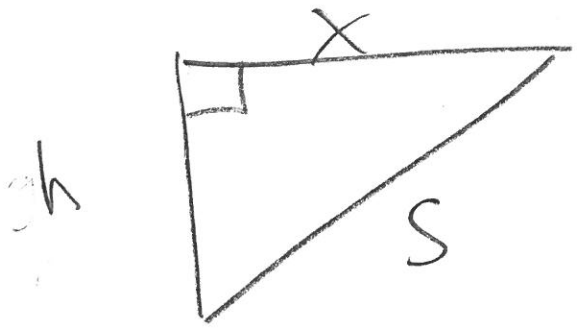
Solutions Airplane Problem



Answers in General

$$h^2 + x^2 = s^2$$

$$2h\frac{dh}{dt} + 2x\frac{dx}{dt} = 2s\frac{ds}{dt}$$

we know $\frac{dh}{dt} = 0$ because h = constant

$$2h(0) + 2x\frac{dx}{dt} = 2s\frac{ds}{dt}$$

$$2x\frac{dx}{dt} = 2s\frac{ds}{dt}$$

$$\frac{dx}{dt} = \frac{2s}{2x}\frac{ds}{dt} \quad \text{OR} \quad \frac{2x}{2s}\frac{dx}{dt} = \frac{ds}{dt}$$

$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$$

Volume of cylinder

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} - \pi r^2 \frac{dh}{dt} = 2\pi r h \frac{dr}{dt}$$

$$\frac{1}{2\pi r h}\left[\frac{dV}{dt} - \pi r^2 \frac{dh}{dt}\right] = \frac{dr}{dt}$$

$$\frac{1}{2\pi r h}\frac{dV}{dt} - \frac{\pi r^2}{2\pi r h}\frac{dh}{dt} = \frac{dr}{dt}$$

$$\boxed{\frac{1}{2\pi r h}\frac{dV}{dt} - \frac{r}{2h}\frac{dh}{dt} = \frac{dr}{dt}}$$

General solutions for all cylinders

example ①

$\frac{ds}{dt} = 500 mph$

$s = 15$

$h = 10$

$$x = \sqrt{15^2 - 10^2}$$

$$= \sqrt{225 - 100}$$

$$\boxed{x = \sqrt{125}}$$

$$\therefore \frac{dx}{dt} = \frac{15}{\sqrt{125}}(500) \rightarrow \boxed{\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}}$$

$$\boxed{\frac{dx}{dt} = 670.8 mph}$$

Example ①

$$x = \sqrt{s^2 - h^2}$$

$$\frac{dx}{dt} = \frac{1}{2}(2s) \cdot \frac{1}{\sqrt{s^2 - h^2}} \frac{ds}{dt}$$

$$\boxed{\frac{dx}{dt} = \frac{s}{\sqrt{s^2 - h^2}} \frac{ds}{dt}}$$

$\frac{ds}{dt} = 500\text{mph}$ $s = 15\text{ miles}$ $h = 10\text{ miles}$

$$\boxed{\frac{dx}{dt} = \frac{15}{\sqrt{15^2 - 10^2}}(500) \approx 670.8\text{ mph}}$$

Example ① hardway

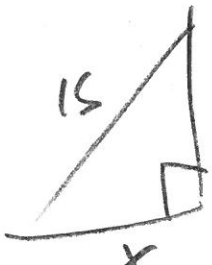
$$s = \sqrt{x^2 + h^2}$$

$$\frac{ds}{dt} = \frac{1}{2}(2x)\frac{1}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

$$\boxed{\frac{ds}{dt} \cdot \frac{\sqrt{x^2+h^2}}{x} = \frac{dx}{dt}}$$

unfortunately we need x



$s = 15 \quad x = ? \quad h = 10$

$\sqrt{15^2 - 10^2} = \sqrt{125}$

$\frac{ds}{dt} = 500 \text{ mph}$

So $500\left(\frac{\sqrt{125}+100}{\sqrt{125}}\right) = \frac{dx}{dt}$

$$\boxed{670.8 = \frac{dx}{dt}}$$

Recall $\boxed{\frac{dr}{dt} = \frac{1}{2\pi rh} \cdot \frac{dV}{dt} - \frac{r}{2h} \frac{dh}{dt}}$

