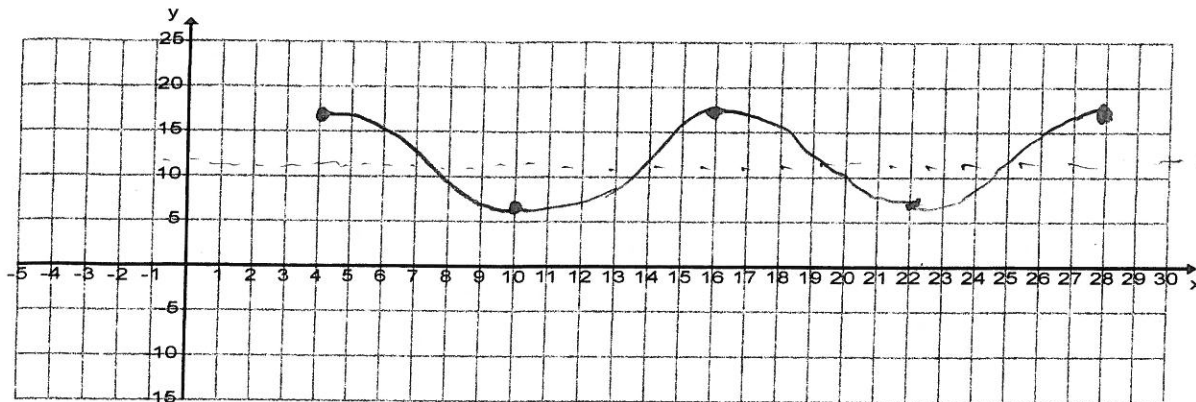


You are given the following model:  $f(x) = 5.2 \cos\left(\frac{2\pi}{12}(x - 4)\right) + 12.28$  This model measures the number of hours of daylight a certain city has over the year.  $x$  represents the month in the year and  $f(x)$  represents the number of hours of daylight this city has.

*Annotations:*  
 - Amplitude: 5.2  
 - Phase Shift: 4  
 - Period: 12  
 - Midline:  $y = 12.28$

1. Sketch and LABEL extreme values as points for the first TWO periods of the model



$b = \frac{2\pi}{\text{Period}}$

2. Complete the related table

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Amplitude of model	Period of model	Midline of the model	Maximum amount of daylight in any month	Minimum amount of daylight in any month
5.2	12	$y = 12.28$	$12.28 + 5.2$ 17.48	$12.28 - 5.2$ 7.08

3. Circle the months that the amount of daylight is at a MAXIMUM

1 2 3 **4** 5 6 7 8 9 10 11 12 13 14 15 **16** 17 18 19 20 21 22 23 24

4. Circle the months that the amount of daylight is at a MINIMUM

1 2 3 4 5 6 7 8 9 **10** 11 12 13 14 15 16 17 18 19 20 21 **22** 23 24

5. Determine when (round to three decimal places) this city has EXACTLY 12 hours of daylight in the FIRST period of this model

$$x = 4 + \frac{12}{2\pi} \cos^{-1}\left(\frac{12 - 12.28}{5.2}\right) \approx 7.103$$

$$x = 16 - \frac{12}{2\pi} \cos^{-1}\left(\frac{12 - 12.28}{5.2}\right) \approx 12.897$$

