\log_5 1 = y$ (6) $\log_2 8 = y$ (7) $\log_7 \frac{1}{7} = y$ (8) $\log_3 \frac{1}{9} = y$

(9) $\log_y 32 = 5$ (10) $\log_9 y = -\frac{1}{2}$ (11) $\log_4 \frac{1}{8} = y$ (12) $\log_9 \frac{1}{81} = y$

2. Evaluate.

(1) $\log_3 1$ (2) $\log_4 4$ (3) $\log_7 7^3$ (4) $b^{\log_b 3}$ (3) $\log_{25} 5^3$ (4) $16^{\log_4 8}$

3. Write the following expressions in terms of logs of x, y and z.

(1) $\log x^2 y$ (2) $\log \frac{x^3 y^2}{z}$ (3) $\log \frac{\sqrt{x}\sqrt[3]{y^2}}{z^4}$ (4) $\log xyz$

(5) $\log \frac{x}{yz}$ (6) $\log \left(\frac{x}{y}\right)^2$ (7) $\log (xy)^{\frac{1}{3}}$ (8) $\log x\sqrt{z}$

(9) $\log \frac{\sqrt[3]{x}}{\sqrt[3]{yz}}$ (10) $\log \sqrt[4]{\frac{x^3 y^2}{z^4}}$ (11) $\log x\sqrt{\frac{\sqrt{x}}{z}}$ (12) $\log \sqrt{\frac{xy^2}{z^8}}$

4. Write the following equalities in exponential form.

(1) $\log_3 81 = 4$ (2) $\log_7 7 = 1$ (3) $\log_{\frac{1}{2}} \frac{1}{8} = 3$ (4) $\log_3 1 = 0$

(5) $\log_4 \frac{1}{64} = -3$ (6) $\log_6 \frac{1}{36} = -2$ (7) $\log_x y = z$ (8) $\log_m n = \frac{1}{2}$

5. Write the following equalities in logarithmic form.

(1) $8^2 = 64$ (2) $10^3 = 10000$ (3) $4^{-2} = \frac{1}{16}$ (4) $3^{-4} = \frac{1}{81}$

(5) $\left(\frac{1}{2}\right)^{-5} = 32$ (6) $\left(\frac{1}{3}\right)^{-3} = 27$ (7) $x^{2z} = y$ (8) $\sqrt{x} = y$

6. True or False?

(1) $\log\left(\frac{x}{y^3}\right) = \log x - 3\log y$ (2) $\log(a - b) = \log a - \log b$ (3) $\log x^k = k \cdot \log x$

(4) $(\log a)(\log b) = \log(a + b)$ (5) $\frac{\log a}{\log b} = \log(a - b)$ (6) $(\ln a)^k = k \cdot \ln a$

(7) $\log_a a^a = a$ (8) $-\ln\left(\frac{1}{x}\right) = \ln x$ (9) $\ln_{\sqrt{x}} x^k = 2k$