Example 1 $\frac{dV}{dt} = 0 \quad r = 3 \quad h = 4$

$\frac{dh}{dt} = -\frac{2}{10}$

$$\frac{dr}{dt} = \frac{1}{2\pi(3)(4)} \cdot 0 - \frac{3}{2(4)} \cdot \frac{-2}{10}$$

$$\frac{dr}{dt} = 0 + \frac{6}{80}$$

$$\frac{dr}{dt} = \frac{6}{80} = \frac{3}{40} \text{ cm/sec}$$

$$\frac{dr}{dt} \approx 0.075 \text{ cm/sec}$$

example ①

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

Recall $\frac{dV}{dt} = 0 \quad r = 3 \quad h = 4 \quad \frac{dh}{dt} = -0.2$

$$0 = 2\pi(3)\frac{dr}{dt}(4) + \pi(3)^2\left(\frac{2}{10}\right)$$

$$0 = 24\pi \frac{dr}{dt} - \frac{18\pi}{10}$$

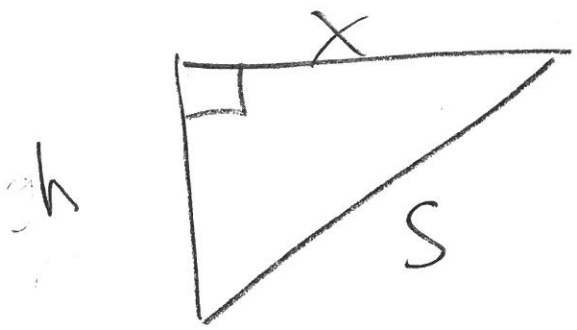
$$\frac{18\pi}{10} = 24\pi \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{18\pi}{240\pi}$$

$$\boxed{\frac{dr}{dt} = \frac{3}{40} \text{ cm/sec}}$$

$$\boxed{\frac{dr}{dt} = 0.075 \text{ cm/sec}}$$

Solutions Airplane Problem



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$$\boxed{\frac{1}{2\pi r h}\frac{dV}{dt} - \frac{r}{2h}\frac{dh}{dt} = \frac{dr}{dt}}$$

General solutions for all cylinders

Example (2)

$$x = \sqrt{s^2 - h^2}$$

$$\frac{dx}{dt} = \frac{1}{2}(2s)\frac{1}{\sqrt{s^2-h^2}}\frac{ds}{dt}$$

$$\boxed{\frac{dx}{dt} = \frac{s}{\sqrt{s^2-h^2}}\frac{ds}{dt}}$$

$\frac{ds}{dt} = 300 \text{ mph} \quad s = 24 \quad h = 18$

$$\boxed{\frac{dx}{dt} = \frac{24}{\sqrt{24^2-18^2}}(300) \approx 453.6 \text{ mph}}$$

Example 3

$$\frac{ds}{dt} = 300 \text{ mph}$$

$$s = 24$$

$$h = 18$$

$$x = \sqrt{24^2 - 18^2}$$

$$= \sqrt{576 - 324}$$

$$\boxed{x = \sqrt{252}}$$

Recall $\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$

$$\frac{dx}{dt} = \frac{24}{\sqrt{252}}(300)$$

$$\boxed{\frac{dx}{dt} = 453.6 \text{ mph}}$$

Example ② hard way

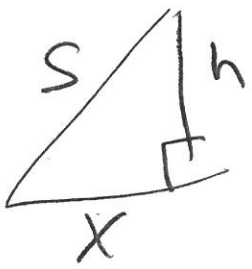
$$S = \sqrt{x^2 + h^2}$$

$$\frac{dS}{dt} = \frac{1}{2}(2x)\frac{1}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

