Sample Quiz #1

$P = 12000$

$r\% = 1.2\%$

$r = \frac{1.2}{100} = 0.012$

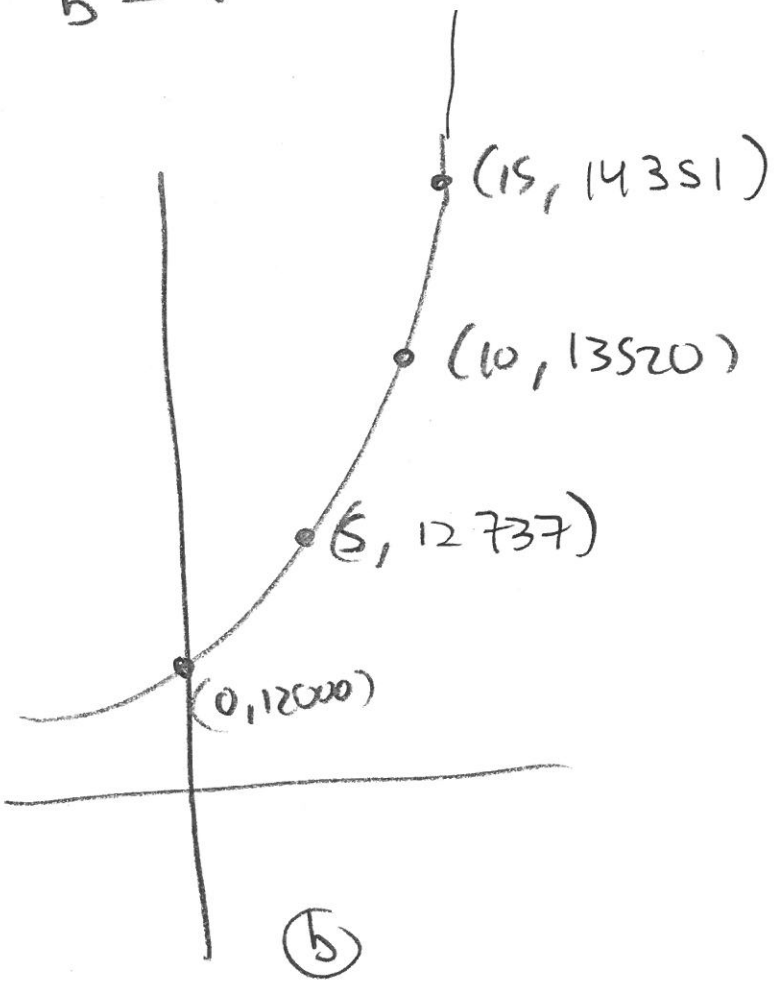
$b = 1 + 0.012$

$b = 1.012$ (a)

(a)

Model

$A(x) = 12000(1.012)^x$



(1c) When does population double?

General

$2P = Pb^x$

$\frac{2P}{P} = b^x$

$2 = b^x$

$\boxed{x = \log_b 2}$

$b = \sqrt[x]{2}$

Particular

$24000 = 12000(1.012)^x$

$\frac{24000}{12000} = \frac{12000(1.012)^x}{12000}$

$2 = 1.012^x$

Method (1) Defn ↓

$\boxed{\log_{1.012} 2 = x}$

$x \approx 58.108$

Method (2)

