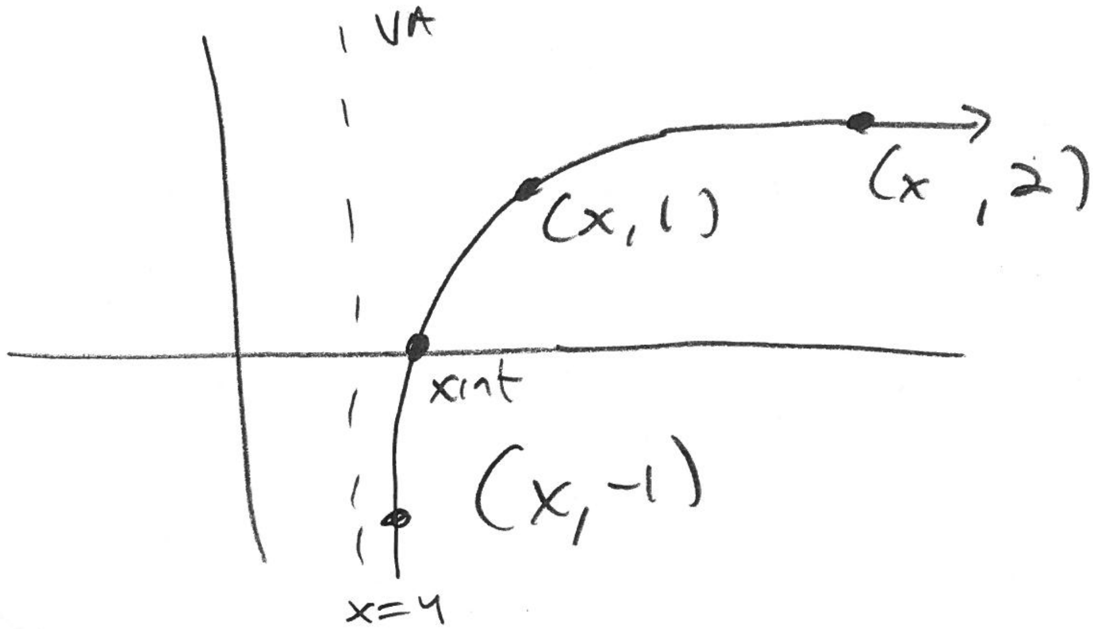
Sample Quiz Solutions   Example ①

$f(x) = \log_3(3x-12)$

$f(x) = \log 5(3(x-4))$



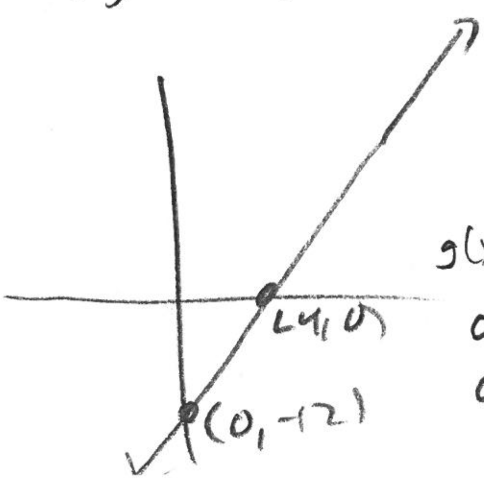
VA / Domain Restriction

$\log_b x$ must have $x > 0$

$\log_5(3x-12)$

$f(x) = \log_5(g(x))$

① $\boxed{\begin{array}{l} D: x > 4 \\ \quad x \in (4, \infty) \\ R: \text{all } y \end{array}}$



$g(x) = 3x - 12$
only + output of $g(x)$
allowed in $f(x) = \log_5(g(x))$

$f(x) = \log_5(3x-12)$

$= \log_5(3(x-4)) = \log_5(s(x))$

$\log_5 1 = 0$ so set $3x - 12 = 1$

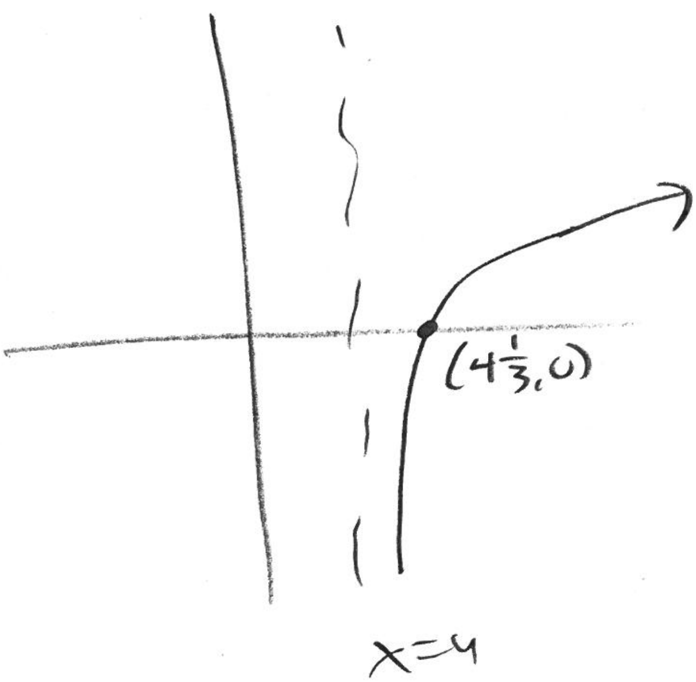
$\downarrow$

$s(x) = 3x - 12$

$3x = 13$

$x = \frac{13}{3} = 4.\overline{33}$

so xint $(4,0)$



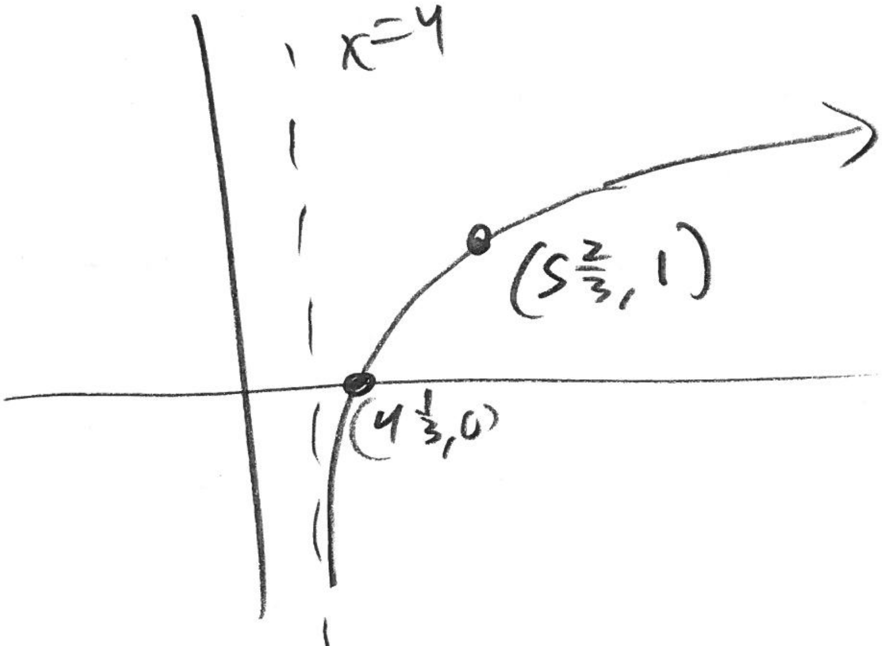
$f(x) = \log_5(3x - 12) = \log_5(3(x-4))$

$= \log_5(g(x))$

$\log_5 5 = 1$ so for $f(x) = 1$

set $g(x) = 5$

$$5 = 3x - 12$$
$$+12 \qquad +12$$
$$17 = 3x$$
$$\frac{17}{3} = x$$
$$\boxed{x = 5\tfrac{2}{3} = \tfrac{17}{3} = 5.\overline{6}}$$