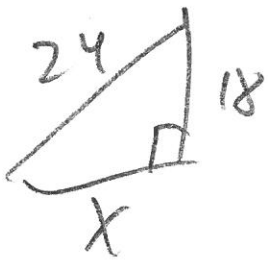
$$\frac{dS}{dt} = \frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

$$\boxed{\frac{dS}{dt} \cdot \frac{\sqrt{x^2+h^2}}{x} = \frac{dx}{dt}}$$

$$\boxed{S = 24 \quad \frac{dS}{dt} = 300 \quad h = 18}$$



$$300\left(\frac{\sqrt{252}+324}{\sqrt{252}}\right) = \frac{dx}{dt}$$



$$\boxed{\frac{dx}{dt} \approx 453.6 \text{ mph}}$$

$$x = \sqrt{24^2 - 18^2}$$

$$x = \sqrt{252}$$

Example 2 Right cylinder

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

$\frac{dV}{dt} = 0 \quad r = 5 \quad h = 12 \quad \frac{dh}{dt} = \frac{4}{10}$

actually straddles vertically but flattens

$$0 = 2\pi(5)(12)\frac{dr}{dt} + \pi(5)^2\left(\frac{4}{10}\right)$$

$$0 = 120\pi \frac{dr}{dt} + \frac{100\pi}{10}$$

$$0 = 120\pi \frac{dr}{dt} + 10\pi$$

$$-10\pi = 120\pi \frac{dr}{dt}$$

$$-\frac{10\pi}{120\pi} = \frac{dr}{dt}$$

$$\frac{dr}{dt} = -\frac{1}{12} \approx -0.083 \text{ cm/sec}$$

Example (2) Right cylinder

Recall $\frac{dr}{dt} = \frac{1}{2\pi rh}\frac{dV}{dt} - \frac{r}{2h}\frac{dh}{dt}$

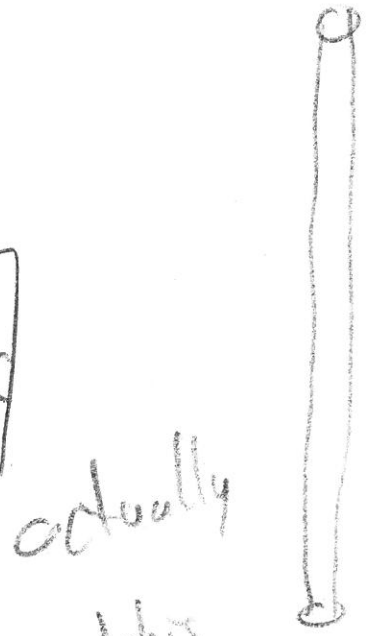
$\frac{dV}{dt} = 0 \quad r = 5 \quad h = 12 \quad \frac{dh}{dt} = +\frac{4}{10}$

$$\frac{dr}{dt} = \frac{1}{2\pi(5)(12)} \cdot 0 - \frac{5}{2(12)}\left(\frac{4}{10}\right)$$

$$\frac{dr}{dt} = 0 - \frac{20}{240}$$

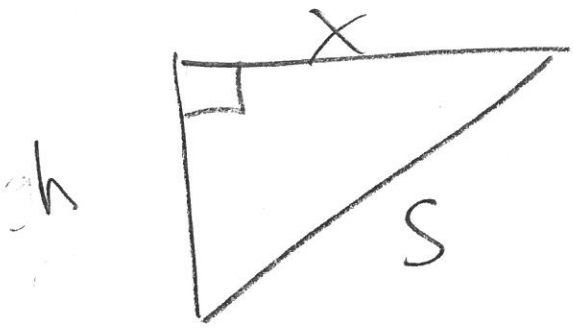
$$\boxed{\frac{dr}{dt} = -\frac{1}{12} \text{ cm/sec}}$$

$$\boxed{\frac{dr}{dt} = -0.0833 \text{ cm/sec}}$$



actually stretches vertically

Solutions Airplane Problem



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$$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$$

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$$\frac{1}{2\pi r h}\left[\frac{dV}{dt} - \pi r^2 \frac{dh}{dt}\right] = \frac{dr}{dt}$$

$$\frac{1}{2\pi r h}\frac{dV}{dt} - \frac{\pi r^2}{2\pi r h}\frac{dh}{dt} = \frac{dr}{dt}$$

$$\boxed{\frac{1}{2\pi r h}\frac{dV}{dt} - \frac{r}{2h}\frac{dh}{dt} = \frac{dr}{dt}}$$

