Name VERSION - 6.5 Summative Assessment Applications of Sinusoidal Models Period___

|-6.5| = amp midline

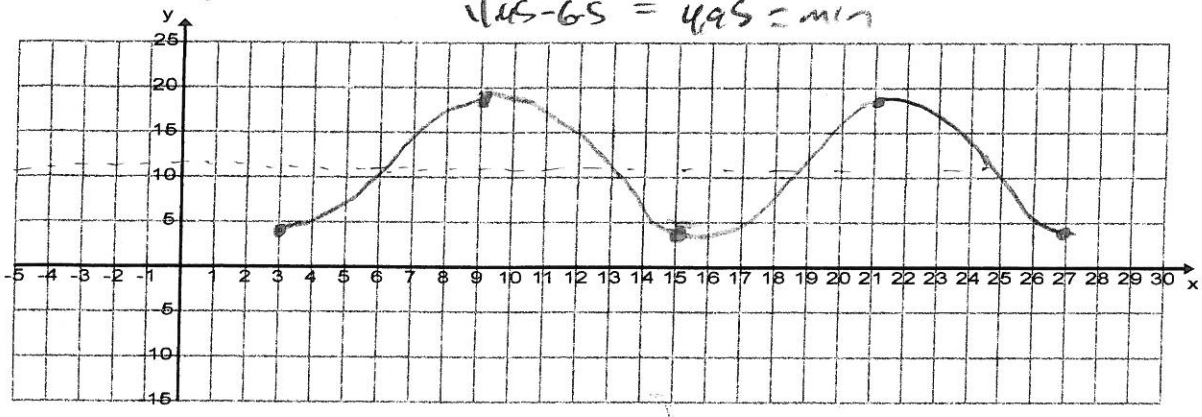
You are given the following model: $f(x) = -6.5\cos\left(\frac{2\pi}{12}(x-3)\right) + 11.45$ This model measures the number of hours of daylight a certain city has over the year. x represents the month in the year and f(x) represents the number of hours of daylight this city has.

period when $b = \frac{2\pi}{12}$

1. Sketch and LABEL extreme values as points for the first TWO periods of the model

midline $y = 11.45$ $11.45 + 6.5 = 17.95$ = max

$11.45 - 6.5 = 4.95$ = min



2. Complete the related table

Amplitude of model	Period of model	Midline of the model	Maximum amount of daylight in any month	Minimum amount of daylight in any month
6.5	12	$y = 11.45$	17.95	4.95

3. Circle the months that the amount of daylight is at a MAXIMUM

1 2 3 4 5 6 7 8 (9) 10 11 12 13 14 15 16 17 18 19 20 (21) 22 23 24

4. Circle the months that the amount of daylight is at a MINIMUM

1 2 (3) 4 5 6 7 8 9 10 11 12 13 14 (15) 16 17 18 19 20 21 22 23 24

5. Determine when (round to three decimal places) this city has EXACTLY 12 hours of daylight in the FIRST period of this model

$x = 3 + \frac{12}{2\pi}\cos^{-1}\left(\frac{12-11.45}{-6.5}\right) \approx 6.162$

$x = 15 - \frac{12}{2\pi}\cos^{-1}\left(\frac{12-11.45}{-6.5}\right) \approx 11.838$

6. Determine when (round to three decimal places) this city has EXACTLY 16 hours of daylight (STATE ALL ANSWERS)

$x = 3 + \frac{12}{2\pi}\cos^{-1}\left(\frac{16-11.45}{-6.5}\right) \approx 7.481$

$x = 15 - \frac{12}{2\pi}\cos^{-1}\left(\frac{16-11.45}{-6.5}\right) \approx 10.519$

Scroll

$x = \begin{cases} 12n + 3 + \frac{12}{2\pi}\cos^{-1}\left(\frac{16-11.45}{-6.5}\right) \\ 12n + 15 - \frac{12}{2\pi}\cos^{-1}\left(\frac{16-11.45}{-6.5}\right) \end{cases}$ $\times \begin{cases} \text{original } 12n \\ \text{original } 12n \end{cases}$

n is an integer

↓ b reflected cosine

Mickey is floating in a wave tank on a raft. His friend Phil is sitting outside of the wave tank and notices that his friend's height above the bottom of the pool is sinusoidal in nature. He starts his stopwatch when MIckey is closest to the bottom of the pool, 3 meters. He stops his stopwatch after 11 seconds when Mickey is at farthest from the bottom of the pool, 15 meters. min (0,3) max (11,15) $(x, f(x)) = (\text{time since min}, \text{height above bottom of pool})$

7. Build the model that will predict Mickey's height in reference to the bottom of the pool in terms of seconds and meters