$f(x) = \log_5(g(x)) = \log_5(3x-12)$

$= \log_5(3(x-4))$

$\log_5 25 = 2$ so set $g(x) = 25$

$\cancel{\log_5} 5^2 = 2$

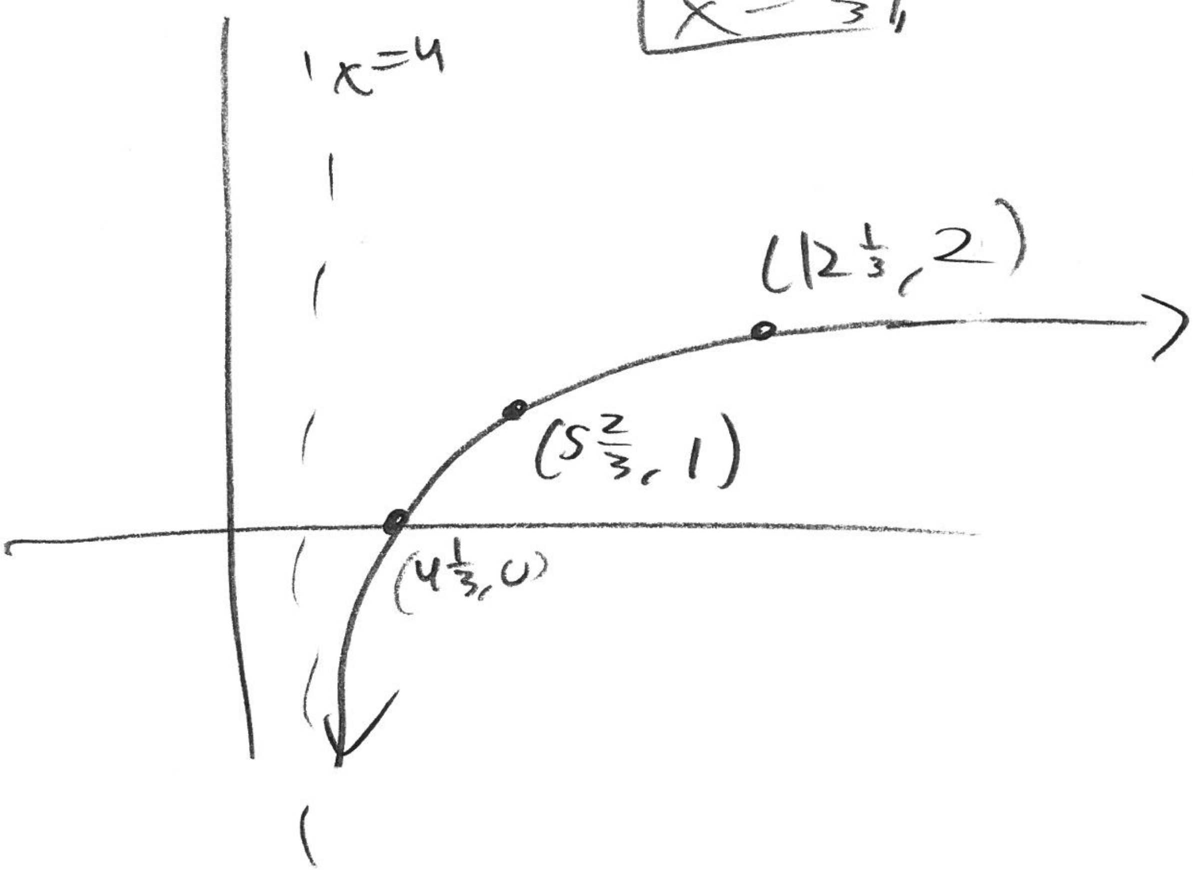
$25 = 3x - 12$

$+12 \qquad +12$

$37 = 3x$

$\frac{37}{3} = \frac{3x}{3}$

$\boxed{x = \frac{37}{3}} = 12\frac{1}{3}$



$f(x) = \log_5(3x-12) = \log_5(3(x-4))$

$= \log_5 g(x)$

$\log_5 \frac{1}{5} = -1$

set $\frac{1}{5} = 3x - 12$

$\quad +12 \qquad +12$

$12\frac{1}{5} = 3x$

$\frac{61}{5} = 3x$

$\frac{61}{5} \cdot \frac{1}{3} = x$

$\frac{61}{15} = x$

Easier way

$\left(\frac{1}{5} = 3x - 12\right) 5$

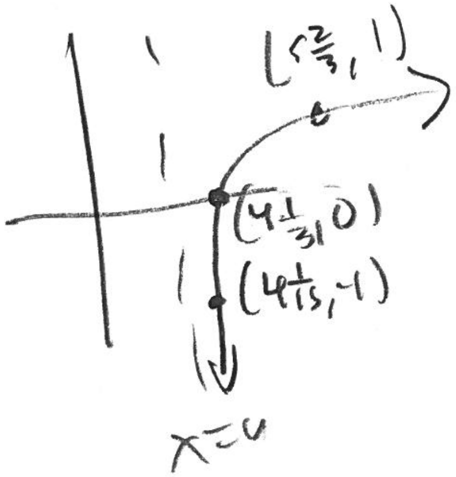
$1 = 15x - 60$

$+60 \qquad +60$

$61 = 15x$

$\frac{61}{15} = \frac{15x}{15}$

$\boxed{x = \frac{61}{15} = 4\frac{1}{15}}$



note as $x \to 4$ $f(x) \to -\infty$

$f(x) = \log_3(x+6)$ Example 2

① Solve these 4 equations

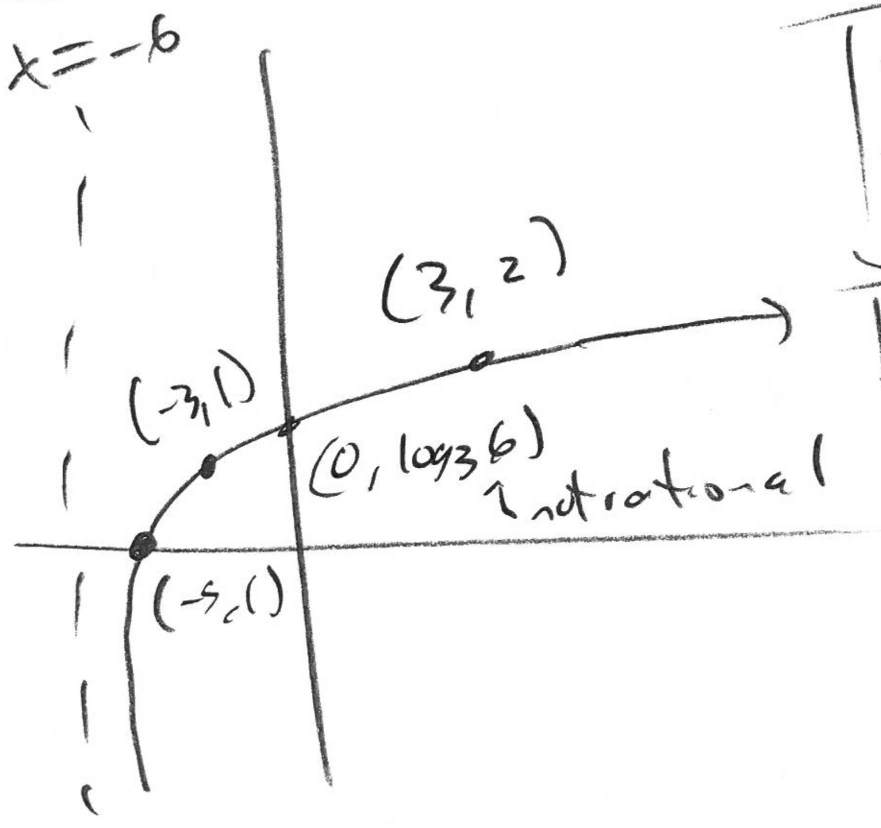
$x+6=0$ → Domain Restriction/VA

$x+6=1$ → x int provided no VS

$x+6=3$ → $(\ , 1)$ x value

$x+6=9$ → $(\ , 2)$ x value

$$\begin{array}{r} x+6=0 \\ -6\ \ -6 \\ \hline x=6 \end{array} \qquad \begin{array}{r} x+6=1 \\ -6\ \ -6 \\ \hline x=-5 \end{array} \qquad \begin{array}{r} x+6=3 \\ -6\ \ -6 \\ \hline x=-3 \end{array} \qquad \begin{array}{r} x+6=9 \\ -6\ \ -6 \\ \hline x=3 \end{array}$$



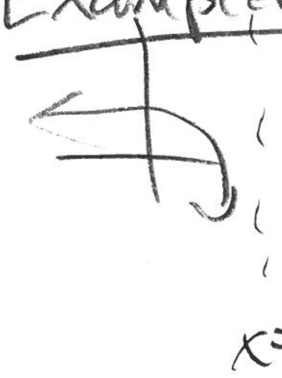
$\boxed{\begin{array}{l} D: x > -6 \\ R: y \in \mathbb{R} \end{array}}$