6. Determine when (round to three decimal places) this city has EXACTLY 16 hours of daylight (STATE ALL ANSWERS)

$$x = 4 + \frac{12}{2\pi} \cos^{-1}\left(\frac{16 - 12.28}{5.2}\right) + 12n$$

$$x = 16 - \frac{12}{2\pi} \cos^{-1}\left(\frac{16 - 12.28}{5.2}\right) + 12n$$

$x \approx 5.478 + 12n$

$n$  is an integer  $x \approx 14.522 + 12n$

with  $n$  is an integer

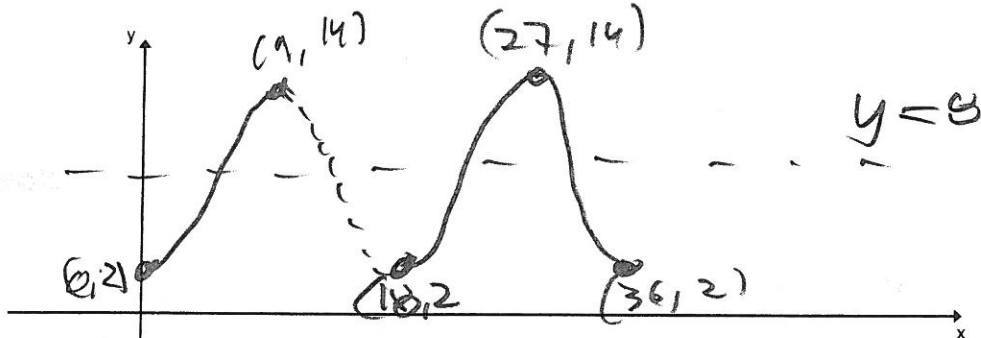
Jerry is floating in a wave tank on a raft. His friend Billy is sitting outside of the wave tank and notices that his friend's height above the bottom of the pool is sinusoidal in nature. He starts his stopwatch when Jerry is closest to the bottom of the pool, 2 meters. He stops his stopwatch after 9 seconds when Jerry is at farthest from the bottom of the pool, 14 meters.  $(0, 2)$  min  $(9, 14)$  max  $\frac{1}{2}$  Period = 9 Period = 18

7. Build the model that will predict Jerry's height in reference to the bottom of the pool in terms of seconds and meters

midline  $\frac{\max + \min}{2} = \frac{14 + 2}{2} = \frac{16}{2} = 8$   $a = -\text{amp}$   $\curvearrowright$  reflected cosine

amp  $= \frac{\max - \min}{2} = \frac{14 - 2}{2} = \frac{12}{2} = 6$   $f(x) = -6 \cos\left(\frac{2\pi}{18}x\right) + 8$

8. Sketch and LABEL extreme values as points for the first TWO periods of the model



$f(x) = -6 \cos\left(\frac{2\pi}{18}x\right) + 8$   
 $\hookrightarrow$  seconds since min  
 m above bottom

9. Determine when (round to three decimal places) Jerry is EXACTLY 12 meters from the bottom of the pool in the FIRST period of this model

$x_1 = \frac{18}{2\pi} \cos^{-1}\left(\frac{12-8}{6}\right) \approx 6.591$

$x_2 = 18 - \frac{18}{2\pi} \cos^{-1}\left(\frac{12-8}{6}\right) \approx 11.409$

approx  $0 \leq x \leq 4.5$  OR  $13.5 \leq x \leq 18$

10. Determine when (round to three decimal places) Jerry is AT MOST 8 meters from the bottom of the pool in the FIRST period of this model (Hint: you will need inequality to properly state this answer)

$x_1 = \frac{18}{2\pi} \cos^{-1}\left(\frac{8-8}{6}\right) \approx 4.5$

$x_2 = 18 - \frac{18}{2\pi} \cos^{-1}\left(\frac{8-8}{6}\right) \approx 13.5$

exact  $0 \leq x \leq \frac{18}{2\pi} \cos^{-1}\left(\frac{8-8}{6}\right)$  OR  $18 - \frac{18}{2\pi} \cos^{-1}\left(\frac{8-8}{6}\right) \leq x \leq 18$

11. Determine when (round to three decimal places) Jerry is EXACTLY 6 meters from the bottom of the pool

(STATE ALL ANSWERS)

$x_1 = \frac{18}{2\pi} \cos^{-1}\left(\frac{6-8}{6}\right) \approx 3.526$

$x_2 = 18 - \frac{18}{2\pi} \cos^{-1}\left(\frac{6-8}{6}\right) \approx 14.474$

exact  $x_1 = 18n + \frac{18}{2\pi} \cos^{-1}\left(\frac{6-8}{6}\right)$

$x_2 = 18n + 18 - \frac{18}{2\pi} \cos^{-1}\left(\frac{6-8}{6}\right)$

$n$  is any integer

approx

$x_1 = 3.526 + 18n$

$x_2 = 14.474 + 18n$

$n$  is any integer

Min max model  $a < 0$

A mathematics teacher is riding a Ferris wheel to bring awareness to innumeracy in the world. If the gym teacher is helping the mathematics teacher record his data and she noticed that the mathematics teacher was at the minimum height after 6 seconds and again every 48 seconds. The owner of the amusement park is very proud of the fact that his Ferris wheel is the biggest in the area with a diameter of 82 feet. The highest point that the mathematics teacher ever reaches is 90 feet.

