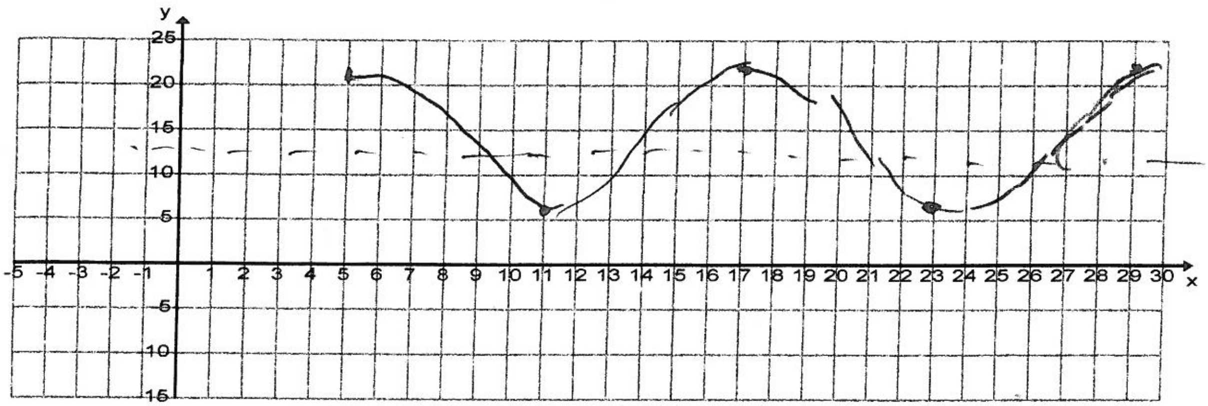
Name VERSION 7.8 Summative Assessment Applications of Sinusoidal Models Period___

amp = |a| Phase Shift midline

You are given the following model: $f(x) = 7.8\cos\left(\frac{2\pi}{12}(x-5)\right) + 13.72$ This model measures the number of hours of daylight a certain city has over the year. x represents the month in the year and f(x) represents the number of hours of daylight this city has.

may to min when $b = \frac{2\pi}{\text{period}}$

1. Sketch and LABEL extreme values as points for the first TWO periods of the model



$13.72 + 7.8$ $13.72 - 7.8$

21.52 5.92

2. Complete the related table

Amplitude of model	Period of model	Midline of the model	Maximum amount of daylight in any month	Minimum amount of daylight in any month
7.8	12	y=13.72	21.52	5.92

3. Circle the months that the amount of daylight is at a MAXIMUM

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

4. Circle the months that the amount of daylight is at a MINIMUM

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

5. Determine when (round to three decimal places) this city has EXACTLY 8 hours of daylight in the FIRST period of this model $X = 5 + \frac{12}{2\pi}\cos^{-1}\left(\frac{8-13.72}{7.8}\right) \approx 9.572$

$$X = 17 - \frac{12}{2\pi}\cos^{-1}\left(\frac{8-13.72}{7.8}\right) \approx 12.428$$

6. Determine when (round to three decimal places) this city has EXACTLY 11 hours of daylight (STATE ALL ANSWERS)

$$X = 12n + 5 + \frac{12}{2\pi}\cos^{-1}\left(\frac{11-13.72}{7.8}\right)$$

$$X = 12n + 17 - \frac{12}{2\pi}\cos^{-1}\left(\frac{11-13.72}{7.8}\right)$$

n is an integer

$X \approx 8.680 + 12n$ provided n is an integers

$X \approx 13.320 + 12n$

Vince is floating in a wave tank on a raft. His friend Brent is sitting outside of the wave tank and notices that his friend's height above the bottom of the pool is sinusoidal in nature. He starts his stopwatch when Vince is closest to the bottom of the pool, 4 meters. He stops his stopwatch after 13 seconds when Vince is at farthest from the bottom of the pool, 16 meters.

$(\underset{\text{seconds}}{x}, \underset{\text{meters above bottom}}{f(x)})$ max $(13, 16)$ min $(0, 4)$