$\boxed{VA\ x = -6}$

$f(x) = \log_9(20-5x)$

$= \log_9(-5(x-4))$

Example ③



looks like ← because $-x$

Set $20-5x=0$

$-20=5x$

$x=4$

UA $x=4$

D: $x<4$

$20-5x=1$

$-20 \quad -20$

$-5x=-19$

$\boxed{x=\frac{19}{5}=4.8}$

$20-5x=5$

$-20 \quad -20$

$-5x=-15$

$x=3$

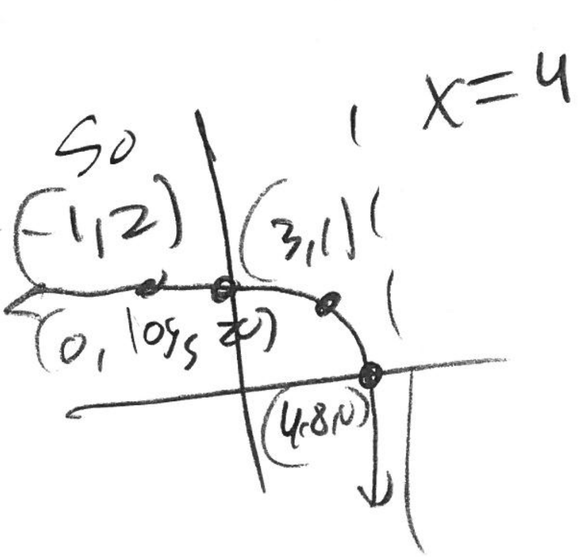
$20-5x=25$

$-20 \quad -20$

$-5x=5$

$x=-1$

So



$\boxed{\begin{array}{l} D: x<4 \\ R: y \in \mathbb{R} \end{array}}$

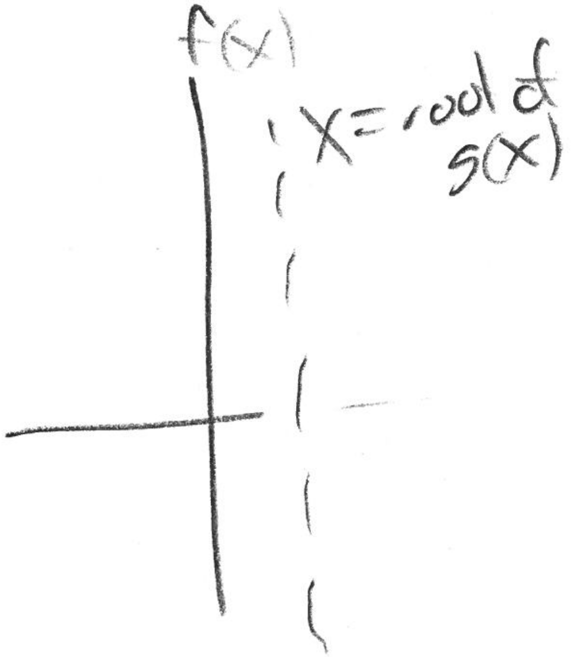
GENERAL GRAPHING LOG RULES

$f(x) = \log_b g(x)$

this is called a <u>composition</u> of functions

① Set $g(x) = 0$ to find Domain Restrictions

<u>Fancy Vocabulary</u>

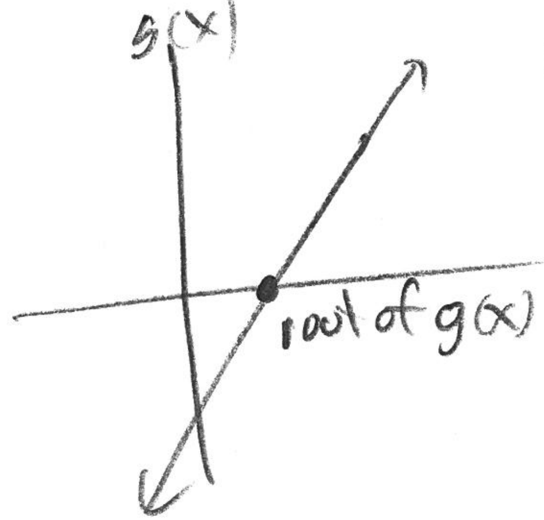


find root of interior function of composition

$g(x) = 0$ at $x =$ root

$x =$ zero

$x =$ xint



$f(x) = \log_b(g(x))$

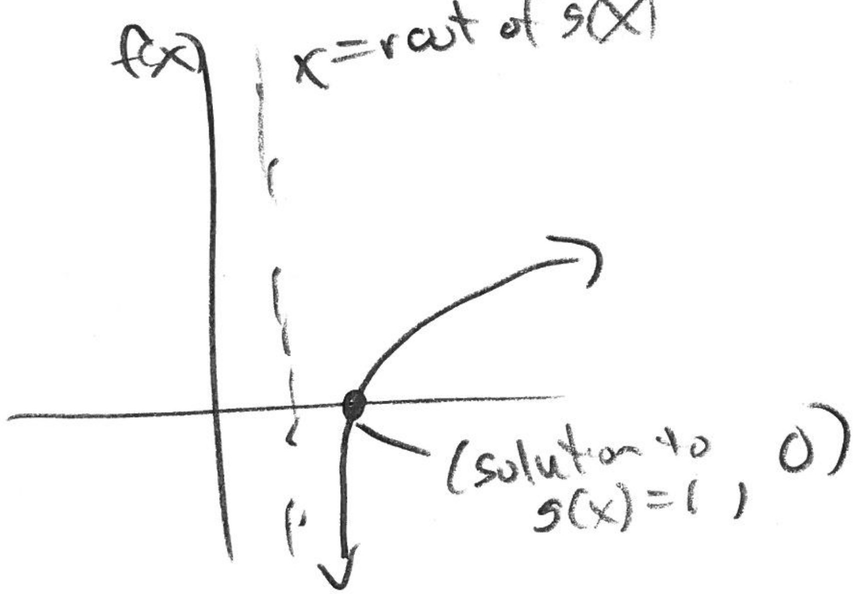
② Set $g(x) = 1$ to find x intercept

Note no <u>vertical</u> shift can be present

$f(x) = \log_b(g(x)) + 0$

no vertical shift

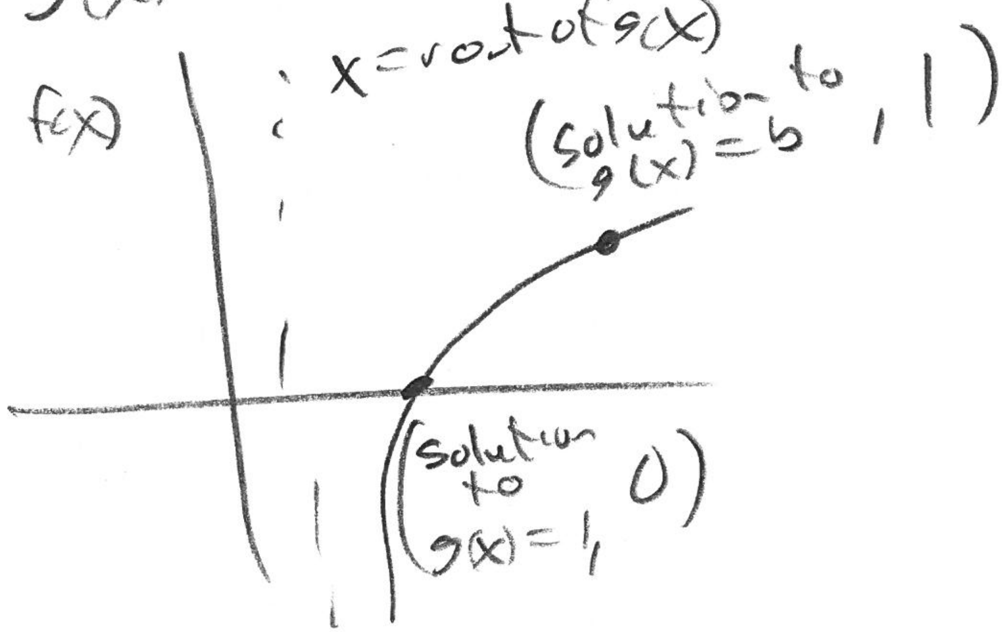
solve $g(x) = 1$



③ set $g(x) = b$ to find where

$f(x) = 1$ because $f(x) = \log_b b \stackrel{!}{=} 1$

$g(x) = b$

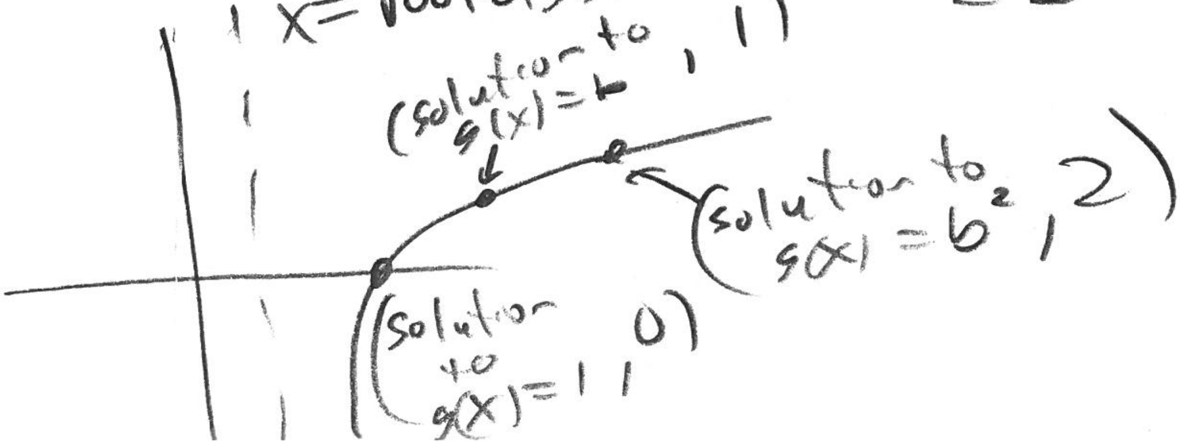