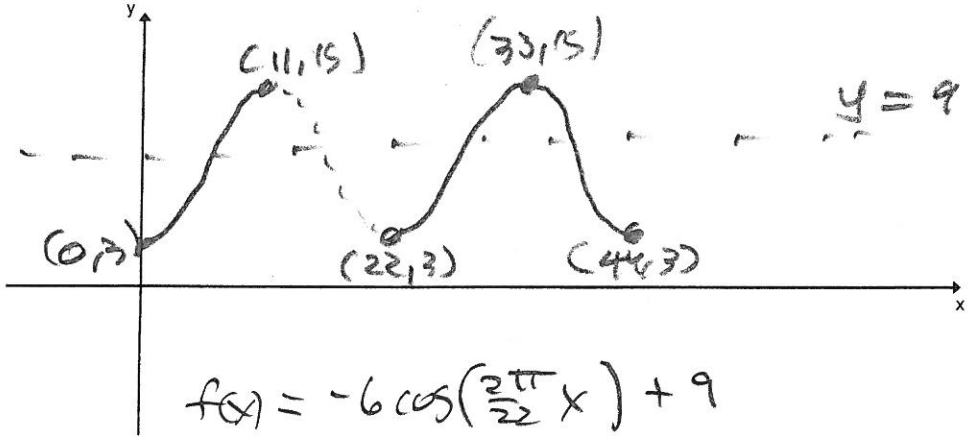
$f(x) = -6\cos\left(\frac{2\pi}{22}(x)\right) + 9$

$amp = \frac{1}{2}(max - min) = \frac{1}{2}(15-3) = \frac{1}{2}(12) = 6$

$midline = \frac{1}{2}(max + min) = \frac{1}{2}(15+3) = \frac{1}{2}(18) = 9$

$\frac{1}{2}$ period = 11sec period 22sec

8. Sketch and LABEL extreme values as points for the first TWO periods of the model



$f(x) = -6\cos\left(\frac{2\pi}{22}x\right) + 9$

9. Determine when (round to three decimal places) Mickey is EXACTLY 10 meters from the bottom of the pool in the FIRST period of this model

$x_1 = \frac{22}{2\pi}\cos^{-1}\left(\frac{10-9}{-6}\right) \approx 6.086$

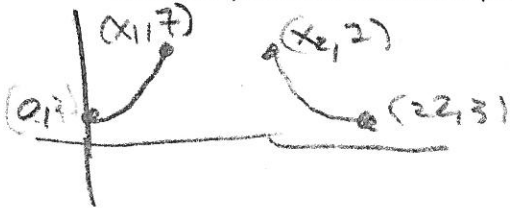
$x_2 = 22 - \frac{22}{2\pi}\cos^{-1}\left(\frac{10-9}{-6}\right) \approx 14.914$

#10 Exact

$0 \le x \le \frac{22}{2\pi}\cos^{-1}\left(\frac{7-9}{-6}\right)$ OR

$22 - \frac{22}{2\pi}\cos^{-1}\left(\frac{7-9}{-6}\right) \le x \le 22$

10. Determine when (round to three decimal places) Mickey is AT MOST 7 meters from the bottom of the pool in the FIRST period of this model (Hint: you will need inequality to properly state this answer) APPROX



$x_1 = \frac{22}{2\pi}\cos^{-1}\left(\frac{7-9}{-6}\right) \approx 4.310$

$x_2 = 22 - \frac{22}{2\pi}\cos^{-1}\left(\frac{7-9}{-6}\right) \approx 17.690$

$0 \le x \le 4.310$ OR $17.690 \le x \le 22$

11. Determine when (round to three decimal places) Mickey is EXACTLY 8 meters from the bottom of the pool (STATE ALL ANSWERS)

$x_1 = \frac{22}{2\pi}\cos^{-1}\left(\frac{8-9}{-6}\right) \approx 4.914$

$x_2 = 22 - \frac{22}{2\pi}\cos^{-1}\left(\frac{8-9}{-6}\right) \approx 17.086$

exact $x_1 = 22n + \frac{22}{2\pi}\cos^{-1}\left(\frac{8-9}{-6}\right)$

$x_2 = 22n + 22 - \frac{22}{2\pi}\cos^{-1}\left(\frac{8-9}{-6}\right)$

n is an integer

approx $x_1 \approx 4.914 + 22n$ n is an integer

$x_2 \approx 17.086 + 22n$ integer

A mathematics teacher is riding a Ferris wheel to bring awareness to innumeracy in the world. If the gym teacher is helping the mathematics teacher record his data and she noticed that the mathematics teacher was at the minimum height after 8 seconds and again every 56 seconds. The owner of the amusement park is very proud of the fact that his Ferris wheel is the biggest in the area with a diameter of 92 feet. The highest point that the mathematics teacher ever reaches is 100 feet. (8 min (64, min) Period 56 diameter 92 max 100 ½ Period 28 radius 46 amp 46