$\log 2 = \log(1.012)^x$

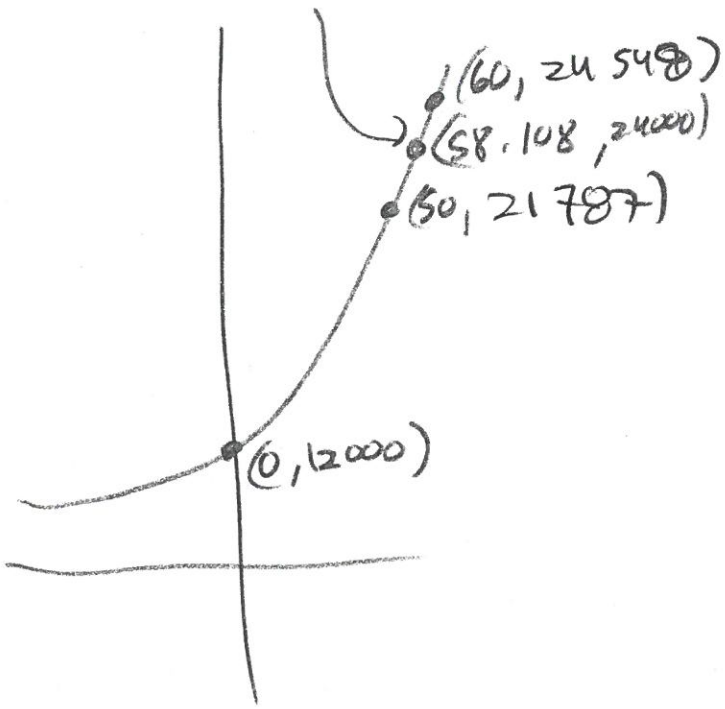
$\log 2 = x \log(1.012)$

$x = \frac{\log 2}{\log(1.012)}$

$x = \log_{1.012} 2$

$\boxed{x \approx 58.108}$

Method (3)



(1d) When does this reach 1.6 times the population?