again $f(x) = \log_b(g(x)) + 0$

④ set $g(x) = b^2$ to find where

$f(x) = 2$ because $f(x) = \log_b b^2$

$= 2$



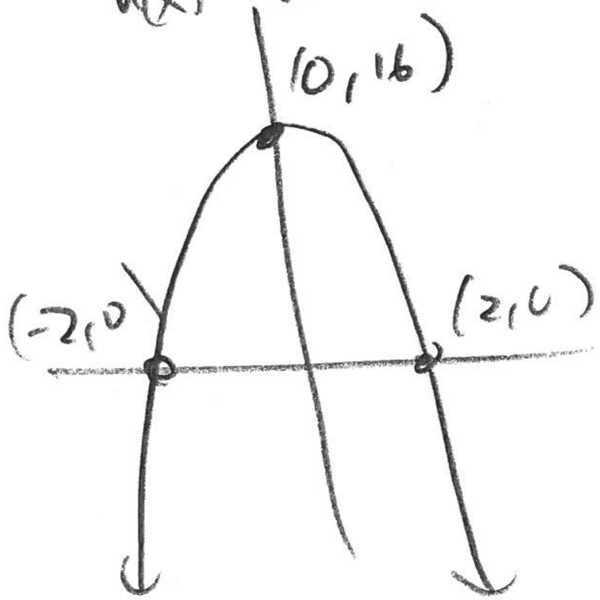
Example 4

$f(x) = \log_6(16 - 4x^2)$

$f(x) = \log_6(h(x))$

note $h(x) = y = 16 - 4x^2$

$h(x) = 16 - x^2$



set $16 - 4x^2 = 0$ to find **2** VA.

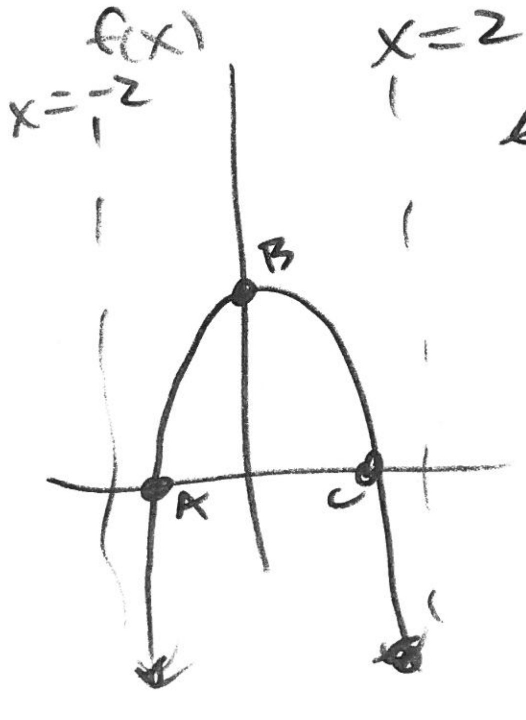
$16 - 4x^2 = 0$

$16 = 4x^2$

$\frac{16}{4} = x^2 = 4$

$x = \pm\sqrt{4}$

$\boxed{x = \pm 2}$



Example (4) cont

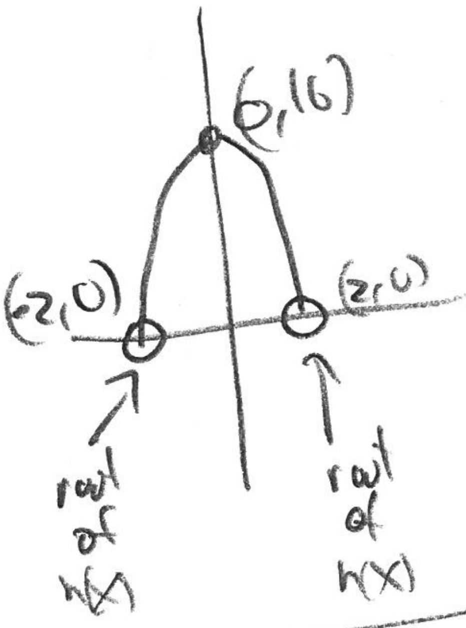
$f(x) = \log_6(16 - 4x^2)$

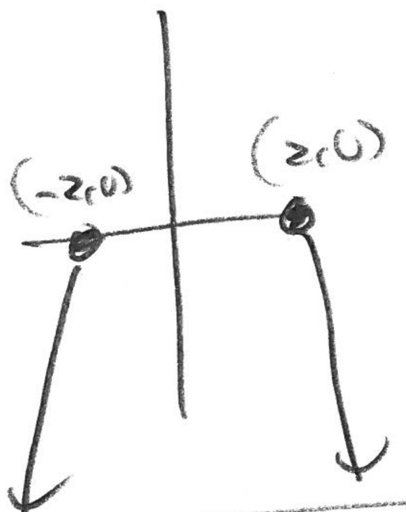
$f(x) = \log_8(h(x))$

this is a composition of functions

$f(x) = \log_6(h(x))$

↳ quadratic **NOT** linear





$h(x) \le 0$
when
$x \le -2$
or
$x \ge 2$

$\boxed{f(x) = \log_6(16 - 4x^2) \text{ is } \underline{\text{NOT}} \text{ defined when } x \le -2 \text{ or } x \ge 2}$

Example ④ cont

$f(x) = \log_6(16-4x^2)$

D: $x \in (-2, 2)$

R: $y \le \log_6 16 \leftarrow$ why

$h(x)$ has max at $x=0$

$f(x) = \log_6(h(x))$ has max at $x=0$

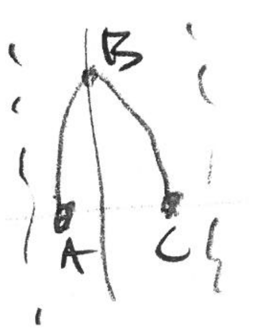
Set $16-4x^2 = 1$ to find x intercepts

$$\begin{aligned} 16-4x^2 &= 1 \\ -16 \quad &\quad -16 \\ \hline -4x^2 &= -15 \end{aligned}$$

