

Problem 1

|    | A   | B    | C            | D |
|----|-----|------|--------------|---|
| 1  | R1  |      | 225 ohms     |   |
| 2  | R2  |      | 350 ohms     |   |
| 3  | dr1 |      | 0.4 ohms/sec |   |
| 4  | dr2 |      | 0.6 ohms/sec |   |
| 5  |     |      |              |   |
| 6  | R   | ?    |              |   |
| 7  |     |      |              |   |
| 8  |     |      |              |   |
| 9  |     |      |              |   |
| 10 |     |      |              |   |
| 11 |     |      |              |   |
| 12 |     |      |              |   |
| 13 |     |      |              |   |
| 14 |     |      |              |   |
| 15 |     |      |              |   |
| 16 |     |      |              |   |
| 17 |     |      |              |   |
| 18 |     |      |              |   |
| 19 |     |      |              |   |
| 20 |     |      |              |   |
| 21 |     |      |              |   |
|    | A1  | "R1" |              |   |

Two resistors are connected in parallel.

The total resistance,  $R$ , can be measured in ohms ( $\Omega$ ) is given

by . 
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

9. If  $R_1$  is increasing at a rate  $0.4 \Omega/s$  and  $R_2$  is increasing at a rate  $0.6 \Omega/s$ , how fast is their total resistance changing when the resistors are  $R_1 = 225 \Omega$  and  $R_2 = 350 \Omega$ , respectively?

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{23}{3150}$$

$$\frac{d}{dt} \left( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \right) \rightarrow \frac{0 \cdot R - \frac{dR}{dt} \cdot 1}{(R)^2} = \frac{0 \cdot R_1 - \frac{dR_1}{dt} \cdot 1}{(R_1)^2} + \frac{0 \cdot R_2 - \frac{dR_2}{dt} \cdot 1}{(R_2)^2}$$

$$\rightarrow \frac{-1}{R^2} \cdot \frac{dR}{dt} = \frac{-1}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{-1}{R_2^2} \cdot \frac{dR_2}{dt}$$

$$\rightarrow \frac{dR}{dt} = -R^2 \left( \frac{-1}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{-1}{R_2^2} \cdot \frac{dR_2}{dt} \right)$$

$$\rightarrow \frac{dR}{dt} = \frac{R^2}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{R^2}{R_2^2} \cdot \frac{dR_2}{dt}$$

$$\frac{dR}{dt} = \frac{R^2}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{R^2}{R_2^2} \cdot \frac{dR_2}{dt}$$

$$\frac{1}{R} = \frac{1}{225} + \frac{1}{350} = \frac{23}{3150}$$

$$R = \frac{3150}{23} \text{ (total resistance)}$$

$$R_1 = 225$$

$$R_2 = 350$$

$$\frac{dR_1}{dt} = 0.4 = \frac{2}{5}$$

$$\frac{dR_2}{dt} = 0.6 = \frac{3}{5}$$

$$\frac{dR}{dt} = \frac{127}{529} \approx 0.2401$$

$$\frac{dR}{dt} = \frac{R^2}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{R^2}{R_2^2} \cdot \frac{dR_2}{dt}$$

$$= \left( \frac{1600}{13} \right)^2 \cdot (0.4) + \left( \frac{1600}{13} \right)^2 \cdot (0.6)$$

$$= \frac{9922500}{529} (0.4) + \frac{9922500}{122500} (0.6)$$

$$= \frac{196}{529} (0.4) + \frac{81}{529} (0.6)$$

$$= \frac{196}{529} \left( \frac{2}{5} \right) + \frac{81}{529} \left( \frac{3}{5} \right)$$

$$= \frac{127}{529} \approx 0.240075614367$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{23}{3150} \quad R = \frac{3150}{23} \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R^{-1} = R_1^{-1} + R_2^{-1}$$

$$\frac{d}{dt} \left( R^{-1} = R_1^{-1} + R_2^{-1} \right) \rightarrow \frac{dR}{dt} (-1R^{-2}) = \frac{dR_1}{dt} (-1R_1^{-2}) + \frac{dR_2}{dt} (-1R_2^{-2})$$

$$\frac{\frac{dR}{dt} (-1R^{-2})}{-1R^{-2}} = \frac{\frac{dR_1}{dt} (-1R_1^{-2}) + \frac{dR_2}{dt} (-1R_2^{-2})}{-1R^{-2}}$$

$$\frac{dR}{dt} = \frac{dR_1}{dt} \cdot \frac{-R_1^{-2}}{-R^{-2}} + \frac{dR_2}{dt} \cdot \frac{-R_2^{-2}}{-R^{-2}}$$

$$\frac{dR}{dt} = \frac{dR_1}{dt} \cdot \frac{R_1^{-2}}{R^{-2}} + \frac{dR_2}{dt} \cdot \frac{R_2^{-2}}{R^{-2}}$$

$$\frac{dR}{dt} = \frac{dR_1}{dt} \cdot \frac{R^2}{R_1^2} + \frac{dR_2}{dt} \cdot \frac{R^2}{R_2^2}$$