General

$1.6P = Pb^x$

$\frac{1.6P}{P} = b^x$

$1.6 = b^x$

$\boxed{\log_b 1.6 = x}$

$\sqrt[x]{1.6} = b$

Particular

$1.6(12000) = 19200$

$192000 = 12000(1.012)^x$

$\frac{192000}{12000} = (1.012)^x$

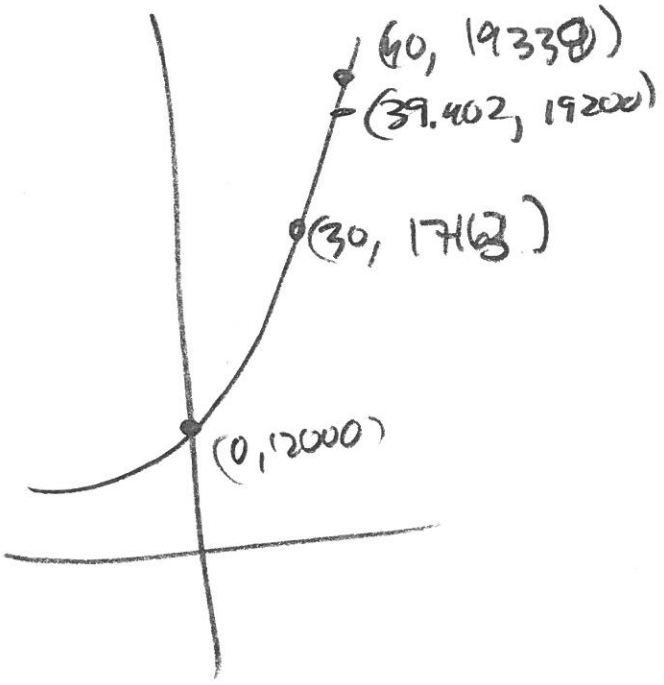
$1.6 = 1.012^x$

Method ① Def. of log

$\boxed{\log_{1.012} 1.6 = x}$

$x \approx 39.402$

Method ③



Method ②

$\log 1.6 = \log 1.012^x$

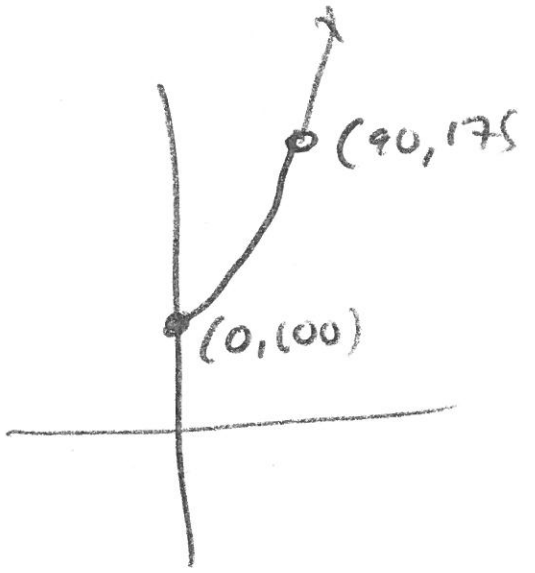
$\log 1.6 = x \log 1.012$

$x = \frac{\log 1.6}{\log 1.012} =$

$\boxed{x = \log_{1.018} 1.6 \approx 39.402}$

(2) $P = 100$

$A(90) = 175$



$175 = 100(b)^{90}$

$\frac{175}{100} = b^{90}$

$\sqrt[90]{\frac{175}{100}} = b$

$\sqrt[90]{1.75} = b$

$b \approx 1.006237324$

$\boxed{b \approx 1.006}$

Exact model $A(x) = 100\left(\sqrt[90]{1.75}\right)^x$

Approx model $A(x) = 100(1.006)^x$

$b \approx 1.006$

$b = 1 + r$

$r = b - 1$

$r = 1.006 - 1$

$r = 0.006$

$r\% \approx 0.6\%$

$b = \sqrt[90]{1.75}$

$\boxed{r = \sqrt[90]{1.75} - 1}$

exact ↗

$r\%$ exact

$r\% = (100\sqrt[90]{1.75} - 100)\%$

$\boxed{P = 100}$

(3) $P = 1500$

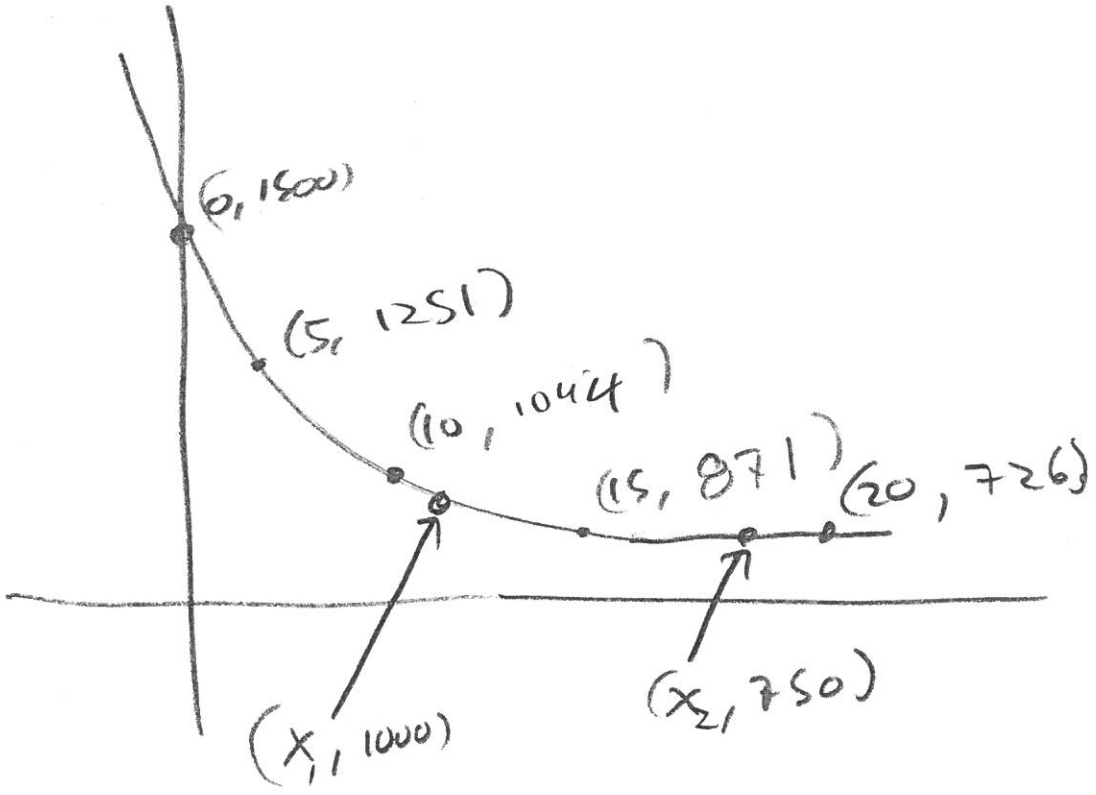
decay $\quad r\% = 3.56 \% \quad r = 0.0356$