$$\frac{-4x^2}{-4} = \frac{-15}{-4}$$

$$x^2 = \frac{15}{4}$$

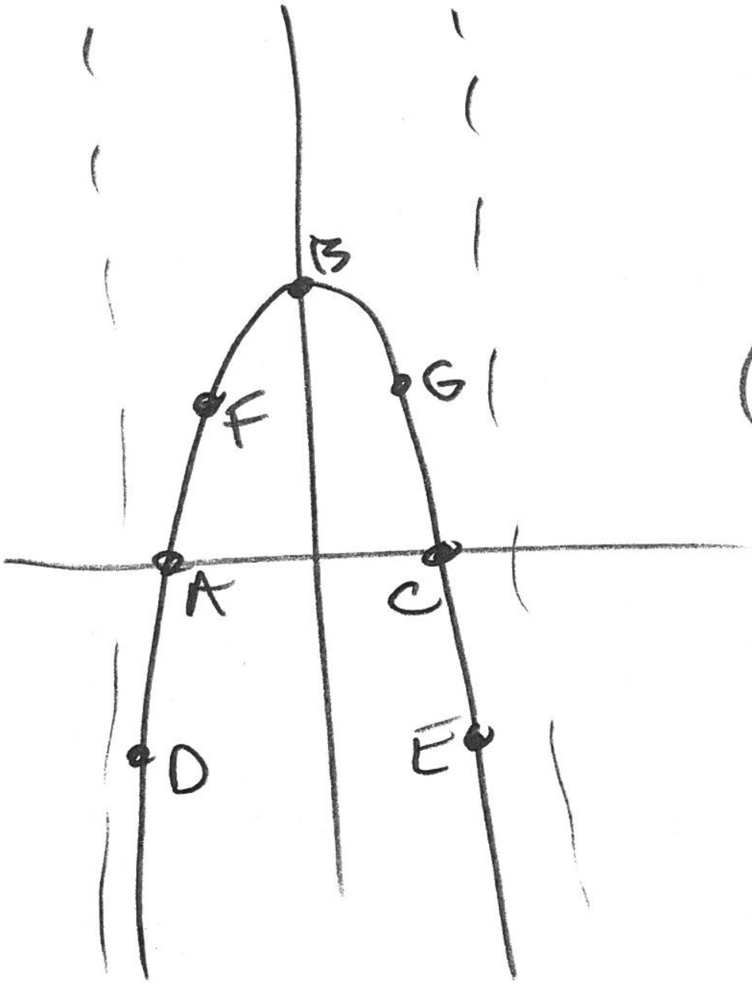
$$\boxed{x = \pm\sqrt{\frac{15}{4}} = \pm\frac{\sqrt{15}}{2} \approx \pm 1.94}$$



$B = (0, \log_6(16-4(0)^2))$
$= (0, \log_6 16) \approx 1.55$

$A \approx (-1.94, 0)$ $\quad C \approx (+1.94, 0)$

Example ④



Solve

$16 - 4x^2 = \frac{1}{6}$

to find

D & E

$\left(16 - 4x^2 = \frac{1}{6}\right)6$

$96 - 24x^2 = 1$

$\underline{-96 \qquad\qquad -96}$

$-24x^2 = -95$

$x^2 = \frac{-95}{-24}$

$x^2 = \frac{95}{24}$

$x = \pm\sqrt{\frac{95}{24}}$

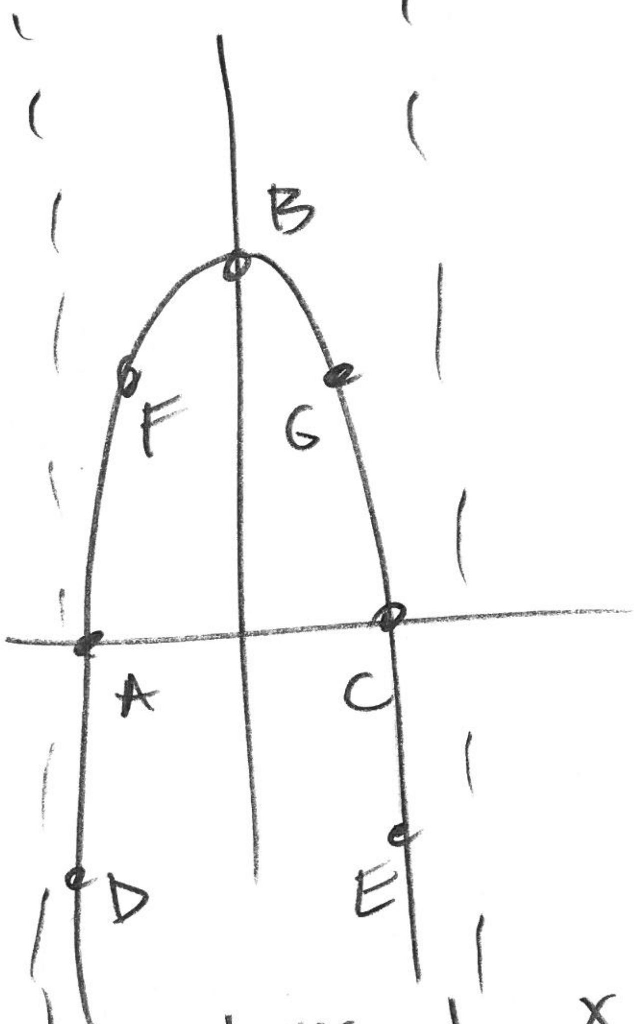
$\approx \pm 1.99$

SO D & E

	$\approx x$	x	y	$\approx y$
D	-1.99	$\sqrt{\frac{95}{24}}$	-1	—
E	$+1.99$	$\sqrt{\frac{95}{24}}$	-1	—

Example (4) Solve $16-4x^2=6$

to find F & G



$$16-4x^2=6$$
$$\underline{-16 \qquad -16}$$
$$-4x^2=-10$$

$$\frac{-4x^2}{-4}=\frac{-10}{-4}$$

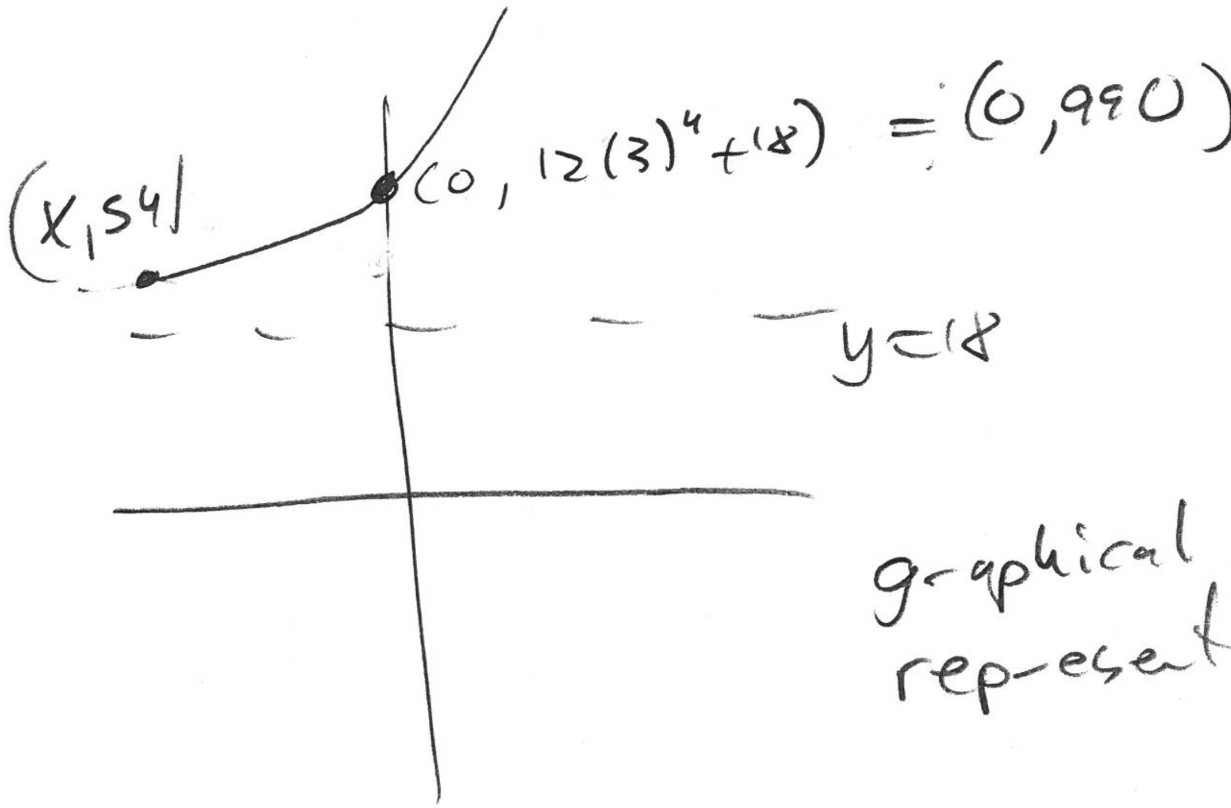
$$x^2=\frac{5}{2}$$

$$\boxed{\begin{array}{l} x=\pm\sqrt{\frac{5}{2}} \\ x\approx\pm 1.58 \end{array}}$$

	$\approx x$	x	y	$\approx y$
F	-1.58	$-\sqrt{\frac{5}{2}}$	1	—
G	1.58	$\sqrt{\frac{5}{2}}$	1	—

