

## Expanding and Condensing Logarithms

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**Expand each logarithm. Justify each step by stating logarithm property used.****Level 2:**

1)  $\log_7 \sqrt[3]{10}$

2)  $\log_9 11^5$

3)  $\log_8 \frac{u}{v}$

4)  $\log_3 \sqrt[3]{x}$

5)  $\ln x^3$

6)  $\log_8 (x \cdot y)$

**Level 3:**

7)  $\log_3 \left(\frac{x}{y}\right)^4$

8)  $\log_4 \frac{8^4}{7}$

9)  $\log_4 \left(\frac{7}{12}\right)^5$

10)  $\log_6 (a \cdot b \cdot c)$

11)  $\log_5 \frac{x^5}{y}$

12)  $\log_6 \sqrt[3]{u^2}$

**Level 4:**

13)  $\log_9 \left(\frac{x^5}{y}\right)^6$

14)  $\log_8 (x \cdot y \cdot z^6)$

15)  $\ln (x \cdot y \cdot z^6)$

16)  $\log_8 (w^3 \sqrt[3]{u})$

17)  $\log_7 \left(\frac{a^5}{b}\right)^4$

18)  $\log_2 (u^3 v^4)$

**Condense each expression to a single logarithm. Justify each step by stating the logarithm property used.****Level 2:**

19)  $\frac{\ln x}{3}$

20)  $\log_4 x - \log_4 y$

21)  $2 \ln a$

22)  $\log_5 u - \log_5 v$

23)  $6 \log_6 7$

24)  $\log_5 x + \log_5 y$

**Level 3:**

25)  $3 \log_2 x - 3 \log_2 y$

26)  $5 \log u - \log v$

27)  $3 \log_7 a - 3 \log_7 b$

28)  $\log_7 12 - 5 \log_7 5$

29)  $5 \log_9 6 - 5 \log_9 11$

30)  $\log_8 x - 5 \log_8 y$

**Level 4:**

31)  $30 \log a + 6 \log b$

32)  $\log_3 c + \frac{\log_3 a}{2} + \frac{\log_3 b}{2}$

33)  $\log_5 z + \frac{\log_5 x}{2} + \frac{\log_5 y}{2}$

34)  $6 \ln x - 36 \ln y$

35)  $5 \log_3 x - 2 \log_3 y$

36)  $3 \log_6 u - 18 \log_6 v$

Alg 2 Task 7.2 ODDS

$$\begin{aligned} \textcircled{1} \log_7 \sqrt[3]{10} &= \log_7 (10)^{1/3} && \text{Exponent Law} \\ &= \frac{1}{3} \log_7 10 && \text{Power Rule} \\ &= \frac{1}{3} [\log_7 (2 \cdot 5)] \\ &= \frac{1}{3} [\log_7 2 + \log_7 5] \\ &= \frac{1}{3} \log_7 2 + \frac{1}{3} \log_7 5 && \leftarrow \text{Prime Notation} \end{aligned}$$

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$$\textcircled{3} \log_8 \frac{4}{v} = \log_8 4 - \log_8 v \quad \text{Quotient Law}$$

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$$\textcircled{5} \ln x^3 = 3 \ln x \quad \text{Power Rule}$$

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$$\begin{aligned} \textcircled{7.1} \log_3 \left(\frac{x}{y}\right)^4 &= 4 \log_3 \left(\frac{x}{y}\right) && \text{Power Rule} \\ &= 4 [\log_3 x - \log_3 y] && \text{Quotient Rule} \\ &= 4 \log_3 x - 4 \log_3 y \end{aligned}$$

$$\begin{aligned} \textcircled{7.2} \log_3 \frac{x^4}{y^4} &= \log_3 x^4 - \log_3 y^4 && \text{Quotient} \\ &= \boxed{4 \log_3 x - 4 \log_3 y} && \text{Power} \end{aligned}$$

$$\textcircled{9.1} \log_4 \left( \frac{7}{12} \right)^5 = 5 \log_4 \left( \frac{7}{12} \right) \text{ Power Rule}$$

$$= 5 [\log_4 [7] - \log_4 [12]] \text{ Quotient Rule}$$

$$= \boxed{5 \log_4 7 - 5 \log_4 12} \text{ Quotient Rule}$$

$$\textcircled{9.2} \log_4 \left( \frac{7^5}{12^5} \right) = \log_4 7^5 - \log_4 12^5 \leftarrow \text{exponent laws}$$

$$= \boxed{5 \log_4 7 - 5 \log_4 12} \text{ Power Rule}$$

$$\textcircled{11} \log_5 \left( \frac{x^5}{y} \right) = \log_5 x^5 - \log_5 y \text{ Quotient Power}$$

$$= \boxed{5 \log_5 x - \log_5 y}$$

$$\textcircled{13} \log_9 \left( \frac{x^5}{y} \right)^6 = \log_9 \left( \frac{x^{30}}{y^6} \right) \text{ Exponent laws}$$