12. Build the model that will predict the mathematics teacher's height in reference to the ground in terms of seconds and feet

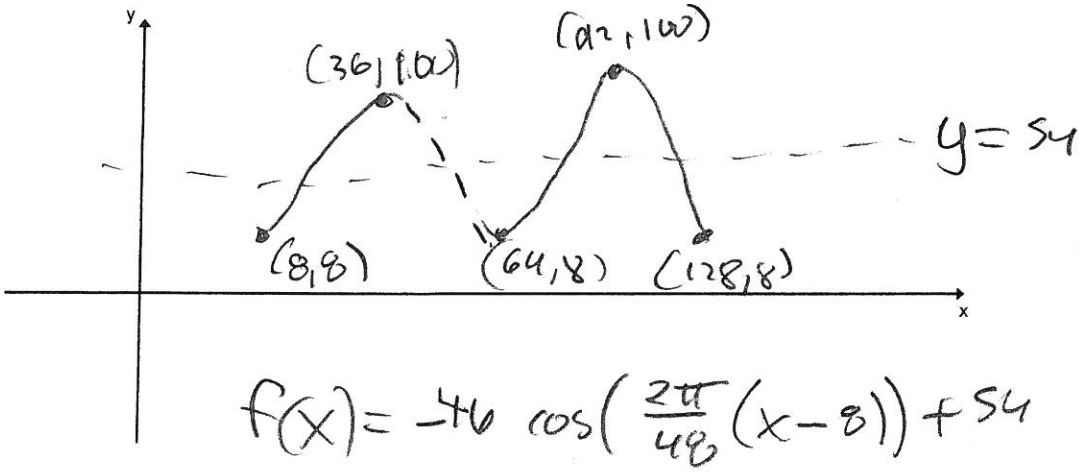
$a<0$ $a=-46$ amp 46 $b=\frac{2\pi}{56}$ ← PL

PHASE shift 8 middle 54 $f(x)=-46\cos\left(\frac{2\pi}{56}(x-8)\right)+54$

midline: max − amp = 100 − 46 = 54

min: max − diam = 100 − 92 = 8

13. Sketch and LABEL extreme values as points for the first TWO periods of the model



$f(x)=-46\cos\left(\frac{2\pi}{48}(x-8)\right)+54$

14. Determine when (round to three decimal places) the mathematics teacher is EXACTLY 65 feet from the ground in the FIRST period of this model

$x_1=8+\frac{56}{2\pi}\cos^{-1}\left(\frac{65-54}{-46}\right)\approx 24.152$

$x_2=64-\frac{56}{2\pi}\cos^{-1}\left(\frac{65-54}{-46}\right)\approx 47.848$

15. Determine when (round to three decimal places) the mathematics teacher is NO more than 35 feet from the ground in the FIRST period of this model (Hint: you will need inequality to properly state this answer)

(x, 35) (x, 35) (64, 8) (8, 8)

Exact $8\le x\le 8+\frac{56}{2\pi}\left(\cos^{-1}\left(\frac{35-54}{-46}\right)\right)$ OR $64-\frac{56}{2\pi}\cos^{-1}\left(\frac{45-54}{-46}\right)\le x\le 64$

Approx $8\le x\le 18.205$ or $53.795\le x\le 64$

16. Determine when (round to three decimal places) the mathematics teacher is EXACTLY 18 feet from the ground (STATE ALL ANSWERS)

Exact $x_1=56n+8+\frac{56}{2\pi}\cos^{-1}\left(\frac{18-54}{-46}\right)$ Approx $x_1=13.989+56n$

$x_2=56n+64-\frac{56}{2\pi}\cos^{-1}\left(\frac{18-54}{-46}\right)$ $x_2\approx 58.011+56n$

n is any integer n is any integer

Grading Rubric

IF page 1 completely correct THEN you receive a 75	IF pages 1 & 2 completely correct Then you receive an 85	IF pages 1, 2 & 3 completely correct Then you receive 95 IF at least one page has EXACT solutions for time and the supporting work then 100
Errors on each page will be accounted for and grades will be adjusted from 75/85/95/100 as necessary based on the nature and severity of the mistake.		

Extra Credit Questions

1. Name one thing in your life, other than grades or money, that you want to maximize and why

2. Name one thing in your life, other than your weight, that you want to minimize and why

3. If you had the opportunity to get two tickets to any event (sporting event, concert, Hall of Fame Induction, etc.) what would that event be and who would you take?

4. Rate 2022, so far on a scale of 1 to 10 (1 = not so good and 10 = being awesome) _____

5. Since the school board has decided to take two weeks of the summer away from the returning students, what will you this summer differently? If you are a senior, then explain what you will be doing during the first 12 days of August this year.