Solving Exponential Equations

① $12(3)^{2x+4}+18=54$



graphical representation

$$12(3)^{2x+4}+18=54$$
$$\quad -18 \quad -18$$

$$12(3)^{2x+4}=36$$

$$\frac{12(3)^{2x+4}}{12}=\frac{36}{12}$$

$$\boxed{3^{2x+4}=3}$$

① cont

method ① $b^{f(x)} = b^1$

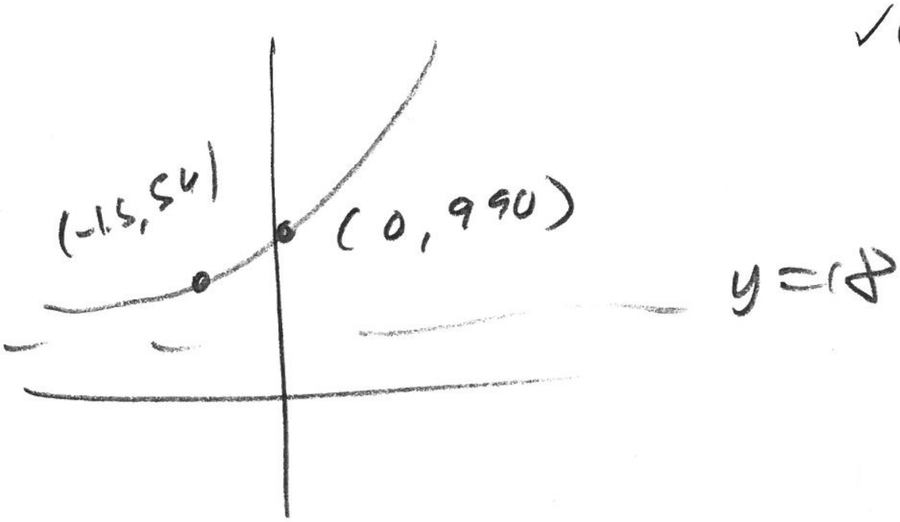
1 to 1 prop of exponential expressions

set $f(x) = 1$

& solve

$$\cancel{3}^{2x+4} = \cancel{3}^1$$

$$2x+4 = 1$$
$$\underline{\quad -4 \quad -4}$$
$$2x = -3$$
$$\frac{2x}{2} = \frac{-3}{2}$$
$$\boxed{x = \frac{-3}{2} = -1.5}$$



✓✓ $12(3)^{2(-1.5)+4} + 18$

$12(3)^{-3+4} + 18$

$12(3)^1 + 18$

$36 + 18$

$= 54$ ✓✓

① Method ②

$$3^{2x+4} = 3$$

<u>Defn of log</u>

$\cancel{\log_3 3} = 2x+4$ iff $3^{2x+4} = 3$

$$1 = 2x+4$$
$$-4 \qquad -4$$

$$-3 = 2x$$

$$\frac{-3}{2} = \frac{2x}{2}$$

$$\boxed{x = -1.5}$$

Method ③ $\quad 3^{2x+4} = 3$ Apply log

$$\log 3^{2x+4} = \log 3$$

$$\frac{(2x+4)\log 3}{\log 3} = \frac{\log 3}{\log 3}$$

$$2x+4 = 1$$

$$2x = -3 \longrightarrow \frac{2x}{2} = \frac{-3}{2}$$

x = -1.

Method ④

$3^{2x+4} = 3$ Apply $\log_3$

$\cancel{\log_3} 3^{2x+4} = \cancel{\log_3 3}^{1}$

$2x+4 = 1$

$\quad -4 \quad -4$

$2x = -3$

$x = -\frac{3}{2}$

Method ⑤ $3^{2x+4} = 3$ Apply Ln

$\ln(3^{2x+4}) = \ln 3$

$\frac{2x+4 \cancel{\ln 3}}{\cancel{\ln 3}} = \frac{\cancel{\ln 3}}{\cancel{\ln 3}}$

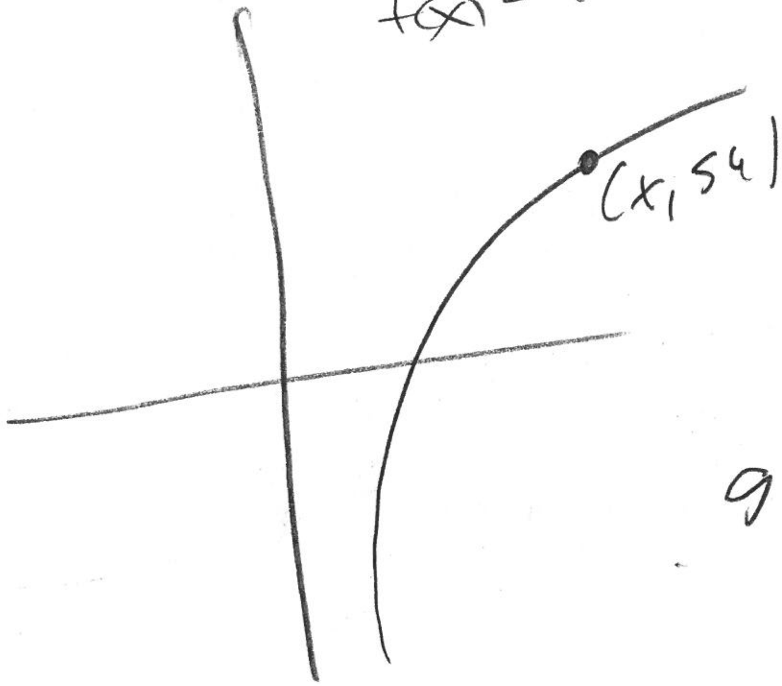
$2x+4 = 1$

$2x = -3$

$\frac{2x}{2} = \frac{-3}{2}$ $x = -\frac{3}{2}$

② $12\ln(6x-4)-18=54$

$f(x)=12\ln(6x-4)-18$



graphical representation

$$12\ln(6x-4)-18=54$$
$$\quad +18 \quad +18$$

$$12\ln(6x-4) = 72$$

$$\frac{12\ln(6x-4)}{12} = \frac{72}{12}$$

$$\boxed{\ln(6x-4) = 6}$$

(2) cont

$\ln(6x-4) = 6$ iff $e^6 = 6x-4$

(Defn)
method (1)

$e^6 + 4 = 6x$

$\frac{4+e^6}{6} = \frac{6x}{6}$

$\boxed{x = \frac{4+e^6}{6} \quad x \approx 67.90}$

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method (2)