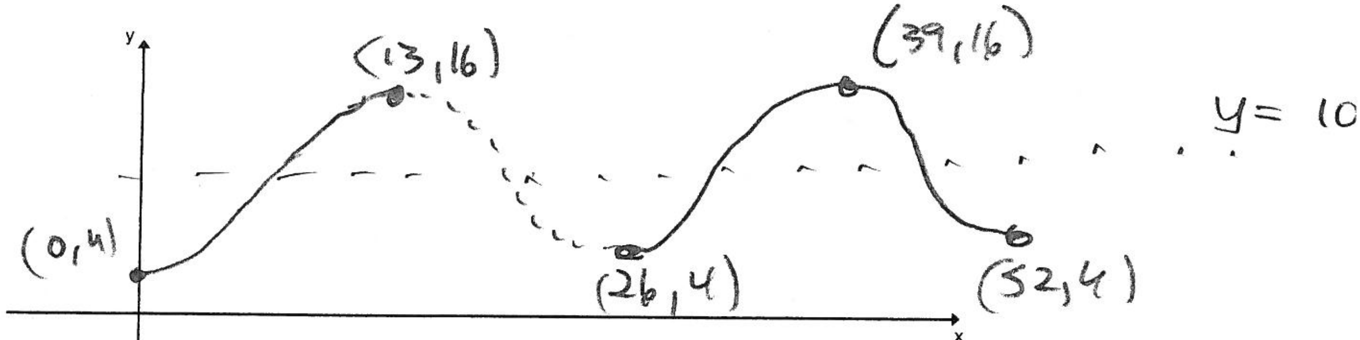
7. Build the model that will predict Jerry's height in reference to the bottom of the pool in terms of seconds and meters

midline $= \frac{16+4}{2} = \frac{20}{2} = 10$ Period $= 2(13) = 26$

amp $= \frac{16-4}{2} = \frac{12}{2} = 6$ $f(x) = -6\cos\left(\frac{2\pi}{26}(x)\right) + 10$

min to max
reflected cosine

8. Sketch and LABEL extreme values as points for the first TWO periods of the model



$f(x) = -6\cos\left(\frac{2\pi}{26}(x)\right) + 10$

9. Determine when (round to three decimal places) Vince is EXACTLY 9 meters from the bottom of the pool in the FIRST period of this model

$x = \frac{26}{2\pi}\cos^{-1}\left(\frac{9-10}{-6}\right) \approx 5.807$

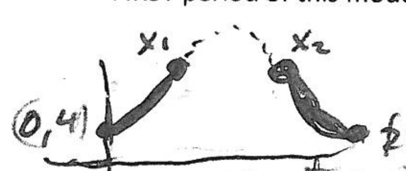
$x = 26 - \frac{26}{2\pi}\cos^{-1}\left(\frac{9-10}{-6}\right) \approx 20.193$

(#10) approx

$0 \le x \le 7.906$

OR $18.094 \le x \le 26$

10. Determine when (round to three decimal places) Vince is AT MOST 12 meters from the bottom of the pool in the FIRST period of this model (Hint: you will need inequality to properly state this answer)



$x_1 = \frac{26}{2\pi}\cos^{-1}\left(\frac{12-10}{-6}\right) \approx 7.906$

$x_2 = 26 - \frac{26}{2\pi}\cos^{-1}\left(\frac{12-10}{-6}\right) \approx 18.094$

exact $\frac{26}{2\pi}\cos\left(\frac{12-10}{6}\right) \le x \le 26 - \frac{26}{2\pi}\cos^{-1}\left(\frac{12-10}{-6}\right)$

11. Determine when (round to three decimal places) Jerry is EXACTLY 7 meters from the bottom of the pool (STATE ALL ANSWERS)

$x_1 = \frac{26}{2\pi}\cos^{-1}\left(\frac{7-10}{-6}\right) \approx 4.333$

$x_2 = 26 - \frac{26}{2\pi}\cos^{-1}\left(\frac{7-10}{-6}\right) \approx 21.667$

approx

$x_1 = 4.333 + 26n$

$x_2 = 21.667 + 26n$

n is an integer

exact $x_1 = 26n + \frac{26}{2\pi}\cos^{-1}\left(\frac{7-10}{-6}\right)$

$x_2 = 26n + 26 - \frac{26}{2\pi}\cos^{-1}\left(\frac{7-10}{-6}\right)$ n is any integer

A mathematics teacher is riding a Ferris wheel to bring awareness to innumeracy in the world. If the gym teacher is helping the mathematics teacher record his data and she noticed that the mathematics teacher was at the minimum height after 9 seconds and again every 32 seconds. The owner of the amusement park is very proud of the fact that his Ferris wheel is the biggest in the area with a diameter of 76 feet. The highest point that the mathematics teacher ever reaches is 85 feet. $x_{min} = 9 + 32n$ n is an integer   diameter = 76

$y_{max} = 85$ Period = 32 $\frac{1}{2}$ period = 16   radius = amp = 38

12. Build the model that will predict the mathematics teacher's height in reference to the ground in terms of seconds and feet