Properties of Logarithms

(1.1) $\log_5 25 = y$

$\log_5 5^2 = y$

$2 \cancel{\log_5 5}$

$\boxed{2 = y}$

OR

$\log_5 25 = y$

$5^y = 25$

$5^y = 5^2$

$y = 2$

(1.2) $\log_3 1 = y$

$y = 0$

OR $3^y = 1$

$\boxed{3^0 = 1}$

$\boxed{y = 0}$

(1.3) $\log_{16} 4 = y$

$16^y = 4$

$16 = 4^2 \rightarrow 16^y = 4$

$(4^2)^y = 4^1$

$4^{2y} = 4^1$

$2y = 1$

$y = \frac{1}{2}$

(1.3) $\log_{16} 4 = y$

$16^y = 4$

$(2^4)^y = 2^2$

$2^{4y} = 2^2$

$4y = 2$

$y = \frac{2}{4} = \frac{1}{2}$

1.4

$\log_2 \frac{1}{8} = y$

$\log_2 8^{-1} = y$

$\log_2 (2^3)^{-1} = y$

$\log_2 2^{-3} = y$

$-3\log_2 2 = y$

$-3(1) = y$

$-3 = y$

1.4

$\log_2\left(\frac{1}{8}\right) =$

$\log_2 1 - \log_2 8$

$0 - \log_2 2^3$

$0 - 3\log_2 2$

$0 - 3(1)$

$0 - 3$

$-3 = y$

1.4

$\log_2 \frac{1}{8} = y$

$2^y = \frac{1}{8}$

$2^y = \left(\frac{1}{2}\right)^3$

$2^y = (2^{-1})^3$

$2^y = 2^{-3}$

$y = -3$

1.5

$\log_5 1 = y$

$5^y = 1$

$y = 0$

1.6 $\log_2 8 = \log_2 2^3 = 3\log_2 2 = 3(1) = 3 = y$

1.7 $\log_7 \frac{1}{7} = \log_7 7^{-1} = -1 \log_7 7 = -1(1) = -1 = y$

$\log_7 \frac{1}{7} = \log_7 1 - \log_7 7$

$= 0 - (1)$

$= -1$

1.8 $\log_3 \frac{1}{9} = \log_3 \frac{1}{3^2} = \log_3 3^{-2} = -2\log_3 3 = y$

$= -2(1) = -2 = y$

OR $3^y = \frac{1}{9}$

$3^y = \frac{1}{3^2}$

$\cancel{3}^y = 3^{-2}$

$y = -2$

1.9 $\log_y 32 = 5 \Rightarrow \log_y 2^5 = 5 = 5\log_y 2 = 5$

If $y=2$ then $5\log_2 2 = 5(1) = 5$ ✓✓

OR $y^5 = 32 \Rightarrow \boxed{y = \sqrt[5]{32} = \sqrt[5]{2^5} = 2^{5/5} = 2^1}$

1.10 $\log_9 y = -\frac{1}{2}$ iff $9^{-\frac{1}{2}} = y$

$\frac{1}{\sqrt{9}} = y$

$\frac{1}{3} = y$

1.11 $\log_4 \frac{1}{8} = y$

$4^y = \frac{1}{8}$

$(2^2)^y = \frac{1}{8}$

$2^{2y} = \frac{1}{2^3}$

$2^{2y} = 2^{-3}$

$2y = -3$

$y = \frac{-3}{2}$

1.12 $\log_9 \frac{1}{81} = y$

$9^y = \frac{1}{81}$ OR $3^{2y} = \frac{1}{3^4}$

$9^y = \frac{1}{9^2}$ $\quad$ $3^{2y} = 3^{-4}$

$9^y = 9^{-2}$ $\quad$ $2y = -4$

$y = -2$ $\quad$ $y = \frac{-4}{2}$

$y = -2$

(2.1) $\log_3 1 = 0$ iff $3^0 = 1$

(2.2) $\log_4 4 = 1$ iff $4^1 = 4$

(2.3) $\log_7 7^3 = 3$ iff $7^3 = 7^3$

(2.3) $\cancel{\log_7 7^3} = 3$

(2.4) $\cancel{b^{\log_b 3}} = 3$ iff $\log_b 3 = \log_b 3$

(2.5) $\log_{25} 5^3 = y$ iff $25^y = 5^3$

$(5^2)^y = 5^3$

$5^{2y} = 5^3$

$2y = 3$

$y = \frac{3}{2}$

(2.5) $\log_{25} 5^3 = \log_{25} 125 = \log_{25} (25 \cdot 5)$

$= \log_{25} 25 + \log_{25} 5 \longrightarrow 25^x = 5$

$= 1 + \frac{1}{2} = \frac{3}{2}$

$(5^2)^x = 5^1$

$5^{2x} = 5^1$

$2x = 1$ $x = \frac{1}{2}$

2.b $16^{\log_4 8}$

Note $\log_4 8 = y$ $\quad 4^y = 8$

$(2^2)^y = 2^3$

$2^{2y} = 2^3$

$2y = 3$

$y = \frac{3}{2}$

$16^{3/2}$

$\sqrt{16^3}$

$(\sqrt{16})^3 = (4)^3 = 64$

2.b $16^{\log_4 8} = (4^2)^{(\log_4 8)}$

$= 4^{2\log_4 8}$

$= 4^{\log_4 8^2}$

$= \cancel{4^{\log_4 64}}$

$= 64$

3.1 $\log x^2y = \log x^2 + \log y$ Product Rule

$= \boxed{2\log x + \log y}$ Power Rule

3.2 $\log \frac{x^3y^2}{z} = \log x^3y^2 - \log z$ Quotient Rule

$= \log x^3 + \log y^2 - \log z$ Product Rule

$= \boxed{3\log x + 2\log y - \log z}$ Power Rule

3.3 $\log \frac{\sqrt{x}\,\sqrt[3]{y^2}}{z^4} = \log \frac{x^{1/2}y^{2/3}}{x^4}$ Exponent Laws