$b = 1 - r$

$= 1 - 0.0356$

$\boxed{b = 0.9644}$ 3a

$\boxed{A(x) = 1500(0.9644)^x}$ Model ← 3a



3b

$x_1 \in [10, 15] \qquad x_2 \in [15, 20]$

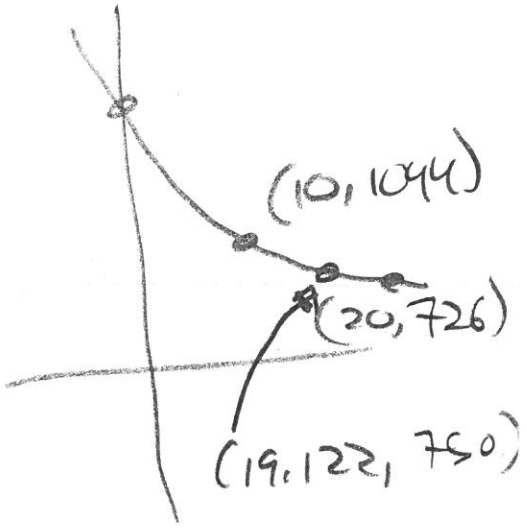
(3c) $P = 1500 \quad \frac{1}{2}P = 750$

$750 = 1500(0.9644)^x$

$\frac{750}{1500} = 0.9644^x$

$0.5 = 0.9644^x$

method (3



method (1 Defn of log

$\log_{0.9644} 0.5 = x$

$x \approx 19.122$

method (2

$0.5 = 0.9644^x$

$\log 0.5 = \log(0.9644^x)$

$\log 0.5 = x \log(0.9644)$

$x = \frac{\log(0.9644)}{\log(0.5)}$

$x = \log_{0.5}(0.9644)$

$x \approx 19.122$

(3d) $P = 1500 \quad A(x) = 1000$?

$$1000 = 1500(0.9644)^x$$

$$\frac{1000}{1500} = 0.9644^x$$

$$\frac{2}{3} = 0.9644^x$$

↗ Donut Round

Method (1) Def. of Log

$$\log_{0.9644}\left(\frac{2}{3}\right) = x$$

$$x \approx 11.186$$

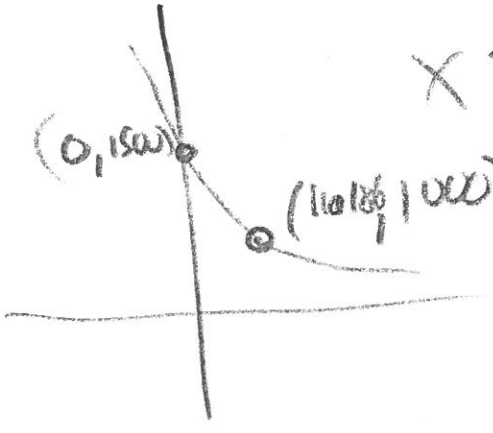
Method (2) Apply log

$$\log\left(\frac{2}{3}\right) = \log 0.9644^x$$

$$\log\left(\frac{2}{3}\right) = x \log(0.9644)$$

$$x = \frac{\log(2/3)}{\log(0.9644)} = \log_{0.9644}(2/3)$$

$$x \approx 11.186$$



(4) $P = 150$

$A(2) = 120$

$120 = 150\,b^2$

$\frac{120}{150} = \frac{150\,b^2}{150}$

$\frac{120}{150} = b^2$

$\sqrt{\frac{120}{150}} = b$

$b = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$b \approx 0.8944$ decay factor

So $r = 1 - b$

$= 1 - 0.8944$

decay rate $r = 0.1056$

decay rate as % $r\% = 10.56\%$

$A(x) = 150(0.8944)^x$

approx ↗

$A(x) = 150\left(\sqrt{\frac{4}{5}}\right)^x$

exact