General solutions for all cylinders

Example (3)

$$\frac{ds}{dt} = 600 \text{ mph}$$

$$s = 20$$

$$h = 16$$

$$x = \sqrt{20^2 - 16^2}$$

$$= \sqrt{400 - 256}$$

$$= \sqrt{144}$$

$$x = 12$$

Recall $\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$

$$= \frac{20}{12}(600)$$

$$\boxed{\frac{dx}{dt} = 1000 \text{ mph}}$$

Example (3)

$$x = \sqrt{s^2 - h^2}$$

$$\frac{dx}{dt} = \frac{1}{2}(2s)\frac{1}{\sqrt{s^2-h^2}}\frac{ds}{dt}$$

$$\boxed{\frac{dx}{dt} = \frac{s}{\sqrt{s^2-h^2}}\frac{ds}{dt}}$$

$\frac{ds}{dt} = 600 \text{ mph}$ $s = 20 \text{ mile}$ $h = 16 \text{ miles}$

$$\boxed{\frac{dx}{dt} = \frac{20}{\sqrt{20^2-16^2}}(600) = 1000 \text{ mph}.}$$

Example (3) hard way

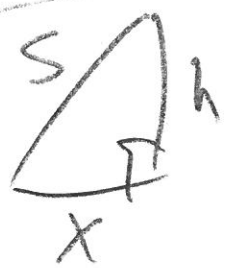
$$s = \sqrt{x^2 + h^2}$$

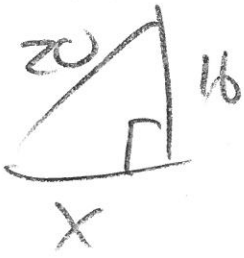
$$\frac{ds}{dt} = \frac{1}{2}(2x)\frac{1}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{x}{\sqrt{x^2+h^2}}\frac{dx}{dt}$$

$$\boxed{\frac{ds}{dt}\cdot\frac{\sqrt{x^2+h^2}}{x} = \frac{dx}{dt}}$$

$\frac{ds}{dt} = 600$ mph $\quad x = ? \quad h = 16 \quad s = 20$





$$600\left(\frac{\sqrt{144+256}}{\sqrt{144}}\right) = \frac{dx}{dt}$$

$$\boxed{\frac{dx}{dt} = 1000 \text{ mph}}$$

$\sqrt{20^2 - 16^2}$

$\sqrt{400 - 256}$

$\sqrt{144}$ (12)

example (3) Right cylinder Problem

Recall $\frac{dr}{dt} = \frac{1}{2\pi rh}\frac{dV}{dt} - \frac{r}{2h}\frac{dh}{dt}$

$r = 7 \quad h = 24 \quad \frac{dV}{dt} = 0 \quad \frac{dh}{dt} = \frac{-6}{10}$

$$\frac{dr}{dt} = \frac{1}{2\pi(7)(24)} \cdot 0 - \frac{7}{2(24)}\left(\frac{-6}{10}\right)$$

$$= 0 + \frac{42}{480}$$

$$\boxed{\frac{dr}{dt} = \frac{7}{80} \text{ cm/sec}}$$

$$\boxed{\frac{dr}{dt} = 0.0875 \text{ cm/sec}}$$

Example (3)

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r \frac{dr}{dt} h + \pi r^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$

$\frac{dV}{dt} = 0 \quad r = 7 \quad h = 24 \quad \frac{dh}{dt} = -\frac{6}{10}$

$$0 = 2\pi(7)(24)\frac{dr}{dt} + \pi(7)^2\left(-\frac{6}{10}\right)$$

$$0 = 336\pi \frac{dr}{dt} + \frac{-294\pi}{10}$$

$$336\pi \frac{dr}{dt} = \frac{294\pi}{10}$$

$$\frac{dr}{dt} = \frac{294}{3360} = \frac{7}{80}$$

$$\boxed{\frac{dr}{dt} = \frac{7}{80} \text{ cm/sec} \simeq 0.0875 \text{ cm/sec}}$$