$\cancel{\ln}(6x-4) = 6$
$\cancel{e} \qquad e$

$6x - 4 = e^6$
$\quad +4 \quad +4$

$6x = 4 + e^6$

$\frac{6x}{6} = \frac{4+e^6}{6}$

$\boxed{x = \frac{4+e^6}{6} \quad x \approx 67.90}$

② cont ✓✓

$$12\ln(6x-4) - 18 = 54$$

$$12\ln\left(6\left(\frac{4+e^6}{6}\right)-4\right) - 18$$

$$12\ln(4+e^6-4) - 18$$

$$*\ 12\ln e^6 - 18$$

$$\cancel{\ln e^{72}} - 18$$

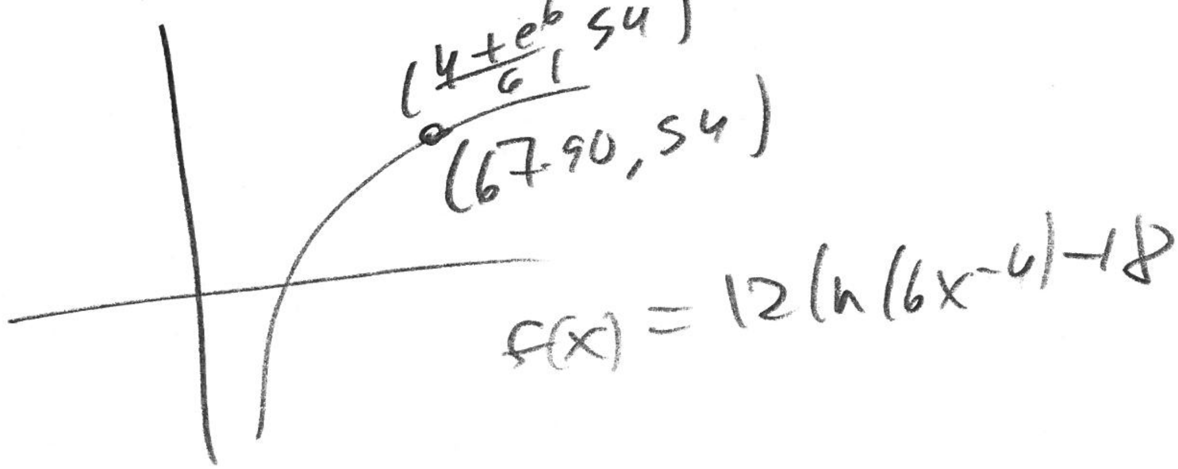
$$72 - 18 = 54$$

or

$$12\cancel{\ln e}^6 - 18$$

$$12(6) - 18$$

$$72 - 18 = 54$$



(3) $-6(10)^{3x-6} - 1 = 59$

$\qquad\qquad +1 \quad +1$

$$\frac{-6(10)^{3x-6}}{-6} = \frac{60}{-6}$$

$$10^{3x-6} = -10$$

<u>impossible!</u>

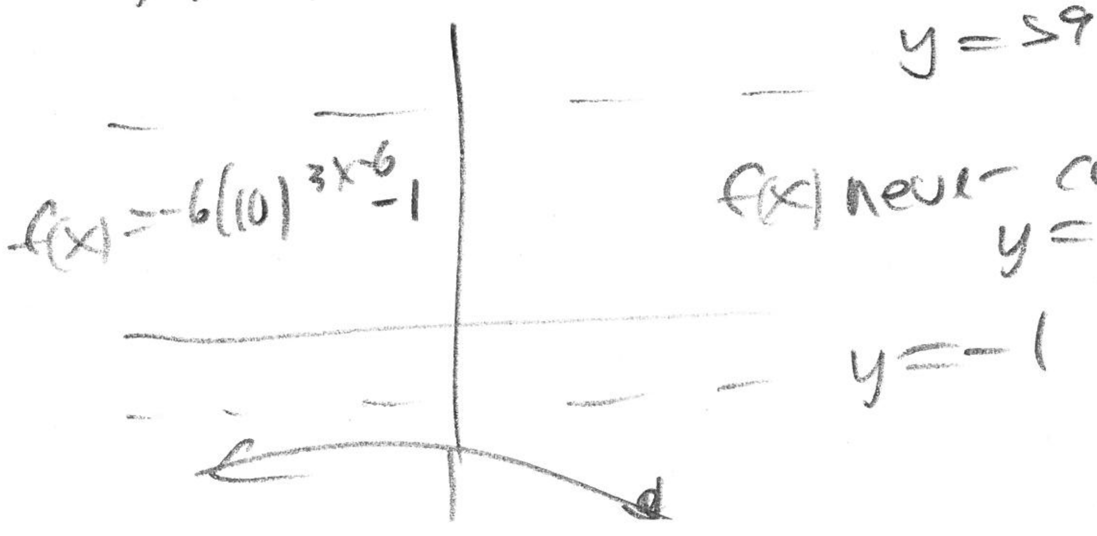
why $10^{3x-6} > 0$

or if true

$10^{3x-6} = -10$ iff $\log_{10}(-10) = 3x-6$

not allowed

why graphically?



f(x) never crosses y = 59

① $3\log(2x+4)+5 = 295$
$\quad\;\; -5 \quad -5$

$3\log(2x+4) = 290$

$\dfrac{3\log(2x+4)}{3} = \dfrac{290}{3}$

$\log(2x+4) = \dfrac{290}{3}$

Def'n $\quad \log(2x+4) = \dfrac{290}{3}$

iff $\quad 10^{\frac{290}{3}} = 2x+4$
$\quad -4 \qquad -4$

$-4+10^{\frac{290}{3}} = 2x$

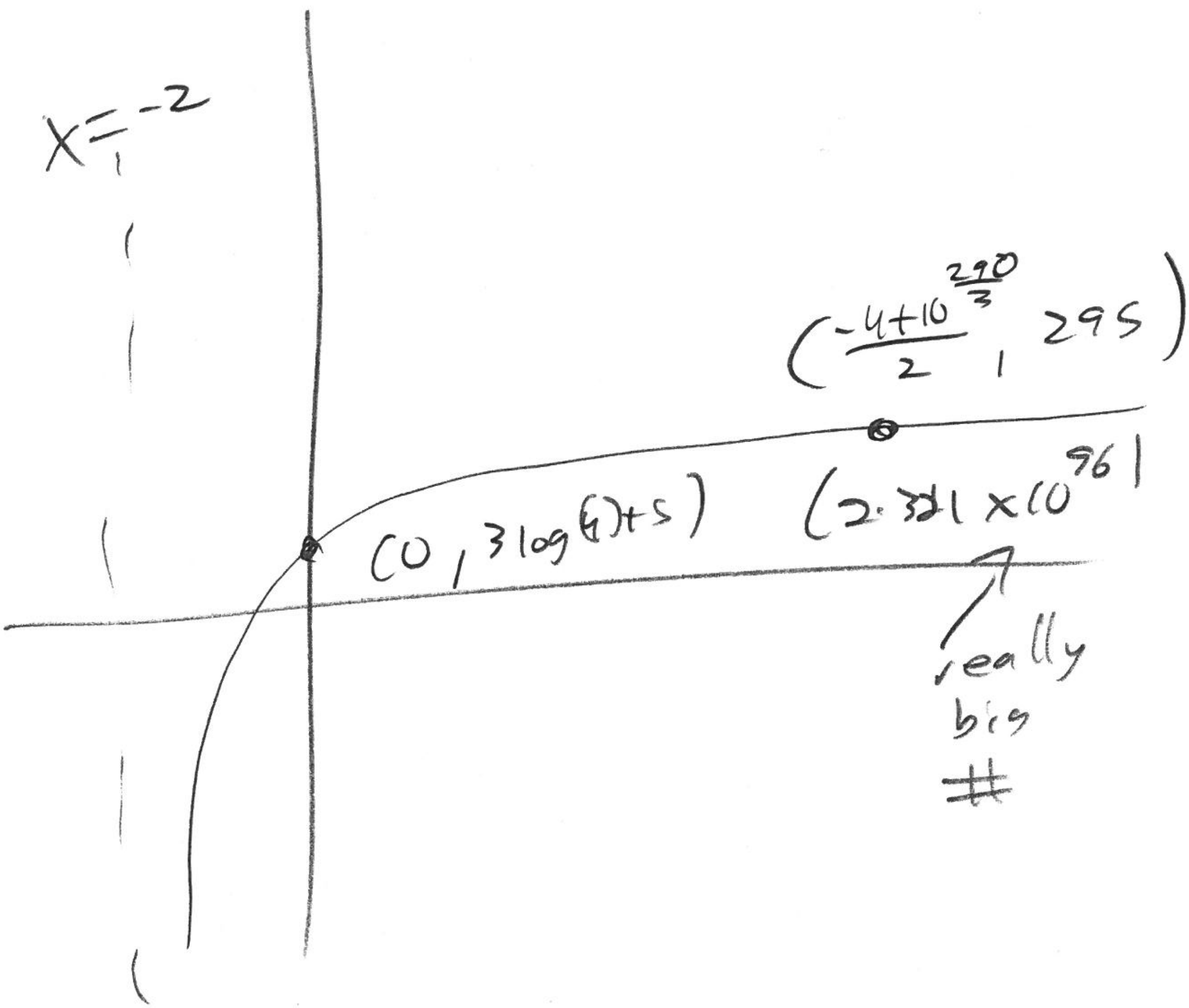
$\dfrac{-4+10^{\frac{290}{3}}}{2} = \dfrac{2x}{2}$

$\dfrac{-4}{2} + \dfrac{10^{\frac{290}{3}}}{2} = x$

$x = \dfrac{-4+10^{\frac{290}{3}}}{2}$

$$\boxed{\begin{aligned} x &= -2+\tfrac{1}{2}\cdot 10^{96.\overline{6}} \\ &\simeq 2.321 \times 10^{96} \end{aligned}}$$

Graphical Representation #1



$$3\log(2x+4)+5 = 295$$

$$3\log\left(2\left(\frac{-4+10^{\frac{290}{3}}}{2}\right)+4\right)+5$$

$$3\log\left(-4+10^{\frac{290}{3}}+4\right)+5$$

$$3\log\left(10^{\frac{290}{3}}\right)+5$$

$$\log\left(\left(10^{\frac{290}{3}}\right)^3\right)+5$$

$$\cancel{\log_{10}} 10^{290}+5 = 290+5 = 295$$

② $2\left(\frac{1}{2}\right)^{5x+10} - 18 = 110$

$\quad +18 \quad +18$

$$\frac{2\left(\frac{1}{2}\right)^{5x+10}}{2} = \frac{128}{2}$$

$$\boxed{\left(\frac{1}{2}\right)^{5x+10} = 64}$$

Method① Defn & Props $\boxed{\text{ADVANCED}}$

$$\boxed{\left(\frac{1}{2}\right)^{5x+10} = 64 \text{ iff } \log_{\frac{1}{2}} 64 = 5x+10}$$