$$\frac{dR}{dt} = \frac{R^2}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{R^2}{R_2^2} \cdot \frac{dR_2}{dt}$$

$$\frac{1}{R} = \frac{1}{225} + \frac{1}{350} = \frac{23}{3150}$$

$$R = \frac{3150}{23} \text{ (total resistance)}$$

$$R_1 = 225$$

$$R_2 = 350$$

$$\frac{dR_1}{dt} = 0.4 = \frac{2}{5}$$

$$\frac{dR_2}{dt} = 0.6 = \frac{3}{5}$$

$$\frac{dR}{dt} = \frac{127}{529} \approx 0.2401$$

$$\frac{dR}{dt} = \frac{R^2}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{R^2}{R_2^2} \cdot \frac{dR_2}{dt}$$

$$= \left( \frac{1600}{13} \right)^2 \cdot (0.4) + \left( \frac{1600}{13} \right)^2 \cdot (0.6)$$

$$= \frac{9922500}{529} (0.4) + \frac{9922500}{122500} (0.6)$$

$$= \frac{196}{529} (0.4) + \frac{81}{529} (0.6)$$

$$= \frac{196}{529} \left( \frac{2}{5} \right) + \frac{81}{529} \left( \frac{3}{5} \right)$$

$$= \frac{127}{529} \approx 0.240075614367$$

Problem 2

|    | A                 | B     | C    | D |
|----|-------------------|-------|------|---|
| 1  | mass              | m     | 2500 |   |
| 2  | velocity or speed | v     | 60   |   |
| 3  | acceleration      | dv/dt | 15   |   |
| 4  |                   |       |      |   |
| 5  |                   |       |      |   |
| 6  |                   |       |      |   |
| 7  |                   |       |      |   |
| 8  |                   |       |      |   |
| 9  |                   |       |      |   |
| 10 |                   |       |      |   |
| 11 |                   |       |      |   |
| 12 |                   |       |      |   |
| 13 |                   |       |      |   |
| 14 |                   |       |      |   |
| 15 |                   |       |      |   |
| 16 |                   |       |      |   |
| 17 |                   |       |      |   |
| 18 |                   |       |      |   |
| 19 |                   |       |      |   |
| 20 |                   |       |      |   |
| 21 |                   |       |      |   |

10. If a 2500 kg car is accelerating at a rate of 15 m/sec<sup>2</sup>, then how fast is its kinetic energy changing when the speed is 60 m/sec.

Recall kinetic energy formula  $E_k = \frac{1}{2}mv^2$

$$E = \frac{1}{2}mv^2 \text{ note: } m \text{ is constant}$$

$$E = \frac{1}{2}(2500)v^2 \rightarrow E = 1250v^2$$

$$\frac{d}{dt}(E = 1250v^2)$$

$$\frac{dE}{dt} = \frac{1}{2} \cdot 2(2500)v \frac{dv}{dt} \rightarrow \frac{dE}{dt} = 2500v \frac{dv}{dt}$$

$$\rightarrow \frac{dE}{dt} = 2500(60)(15) = 2250000 \frac{\text{kg m}^2}{\text{sec}^3}$$

$$= 2250000 \frac{1 \text{ Joule}}{\text{sec}} \quad \text{NOTE: } 1\text{J} = \frac{1 \text{ kg m}^2}{\text{sec}^2}$$

(I had to look up this so no pts off for not knowing what a Joule was)

Problem 3

|    | A     | B | C     |
|----|-------|---|-------|
| 1  | p     |   | 1     |
| 2  | v     |   | 500   |
| 3  | k     | ? |       |
| 4  | dv/dt |   | -4000 |
| 5  |       |   |       |
| 6  |       |   |       |
| 7  |       |   |       |
| 8  |       |   |       |
| 9  |       |   |       |
| 10 |       |   |       |
| 11 |       |   |       |
| 12 |       |   |       |
| 13 |       |   |       |
| 14 |       |   |       |
| 15 |       |   |       |
| 16 |       |   |       |
| 17 |       |   |       |
| 18 |       |   |       |
| 19 |       |   |       |
| 20 |       |   |       |
| 21 |       |   |       |

An internal combustion engine uses a piston to compress combustible air in a cylinder before igniting it to generate energy in the form of mechanical work. Suppose the volume of such a cylinder is 500 cm<sup>3</sup> at 1 atm of pressure. Suddenly, the piston begins to compress the volume of the air-fuel mixture at a rate of -4000 cm<sup>3</sup>/second.

11. What is the corresponding rate of change of pressure in the cylinder? Because we'll assume the temperature doesn't change in this scenario, use Boyle's law,  $PV = k$  which says that  $k$  is a constant.

$$PV=k \rightarrow \frac{d}{dt}(PV=k) \rightarrow \frac{dP}{dt} \cdot V + \frac{dV}{dt} \cdot P = 0$$

$$\frac{dP}{dt} (500) + (-4000)(1) = 0$$

$$500 \frac{dP}{dt} - 4000 = 0$$

$$500 \frac{dP}{dt} = 4000$$

$$\frac{dP}{dt} = 4000 / 500 = 8 \text{ atm/s}$$