PHASE SHIFT 6

min (6, min)

Per. tot 48

max = 90

amp = 41

min = (54, min)

$\frac{1}{2}$  Per. 24

diameter = 82

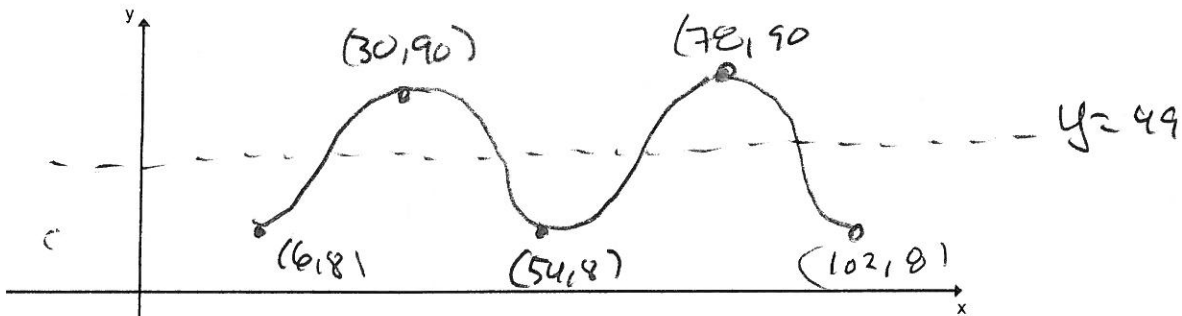
radius = 41

12. Build the model that will predict the mathematics teacher's height in reference to the ground in terms of seconds and feet

$$f(x) = -41 \cos\left(\frac{2\pi}{48}(x-6)\right) + 8$$

$$\begin{aligned} d &= \text{max} - \text{amp} \\ &= 90 - 41 \\ &= 49 \\ \text{min} &= \text{max} \\ &\quad - \text{diameter} \\ &= 90 - 82 \\ &= 8 \end{aligned}$$

13. Sketch and LABEL extreme values as points for the first TWO periods of the model



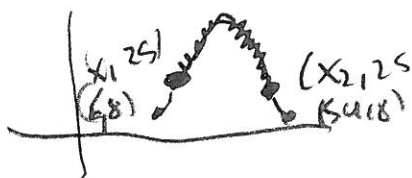
$$f(x) = -41 \cos\left(\frac{2\pi}{48}(x-6)\right) + 8$$

14. Determine when (round to three decimal places) the mathematics teacher is EXACTLY 75 feet from the ground in the FIRST period of this model

$$x_1 = 6 + \frac{48}{2\pi} \cos^{-1}\left(\frac{75-49}{-41}\right) \approx 23.248$$

$$x_2 = 54 - \frac{48}{2\pi} \cos^{-1}\left(\frac{75-49}{-41}\right) \approx 36.752$$

15. Determine when (round to three decimal places) the mathematics teacher is NO less than 25 feet from the ground in the FIRST period of this model (Hint: you will need inequality to properly state this answer)



Exact

$$6 + \frac{48}{2\pi} \cos^{-1}\left(\frac{25-49}{-41}\right) \leq x \leq 54 - \frac{48}{2\pi} \cos^{-1}\left(\frac{25-49}{-41}\right)$$

Approx

$$13.223 \leq x \leq 46.777$$

16. Determine when (round to three decimal places) the mathematics teacher is EXACTLY 14 feet from the ground (STATE ALL ANSWERS)

Exact

$$x_1 = 48n + 6 + \frac{48}{2\pi} \cos^{-1}\left(\frac{14-49}{-41}\right)$$

$$x_2 = 48n + 54 - \frac{48}{2\pi} \cos^{-1}\left(\frac{14-49}{-41}\right)$$

Approx

$$x_1 \approx 10.185 + 48n$$

$$x_2 \approx 49.815 + 48n$$

## Grading Rubric

IF page 1 completely correct THEN you receive a 75	IF pages 1 & 2 completely correct Then you receive an 85	IF pages 1, 2 & 3 completely correct Then you receive 95 IF at least one page has EXACT solutions for time and the supporting work then 100
Errors on each page will be accounted for and grades will be adjusted from 75/85/95/100 as necessary based on the nature and severity of the mistake.		

### Extra Credit Questions

1. Name one thing in your life, other than grades or money, that you want to maximize and why
  
  
  
  
  
  
  
  
  
  
2. Name one thing in your life, other than your weight, that you want to minimize and why
  
  
  
  
  
  
  
  
  
  
3. If you had the opportunity to get two tickets to any event (sporting event, concert, Hall of Fame Induction, etc.) what would that event be and who would you take?
  
  
  
  
  
  
  
  
  
  
4. Rate 2022, so far on a scale of 1 to 10 ( 1 = not so good and 10 = being awesome) \_\_\_\_\_
  
  
  
  
  
  
  
  
  
  
5. Since the school board has decided to take two weeks of the summer away from the returning students, what will you this summer differently? If you are a senior, then explain what you will be doing during the first 12 days of August this year.