$= \log x^{\frac{1}{2}}y^{2/3} - \log x^4$ Quotient Law

$= \log x^{\frac{1}{2}} + \log y^{2/3} - \log x^4$ Product Law

$= \boxed{\frac{1}{2}\log x + \frac{2}{3}\log y - 4\log x}$ Power Rule

3.4 $\log xyz = \boxed{\log x + \log y + \log z}$ Product Rule

3.5 $\log \frac{x}{yz} = \log x - \log yz$ Quotient Law

$= \log x - [\log y + \log z]$ Product Rule

$= \boxed{\log x - \log y - \log z}$

(3.6) $\log\left(\frac{x}{y}\right)^2 = \log \frac{x^2}{y^2}$ Exponent Law

$\log x^2 - \log y^2$ Quotient Law

$\boxed{2\log x - 2\log y}$ Power Rule

(3.6) $\log\left(\frac{x}{y}\right)^2 = 2\log\left(\frac{x}{y}\right)$ Power Rule

$= 2\left[\log x - \log y\right]$ Quotient Rule

$= \boxed{2\log x - 2\log y}$

(3.7) $\log(xy)^{\frac{1}{3}} = \log x^{\frac{1}{3}} y^{\frac{1}{3}}$ Exponent Laws

$= \log x^{\frac{1}{3}} + \log y^{\frac{1}{3}}$ Product Rule

$= \boxed{\frac{1}{3}\log x + \frac{1}{3}\log y}$ Power Rule

(3.7) $\log(xy)^{\frac{1}{3}} = \frac{1}{3}\log(xy)$ Power Rule

$= \boxed{\frac{1}{3}\left[\log x + \log y\right]}$ Product Rule

$= \boxed{\frac{1}{3}\log x + \frac{1}{3}\log y}$

(3.8) $\log x\sqrt{z} = \log x z^{\frac{1}{2}}$ Exponent'l Law

$= \log x + \log z^{\frac{1}{2}}$ Product Rule

$= \boxed{\log x + \frac{1}{2}\log z}$ Power Rule

(3.9) $\log \dfrac{\sqrt[3]{x}}{\sqrt[3]{yz}} = \log \dfrac{x^{\frac{1}{3}}}{(yz)^{1/3}} = \log \dfrac{x^{\frac{1}{3}}}{y^{\frac{1}{3}} z^{\frac{1}{3}}}$

exponent laws

$\log x^{\frac{1}{3}} - \log y^{\frac{1}{3}} z^{\frac{1}{3}}$ Quotient Law

$\log x^{\frac{1}{3}} - \left[\log y^{\frac{1}{3}} + \log z^{\frac{1}{3}}\right]$ Product Rule

$\boxed{\frac{1}{3}\log x - \left[\frac{1}{3}\log y + \frac{1}{3}\log z\right]}$ Power Rule

$\boxed{\frac{1}{3}\log x - \frac{1}{3}\log y - \frac{1}{3}\log z}$

(3.9) $\log \sqrt[3]{\dfrac{x}{yz}} = \log\left(\dfrac{x}{yz}\right)^{\frac{1}{3}}$ Exponent'l Laws

$= \frac{1}{3}\log\left(\dfrac{x}{yz}\right) = \frac{1}{3}\left[\log x - \log yz\right]$ Quotient Law

Power Rule

$= \frac{1}{3}\left[\log x - \left[\log y + \log z\right]\right]$ Product Law

$= \boxed{\frac{1}{3}\log x - \frac{1}{3}\log y - \frac{1}{3}\log z}$

3.10 $\log \sqrt[4]{\dfrac{x^3y^2}{z^4}} = \log\left(\dfrac{x^3y^2}{z^4}\right)^{1/4}$

$= \log \dfrac{x^{\frac{3}{4}}y^{\frac{2}{4}}}{z^{\frac{4}{4}}} = \log \dfrac{x^{\frac{3}{4}}y^{\frac{1}{2}}}{z^1}$ exponential laws

