

Expanding and Condensing Logarithms

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Date _____ Period _____

Expand each logarithm. Justify each step by stating logarithm property used.**Level 2:**

1) $\log_6 \frac{u}{v}$

2) $\log_5 \sqrt[3]{a}$

3) $\log_7 5^4$

4) $\log_4 u^6$

5) $\log(a \cdot b)$

6) $\log_5 \frac{6}{7}$

Level 3:

7) $\log_4 \sqrt{x^3}$

8) $\log_6 (3 \cdot 11)^6$

9) $\log_6 (ab^3)$

10) $\log_4 (a \cdot b \cdot c)$

11) $\log_5 (10 \cdot 11^3)$

12) $\log_7 (x \cdot y)^6$

Level 4:

13) $\log_2 (x^3 \cdot y)^3$

14) $\log_3 (z^4 \sqrt{x})$

15) $\log_9 (z\sqrt{x \cdot y})$

16) $\log_8 \left(\frac{a}{b^4}\right)^5$

17) $\log_8 (x^3 \cdot y)^2$

18) $\log_2 \left(\frac{a}{b^4}\right)^2$

Condense each expression to a single logarithm. Justify each step by stating the logarithm property used.**Level 2:**

19) $6\log_5 10$

20) $\frac{\log x}{3}$

21) $\log_7 u - \log_7 v$

22) $\log_6 x - \log_6 y$

23) $\log_4 2 + \log_4 7$

24) $\log_3 a + \log_3 b$

Level 3:

25) $5\log_7 11 - \log_7 8$

26) $\log_3 x + 2\log_3 y$

27) $2\log_8 x$

28) $2\log_9 a$

29) $\log u + \log v + \log w$

30) $\log_2 12 + \log_2 7 + \log_2 5$

Level 4:

31) $\frac{\log_5 x}{2} + \frac{\log_5 y}{2} + \frac{\log_5 z}{2}$

32) $3\log_6 a - 6\log_6 b$

33) $\frac{\log_7 x}{3} + \frac{\log_7 y}{3} + \frac{\log_7 z}{3}$

34) $3\log_4 a - 3\log_4 b$

35) $3\log_3 u + 15\log_3 v$

36) $3\log_5 u + 12\log_5 v$

Alg 7-2 task 7-2 odds

① $\log_6 \frac{u}{v} = \log_6 u - \log_6 v$ Quotient Law

② $\log_7 S^4 = 4 \log_7 S$ Power Rule

③ $\log(a \cdot b) = \log a + \log b$ Product Rule

④ $\log_4 \sqrt{x^3} = \log_4 x^{3/2}$ Exponent Laws

$= \boxed{\frac{3}{2} \log_4 x}$ Power Rule

⑤ $\log_6 (a b^3) = \log_6 a + \log_6 b^3$ Product Rule

$= \boxed{\log_6 a + 3 \log_6 b}$ Power Rule

⑥ $\log_5 (10 \cdot 11^3) = \log_5 10 + \log_5 11^3$ Product Rule

$= \boxed{\log_5 10 + 3 \log_5 11}$ Power Rule

But

$\log_5 10 = \log_5 (5 \cdot 2)$
 $= \log_5 5 + \log_5 2$
 $= 1 + \log_5 2$

$= \log_5 (5 \cdot 2) + 3 \log_5 11$ Alg

$= \log_5 5 + \log_5 2 + 3 \log_5 11$ Product

$= \boxed{1 + \log_5 2 + 3 \log_5 11}$ log of a base

best form ↑

$$\begin{aligned} \textcircled{12.1} \quad \log_2 (x^3 y)^3 &= \log_2 x^9 y^3 && \text{Exponent} \\ & && \text{Laws} \\ &= \log_2 x^9 + \log_2 y^3 && \text{Product Law} \\ &= \boxed{9 \log_2 x + 3 \log_2 y} && \text{Power Law} \end{aligned}$$

$$\begin{aligned} \textcircled{13.2} \quad \log_2 (x^3 y)^3 &= 3 \log_2 (x^3 y) && \text{Power} \\ & && \text{Rule} \\ &= 3 [\log_2 x^3 + \log_2 y] && \text{Product Rule} \\ &= 3 [3 \log_2 x + \log_2 y] && \text{Power Rule} \\ &= \boxed{9 \log_2 x + 3 \log_2 y} \end{aligned}$$

$$\begin{aligned} \textcircled{15.1} \quad \log_9 (z \sqrt{x \cdot y}) &= \log_9 z (xy)^{1/2} && \text{Exponent} \\ & && \text{Laws} \\ &= \log_9 z x^{1/2} y^{1/2} \\ &= \log_9 z + \log_9 x^{1/2} + \log_9 y^{1/2} && \text{Product Prop.} \\ &= \boxed{\log_9 z + \frac{1}{2} \log_9 x + \frac{1}{2} \log_9 y} && \text{Power Prop.} \end{aligned}$$

$$(15.2) \log_9(z \sqrt{xy}) = \log_9 z (xy)^{\frac{1}{2}} \quad \text{Exponent Prop}$$