$\log_{\frac{1}{2}} 2^6 = 5x+10$

$6\log_{\frac{1}{2}} 2 = 5x+10$

$6\left(\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{-1}\right) = 5x+10$

$-6\log_{\frac{1}{2}} \frac{1}{2} = 5x+10$

$-6 = 5x+10$

$-10 \quad\quad -10$

$\frac{-16}{5} = \frac{5x}{5}$

$x = \frac{-16}{5}$

Method 2 $\quad \left(\frac{1}{2}\right)^{5x+10} = 64$

$$\log\left(\frac{1}{2}\right)^{5x+10} = \log 64$$

$$(5x+10)\log\frac{1}{2} = \log 64$$

$$\frac{(5x+10)\log\frac{1}{2}}{\log\frac{1}{2}} = \frac{\log 64}{\log\frac{1}{2}}$$

$$5x+10 = \frac{\log 64}{\log\frac{1}{2}}$$

$$5x+10-10 = -10 + \frac{\log 64}{\log\frac{1}{2}}$$

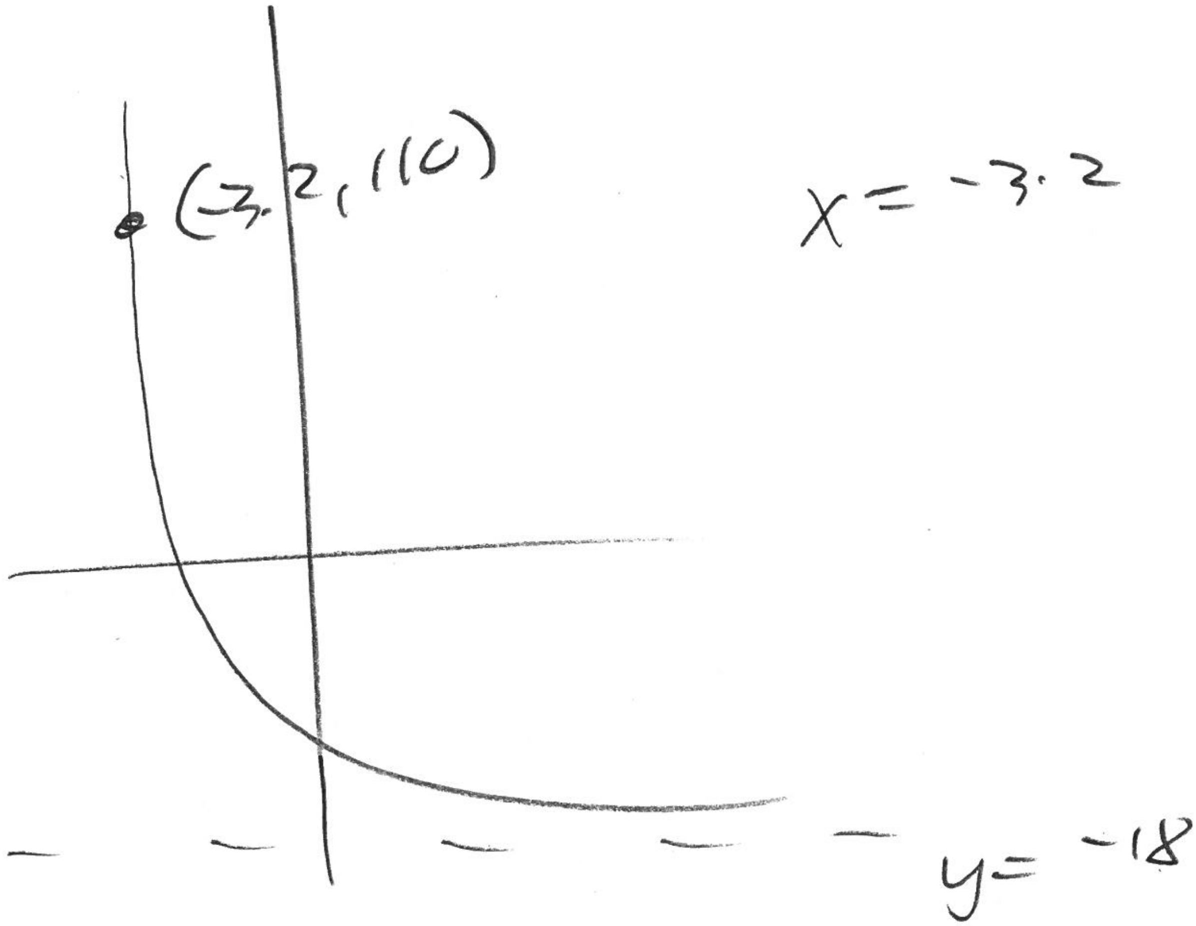
$$5x = -10 + \frac{\log 64}{\log\frac{1}{2}}$$

$$\frac{1}{5}(5x) = \left(-10 + \frac{\log 64}{\log\frac{1}{2}}\right)\frac{1}{5}$$

$$\boxed{\begin{aligned} x &= \frac{-10}{5} + \frac{1}{5}\,\frac{\log 64}{\log\frac{1}{2}} \\ x &= -3.2 \end{aligned}}$$

② graphical

$2\left(\frac{1}{2}\right)^{5x+10} - 18 = -110$



$x = -3.2$

✓✓

$2\left(\frac{1}{2}\right)^{5(-3.2)+10} - 18 =$

$2\left(\frac{1}{2}\right)^{-16+10} - 18 =$

$2\left(\frac{1}{2}\right)^{-6} - 18 =$

$2 \cdot 2^6 - 18 =$

$2^7 =$

$128 - 18 = 110$

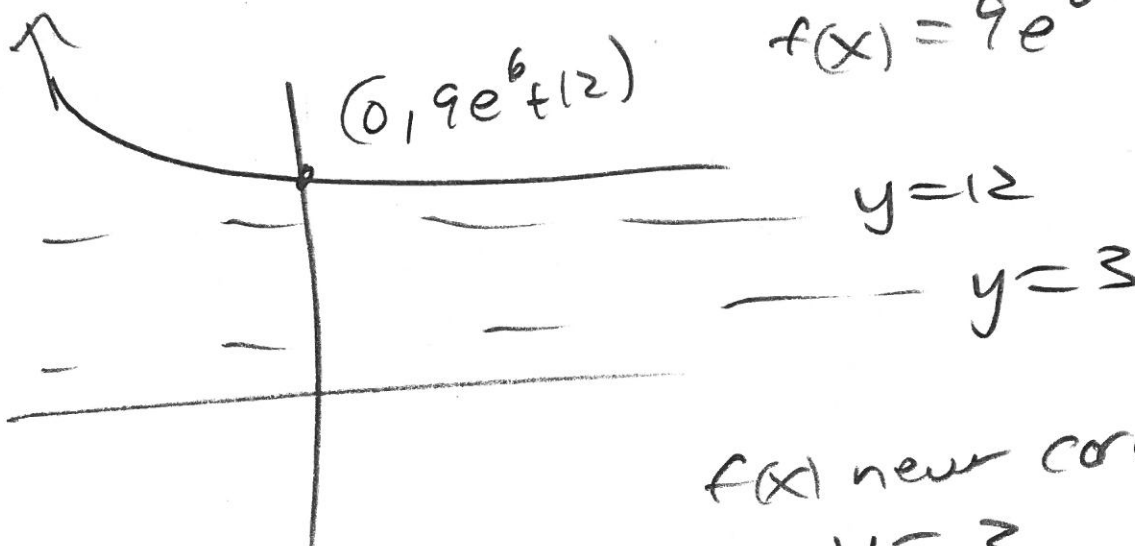
③ $9e^{6-2x}+12=3$

$$\begin{array}{rcl} 9e^{6-2x}+12 & = & 3 \\ -12 & & -12 \\ \hline \dfrac{9e^{6-2x}}{9} & = & \dfrac{-9}{9} \end{array}$$

$$e^{6-2x} = -1$$

no solution? why

$f(x)=9e^{6-2x}+12$



$f(x)$ never corosses $y=3$

or if $e^{6-2x}=-1$ then

$$\ln -1 = 6-2x$$

impossible