$\log x^{\frac{3}{4}}y^{\frac{1}{2}} - \log z^1$ Quotient Law

$\log x^{\frac{3}{4}} + \log y^{\frac{1}{2}} - \log z$ Product law

$\boxed{\frac{3}{4}\log x + \frac{1}{2}\log y - \log z}$ Power Rule

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3.10 $\log\left(\dfrac{x^3y^2}{z^4}\right)^{\frac{1}{4}} = \frac{1}{4}\log\dfrac{x^3y^2}{z^4}$ Power Rule

$= \frac{1}{4}\left[\log x^3y^2 - \log z^4\right]$ Quotient Rule $= \frac{1}{4}\left[\log x^3 + \log y^2 - \log z^4\right]$ Product Rule

$= \frac{1}{4}\left[3\log x + 2\log y - 4\log z\right]$ Power Rule

$\frac{3}{4}\log x + \frac{2}{4}\log y - \frac{4}{4}\log z = \boxed{\frac{3}{4}\log x + \frac{1}{2}\log y - \log z}$

3.11 $\log x\sqrt{\frac{\sqrt{x}}{z}} = \log x\left(\frac{(x)^{1/2}}{z}\right)^{1/2}$

$= \log \frac{x\, x^{\frac{1}{4}}}{z^{\frac{1}{2}}} = \log\left(\frac{x^{\frac{5}{4}}}{z^{\frac{1}{2}}}\right)$ exponent laws

$\log\left[\frac{x^{\frac{5}{4}}}{z^{\frac{1}{2}}}\right] = \log x^{\frac{5}{4}} - \log z^{\frac{1}{2}}$

$= \boxed{\frac{5}{4}\log x - \frac{1}{2}\log z}$

3.11 $\log x\sqrt{\frac{\sqrt{x}}{z}} = \log x + \log\sqrt{\frac{\sqrt{x}}{z}}$ Product Rule

$= \log x + \log\frac{\sqrt{\sqrt{x}}}{\sqrt{z}} = \log x + \log\sqrt{\sqrt{x}} - \log\sqrt{z}$

$= \log x^1 + \log (x^{1/2})^{1/2} - \log z^{1/2}$

$= \log x + \log x^{\frac{1}{4}} - \log z^{\frac{1}{2}}$

$= 1\log x + \frac{1}{4}\log x - \frac{1}{2}\log z$

$= (1+\frac{1}{4})\log x - \frac{1}{2}\log z$

$= \boxed{\frac{5}{4}\log x - \frac{1}{2}\log z}$

(3.12) $\log\sqrt{\frac{xy^2}{z^8}} = \log\left(\frac{xy^2}{z^8}\right)^{1/2}$

$= \log\left(\frac{x^{\frac{1}{2}}y^{\frac{2}{2}}}{z^{\frac{8}{2}}}\right) = \log\left(\frac{x^{\frac{1}{2}}y^{1}}{z^{4}}\right)$

exponent laws

$\log\sqrt{\frac{xy^2}{z^8}} = \log x^{\frac{1}{2}}y^1 - \log z^4$ Quotient Laws

$= \log x^{\frac{1}{2}} + \log y^1 - \log z^4$ Product Rule

$= \boxed{\frac{1}{2}\log x + \log y - 4\log z}$

(3.12) $\log\sqrt{\frac{xy^2}{z^8}} = \log\left(\frac{xy^2}{z^8}\right)^{\frac{1}{2}}$ exponent Law

$\frac{1}{2}\log\left(\frac{xy^2}{z^8}\right)$ Power Rule $= \frac{1}{2}\left[\log xy^2 - \log z^8\right]$ Quotient Rule

$\frac{1}{2}\left[\log x + \log y^2 - \log z^8\right]$ Product Rule

$\frac{1}{2}\left[\log x + 2\log y - 8\log z\right]$ Power Rule

$\frac{1}{2}\log x + \frac{2}{2}\log y - \frac{8}{2}\log z = \boxed{\frac{1}{2}\log x + \log y - 4\log z}$

4.1 $\log_3 81 = 4$ iff $3^4 = 81$

4.2 $\log_7 7 = 1$ iff $7^1 = 7$

4.3 $\log_{\frac{1}{2}} \frac{1}{8} = 3$ iff $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