$$= \log_9 z + \log_9 (xy)^{\frac{1}{2}} \quad \text{Product Prop.}$$

$$= \log_9 z + \frac{1}{2} [\log_9 (xy)] \quad \text{Power Prop}$$

$$= \log_9 z + \frac{1}{2} [\log_9 x + \log_9 y] \quad \text{Product Prop}$$

$$= \boxed{\log_9 z + \frac{1}{2} \log_9 x + \frac{1}{2} \log_9 y}$$

$$(17.1) \log_8(x^3 y)^2 = \log_8 x^6 y^2 \quad \text{Exponent Law}$$

$$= \log_8 x^6 + \log_8 y^2 \quad \text{Product Prop}$$

$$= \boxed{6 \log_8 x + 2 \log_8 y} \quad \text{Power Prop}$$

$$(17.2) \log_8(x^3 y)^2 = 2 [\log_8(x^3 y)] \quad \text{Power Prop}$$

$$= 2 [\log_8 x^3 + \log_8 y] \quad \text{Product Prop}$$

$$= 2 [3 \log_8 x + \log_8 y] \quad \text{Power Prop}$$

$$= \boxed{6 \log_8 x + 2 \log_8 y}$$

$$\textcircled{19} \quad 6 \log_5 10 = \boxed{\log_5 10^6} \quad \begin{array}{l} \leftarrow \text{Power Prop} \\ \text{condensed} \end{array}$$

$$\begin{aligned} \text{But } \log_5 10^6 &= \log_5 (5 \cdot 2)^6 && \text{exponent} \\ &= \log_5 (5^6 2^6) && \text{laws} \\ &= \log_5 5^6 + \log_5 2^6 && \text{Product} \\ &= 6 \log_5 5 + 6 \log_5 2 && \text{Rule} \\ &= 6(1) + 6 \log_5 2 && \text{Power} \\ &= 6 + 6 \log_5 2 && \text{Rule} \\ &= \boxed{6 + 6 \log_5 2} && \text{log of base} \\ &&& \leftarrow \text{expanded} \end{aligned}$$

$$\textcircled{21} \quad \log_7 u - \log_7 v = \log_7 \left(\frac{u}{v} \right) \quad \begin{array}{l} \text{Quotient} \\ \text{Law} \end{array}$$

$$\textcircled{23} \quad \log_4 2 + \log_4 7 = \log_4 \left(\frac{2}{7} \right) \quad \begin{array}{l} \text{Quotient} \\ \text{Law} \end{array}$$

$$\begin{aligned} \textcircled{25} \quad 5 \log_7 11 - \log_7 8 &= \log_7 11^5 - \log_7 8 && \text{Power} \\ &= \boxed{\log_7 \left(\frac{11^5}{8} \right)} && \text{Rule} \\ &&& \text{quotient Law} \end{aligned}$$

$$(27) \quad 2 \log_8 X = \boxed{\log_8 X^2} \quad \text{Power Rule}$$

$$(29) \quad \log u + \log v + \log w = \log(uvw)$$

Product Prop

$$(31.1) \quad \frac{\log_5 X}{2} + \frac{\log_5 Y}{2} + \frac{\log_5 Z}{2} = \frac{1}{2} (\log_5 X + \log_5 Y + \log_5 Z)$$

Product Product

$$= \frac{1}{2} (\log_5 XYZ)$$

$$= \log_5 \sqrt{XYZ} = \log_5 (XYZ)^{\frac{1}{2}}$$

Power Prop.

$$(31.2) \quad \frac{1}{2} \log_5 X + \frac{1}{2} \log_5 Y + \frac{1}{2} \log_5 Z$$

Power Rule

$$\log_5 X^{\frac{1}{2}} + \log_5 Y^{\frac{1}{2}} + \log_5 Z^{\frac{1}{2}}$$

$$\log_5 X^{\frac{1}{2}} Y^{\frac{1}{2}} Z^{\frac{1}{2}} = \log_5 (XYZ)^{\frac{1}{2}}$$
$$= \log_5 \sqrt{XYZ}$$

Product Rule

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$$\frac{\log_7 X}{3} + \frac{\log_7 Y}{3} + \frac{\log_7 Z}{3}$$

$$= \frac{1}{3} (\log_7 X + \log_7 Y + \log_7 Z)$$

$$= \frac{1}{3} (\log_7 (XYZ)) \quad \text{Product Prop}$$

$$= \log_7 (XYZ)^{\frac{1}{3}}$$

$$= \log_7 \sqrt[3]{XYZ}$$

Power Prop

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$$\frac{1}{3} \log_7 X + \frac{1}{3} \log_7 Y + \frac{1}{3} \log_7 Z$$

$$\log_7 X^{\frac{1}{3}} + \log_7 Y^{\frac{1}{3}} + \log_7 Z^{\frac{1}{3}}$$

Power Rule

$$\log_7 (X^{\frac{1}{3}} Y^{\frac{1}{3}} Z^{\frac{1}{3}})$$

Product Rule

$$\log_7 \sqrt[3]{XYZ}$$

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$$3 \log_3 U + 15 \log_3 V$$

Power Rule

$$\log_3 U^3 + \log_3 V^{15}$$

$$\log_3 (U^3 V^{15})$$

Product Rule