$$= \log_9 x^{30} - \log_9 y^6 \text{ Quotient Power}$$

$$= \boxed{30 \log_9 x - 6 \log_9 y} \text{ Power}$$

$$(15) \ln(x \cdot y \cdot z^6)$$

$$\ln(xy) + \ln z^6 \quad \text{Product Rule}$$

$$\ln x + \ln y + \ln z^6 \quad \text{Product Rule}$$

$$\boxed{\ln x + \ln y + 6 \ln z} \quad \text{Power Rule}$$

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$$(17) \log_7 \left( \frac{a^5}{b} \right)^4 = \log_7 \left( \frac{a^{20}}{b^4} \right) \quad \text{Exponent Laws}$$

$$= \log_7 a^{20} - \log_7 b^4 \quad \text{Quotient Rule}$$

$$\boxed{20 \log_7 a - 4 \log_7 b}$$

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$$(17.2) \log_7 \left( \frac{a^5}{b} \right)^4 = 4 \log_7 \left( \frac{a^5}{b} \right) \quad \text{Power}$$

$$= 4 [\log_7 a^5 - \log_7 b] \quad \text{Quotient}$$

$$= 4 [5 \log_7 a - \log_7 b] \quad \text{Power}$$

$$\boxed{20 \log_7 a - 4 \log_7 b}$$

$$\textcircled{19} \quad \frac{\ln x}{3} = \frac{1}{3} \ln x = \ln x^{\frac{1}{3}} \quad \text{Power Rule}$$

algebra

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$$\textcircled{21} \quad 2 \ln a = \ln a^2 \quad \text{Power Rule}$$

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$$\textcircled{23} \quad 6 \log_6 7 = \log_6 7^6 \quad \text{Power Rule}$$

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$$\textcircled{25.1} \quad \begin{aligned} & 3 \log_2 x - 3 \log_2 y && \text{Power Rule} \\ & \log_2 x^3 - \log_2 y^3 && \\ & \log_2 \left( \frac{x^3}{y^3} \right) && \text{Quotient Rule} \end{aligned}$$

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$$\textcircled{25.2} \quad \begin{aligned} & 3 \log_2 x - 3 \log_2 y && \\ & 3(\log_2 x - \log_2 y) && \text{Dist. Prop} \\ & 3 \left( \log_2 \frac{x}{y} \right) && \text{Quotient Rule} \\ & \log_2 \left( \frac{x}{y} \right)^3 && \text{Power Rule} \\ & \log_2 \frac{x^3}{y^3} && \text{Exponent Law} \end{aligned}$$

$$(27.1) \quad 3 \log_7 9 - 3 \log_7 b$$

$$\log_7 9^3 - \log_7 b^3$$

Power Rule

Quotient Rule

$$\boxed{\log_7 \left( \frac{9^3}{b^3} \right)}$$

(27.2)

$$3 \log_7 9 - 3 \log_7 b$$

$$3(\log_7 9 - \log_7 b)$$

Dist Prop

Quotient Rule

$$3 \left( \log_7 \frac{9}{b} \right)$$

Power Rule

$$\log_7 \left( \frac{9}{b} \right)^3$$

Exponent Law

$$\boxed{\log_7 \left( \frac{9^3}{b^3} \right)}$$

$$\textcircled{29.1} \quad 5 \log_9 6 - 5 \log_9 11$$

$$\log_9 6^5 - \log_9 11^5$$

Power Rule

Quotient Rule

$$\boxed{\log_9 \left( \frac{6^5}{11^5} \right)}$$

$$\textcircled{29.2} \quad 5 \log_9 6 - 5 \log_9 11$$

$$5(\log_9 6 - \log_9 11)$$

Dist. Prop

Quotient Rule

$$5 \left( \log_9 \left( \frac{6}{11} \right) \right)$$

$$\log_9 \left( \frac{6}{11} \right)^5$$

Power Rule

$$\boxed{\log_9 \frac{6^5}{11^5}}$$

$$(31) \quad 30 \log a + 6 \log b$$

$$\log a^{30} + \log b^6$$

Power Rule

$$\boxed{\log \left( \frac{a^{30}}{b^6} \right)}$$

Quotient Rule

$$(33) \quad \log_5 z + \frac{\log_5 x}{z} + \frac{\log_5 y}{z}$$

$$\log_5 z + \frac{1}{z} \log_5 x + \frac{1}{z} \log_5 y \quad (\text{Alg})$$

$$\log_5 z + \log_5 x^{\frac{1}{z}} + \log_5 y^{\frac{1}{z}} \quad \text{Power Rule}$$

$$\log_5 (z x^{\frac{1}{z}} y^{\frac{1}{z}}) \quad \text{Product Rule}$$

$$\boxed{\log_5 z \sqrt{xy}}$$



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$$5 \log_3 X - 2 \log_3 Y$$

$$\log_3 X^5 - \log_3 Y^2$$

Power Rule

quotient Rule

$$\log_3 \left( \frac{X^5}{Y^2} \right)$$