4.4 $\log_3 1 = 0$ iff $3^0 = 1$

4.5 $\log_4 \frac{1}{64} = -3$ iff $4^{-3} = \frac{1}{64}$

4.6 $\log_6 \frac{1}{36} = -2$ iff $6^{-2} = \frac{1}{36}$

4.7 $\log_x y = z$ iff $x^z = y$

4.8 $\log_m n = \frac{1}{2}$ iff $m^{\frac{1}{2}} = n$

5.1 $8^2 = 64$ iff $\log_8 64 = 2$

5.2 $10^3 = 1000$ iff $\log_{10} 1000 = 3$

$\log 1000 = 3$

5.3 $4^{-2} = \frac{1}{16}$ iff $\log_4 \frac{1}{16} = -2$

5.4 $3^{-4} = \frac{1}{81}$ iff $\log_3 \frac{1}{81} = -4$

5.5 $\left(\frac{1}{2}\right)^{-5} = 32$ iff $\log_{\frac{1}{2}} 32 = -5$

5.6 $\left(\frac{1}{3}\right)^{-3} = 27$ iff $\log_{\frac{1}{3}} 27 = -3$

5.7 $x^{27} = y$ iff $\log_x y = 27$

5.8 $\sqrt{x} = y \Leftrightarrow x^{\frac{1}{2}} = y$ iff $\log_x y = \frac{1}{2}$

(6) True or False

(6.1) $\log\left(\frac{x}{y^3}\right) \overset{?}{=} \log x - 3\log y$

True

RHS

$\log x - \log y^3$ (Power Rule

$\boxed{\log\left(\frac{x}{y^3}\right)}$ same as LHS

(6.2) False $\log(a-b) \neq \log a - \log b$

$RHS = \log a - \log b$

$= \log\left(\frac{a}{b}\right) \neq \log(a-b)$ LHS

(6.2) False Apply 10 as base

$$\cancel{10}^{\log_{10}(a-b)} \neq 10^{\log_{10} a - \log_{10} b}$$

$$a-b \neq \frac{\cancel{10}^{\log_{10} a}}{\cancel{10}^{\log_{10} b}}$$

$$\boxed{a-b \neq \frac{a}{b}}$$

6-3 $\log x^K$ is $K\log X$ TRUE

RHS

$K\log X = \log x^K$ Power Rule

LHS

6-4 $(\log a)(\log b) \neq \log(a+b)$ False

Apply 10 as base

$$10^{(\log_{10} a \cdot \log_{10} b)} \neq \cancel{10^{\log_{10}}}(a+b)$$

$$\left(\cancel{10^{\log_{10}}} a\right)^{\log_{10} b} \neq a+b$$

$$a^{\log_{10} b} \neq a+b$$

6-5 $\dfrac{\log a}{\log b} \neq \log(a-b)$

$\log_b a \neq \log(a-b)$

change of base

False

6-4 $(\log a)(\log b) \neq \log(a+b)$

$\log b \neq \dfrac{\log(a+b)}{\log a}$

$\log b \neq \log_a(a+b)$

change of base

6.6 False $(\ln a)^k \neq k \ln a$

LHS $= (\ln a)^k$

$\boxed{\text{RHS} = [k \ln a] = \ln a^k}$

$\underline{\text{test}}$ $(\ln e^1)^k = 1^k = 1$ LHS

Let $a = e$ $\ln e^k = k$ RHS

$k \neq 1$ always

6.7 $\log_a a^a = a$

LHS $\log_a a^a = a \log_a a$

$= a(1) = a$

TRUE

6.8 $-\ln\frac{1}{x} = \ln x$

$-1\ln\left(\frac{1}{x}\right) = \ln x$

$\ln\left(\frac{1}{x}\right)^{-1} = \ln x$

$\ln\frac{1^{-1}}{(x^{-1}} = \ln x$

$\boxed{\ln\frac{x}{1} = \ln x}$ TRUE

Test $x = e$

LHS $-\ln\frac{1}{e}$ RHS $\cancel{\ln e}^{1}$

$\ln\left(\frac{1}{e}\right)^{-1}$ 1

$\cancel{\ln e^{+1}}$

1

6.9 $\ln \sqrt{x}\, x^k \stackrel{?}{=} 2k$

$\ln \sqrt{x} + \ln x^k$:

$\ln x^{\frac{1}{2}} + \ln x^k$

$\frac{1}{2} \ln x + k \ln x$

$\left(\frac{1}{2} + k\right) \ln x \quad \neq 2k$

But $\log_{\sqrt{x}} x^k = 2k$

implies $(\sqrt{x})^{2k} = x^k$

$(x^{\frac{1}{2}})^{2k} = x^k$

$x^{\frac{2}{2}k} = x^k$

$x^k = x^k$ true