min = max − diameter = 85 − 76 = 9

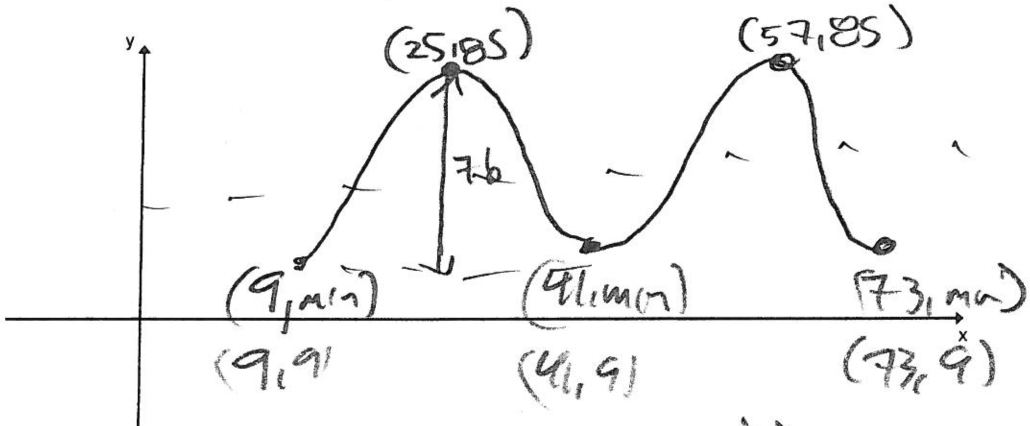
midline = max − amp = 85 − 38 = 47

midline = $\frac{1}{2}$(max + min) = $\frac{1}{2}$(85 + 9) = $\frac{1}{2}$(94) = 47

min/max → −cos

$y = -38\cos\left(\frac{2\pi}{32}(x-9)\right) +$

13. Sketch and LABEL extreme values as points for the first TWO periods of the model



y = 47

$f(x) = -38\cos\left(\frac{2\pi}{32}(x-9)\right) + 47$

14. Determine when (round to three decimal places) the mathematics teacher is EXACTLY 40 feet from the ground in the FIRST period of this model

$x = 9 + \frac{32}{2\pi}\cos^{-1}\left(\frac{40-47}{-38}\right) \approx 16.056$

$x = 41 - \frac{32}{2\pi}\cos^{-1}\left(\frac{40-47}{-38}\right) \approx 33.944$

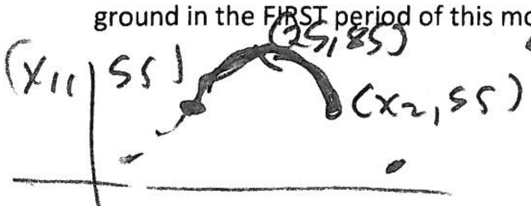
#15 EXACT

$9 + \frac{32}{2\pi}\cos^{-1}\left(\frac{55-47}{-38}\right) \le x \le 41 - \frac{32}{2\pi}\cos^{-1}\left(\frac{55-47}{-38}\right)$

APPROX

$18.080 \le x \le 31.920$

15. Determine when (round to three decimal places) the mathematics teacher is NO less than 55 feet from the ground in the FIRST period of this model (Hint: you will need inequality to properly state this answer)



$x_1 = 9 + \frac{32}{2\pi}\cos^{-1}\left(\frac{55-47}{-38}\right) \approx 18.080$

$x_2 = 41 - \frac{32}{2\pi}\cos^{-1}\left(\frac{55-47}{-38}\right) \approx 31.920$

16. Determine when (round to three decimal places) the mathematics teacher is EXACTLY 19 feet from the ground (STATE ALL ANSWERS)

$x_1 = 9 + \frac{32}{2\pi}\cos^{-1}\left(\frac{19-47}{-38}\right) \approx 12.781$

$x_2 = 41 - \frac{32}{2\pi}\cos^{-1}\left(\frac{19-47}{-38}\right) \approx 37.219$

exact

$x_1 = 32n + 9 + \frac{32}{2\pi}\cos^{-1}\left(\frac{19-47}{-38}\right)$

$x_2 = 32n + 41 - \frac{32}{2\pi}\cos^{-1}\left(\frac{19-47}{-28}\right)$

n is any integer

OR

$x_1 = 12.781 + 32n$

$x_2 = 37.219 + 32n$

n is any integer

Grading Rubric

IF page 1 completely correct THEN you receive a 75	IF pages 1 & 2 completely correct Then you receive an 85	IF pages 1, 2 & 3 completely correct Then you receive 95 IF at least one page has EXACT solutions for time and the supporting work then 100
Errors on each page will be accounted for and grades will be adjusted from 75/85/95/100 as necessary based on the nature and severity of the mistake.		

Extra Credit Questions

1. Name one thing in your life, other than grades or money, that you want to maximize and why

2. Name one thing in your life, other than your weight, that you want to minimize and why

3. If you had the opportunity to get two tickets to any event (sporting event, concert, Hall of Fame Induction, etc.) what would that event be and who would you take?

4. Rate 2022, so far on a scale of 1 to 10 (1 = not so good and 10 = being awesome) ________

5. Since the school board has decided to take two weeks of the summer away from the returning students, what will you this summer differently? If you are a senior, then explain what you will be doing during the first 12 days of August this year.