

# Solutions to Summative Assessment version 8000

①  $P = 8000$

$r = 7.2\%$   $r = 0.072$

$n = \text{monthly}$

$n = 12$

$$\text{Model } A(x) = 8000 \left(1 + \frac{0.072}{12}\right)^{12x}$$

② Balance after 12 years =  $A(12)$

$$A(12) = 8000 \left(1 + \frac{0.072}{12}\right)^{12(12)}$$

$$= 8000 \left(1 + \frac{0.072}{12}\right)^{144}$$

$$= 8000 (1.006)^{144}$$

$A(12) \approx$

③ Balance of 9750 occurs when?

$$A(x) = 9750$$

$$9750 = 8000 \left(1 + \frac{0.072}{12}\right)^{12x}$$

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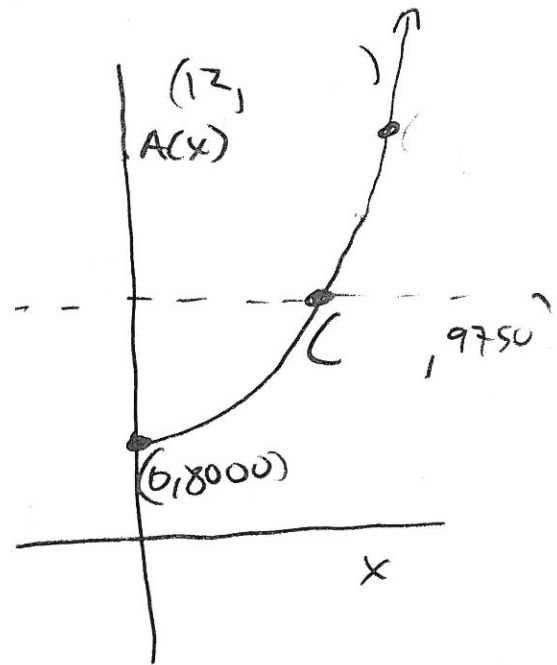
$$\frac{9750}{8000} = \frac{8000 \left(1 + \frac{0.072}{12}\right)^{12x}}{8000}$$

$$\frac{9750}{8000} = \left(1 + \frac{0.072}{12}\right)^{12x}$$

$$\log \left(1 + \frac{0.072}{12}\right) \left(\frac{9750}{8000}\right) = 12x$$

$$x = \frac{\log \left(1 + \frac{0.072}{12}\right) \left(\frac{9750}{8000}\right)}{12}$$

$\approx$



OR

$$\log \left(\frac{9750}{8000}\right) = \log \left(1 + \frac{0.072}{12}\right)^{12x}$$

$$\log \left(\frac{9750}{8000}\right) = 12x \left(\log \left(1 + \frac{0.072}{12}\right)\right)$$

$$\log \left(\frac{9750}{8000}\right) = 12x \log \left(1 + \frac{0.072}{12}\right)$$

$$x = \frac{\log \left(\frac{9750}{8000}\right)}{12 \log \left(1 + \frac{0.072}{12}\right)}$$

$x \approx$

Solutions version 8000

Given Model  $A(x) = 3600 \left(1 + \frac{0.072}{4}\right)^{4x}$

$P = 3600$

$r = 0.072 \rightarrow r\% = 7.2\%$

$n = 4 \rightarrow$  Quarterly

④  $r = 0.072$   $r\% = 100r = 7.2\%$

⑤ Amount of Investment 3600

⑥ How often is interest compounded?  $n = 4$   
Quarterly

⑦  $P = 3600$   
 $r = 0.072$   
 $n = \text{weekly} = 52$

$$A(x) = 3600 \left(1 + \frac{0.072}{4}\right)^{4x}$$
$$= 3600 (1.018)^{4x}$$

new model  $\rightarrow$   
weekly

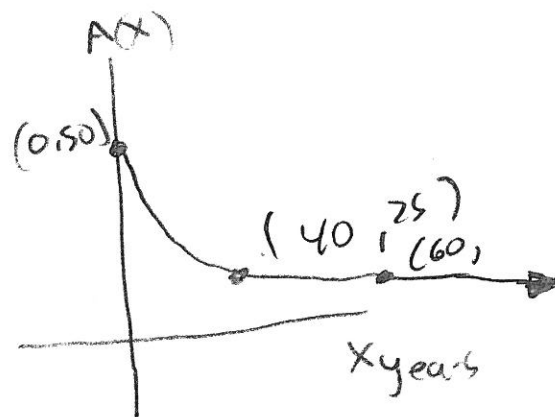
# Solutions version 8000

HalfLife 40 years

Initial Amount = 50 grams

Natural Decay Model

$$A(x) = P e^{rx}$$



$$\frac{1}{2} \text{ life Population} = \frac{1}{2} 50 = 25 \text{ grams}$$

$A(x) = 25$  grams occurs at 40 years

$$\text{So } A(40) = 25 \text{ grams}$$

$$25 = 50 e^{40r}$$

$$\frac{25}{50} = \frac{50 e^{40r}}{50}$$

$$\frac{1}{2} = e^{40r}$$

$$\ln \frac{1}{2} = 40r$$

$$\textcircled{9} \quad r = \frac{\ln \frac{1}{2}}{40} \approx -0.0173$$

$$\textcircled{9} \quad r\% = 100r = 1.73\%$$

$$\textcircled{9} \quad \text{Model} \\ A(x) = 50 e^{\frac{\ln \frac{1}{2}}{40} x}$$

$\textcircled{10}$  What is the amount after 60 years?

$$x = 60$$

$$A(60) = 50 e^{\frac{\ln \frac{1}{2}}{40} \cdot 60}$$

$$A(60) = 50 e^{\frac{3}{2} \ln \frac{1}{2}}$$

$$A(60) \approx$$

$$A(x) = 3.6 e^{0.0078x}$$

$\downarrow$  measured in millions  
 $\downarrow$  3.6 million  
 $\downarrow$  measured in years since 2015  
 $r = 0.0078$   
 $100r = r\% = 0.78\%$

$x=0$  means 2015

$x=2035$  means year 2015 + 2035 = 4050

$x=20$  means year 2015 + 20 = 2035

(11)  $r = 0.0078$   
 $r\% = 0.78\%$

(12) When will population reach 4,500,000?

Note  $\frac{4,500,000}{1,000,000} = 4.5$  million

$4.5 = 3.6 e^{0.0078x}$

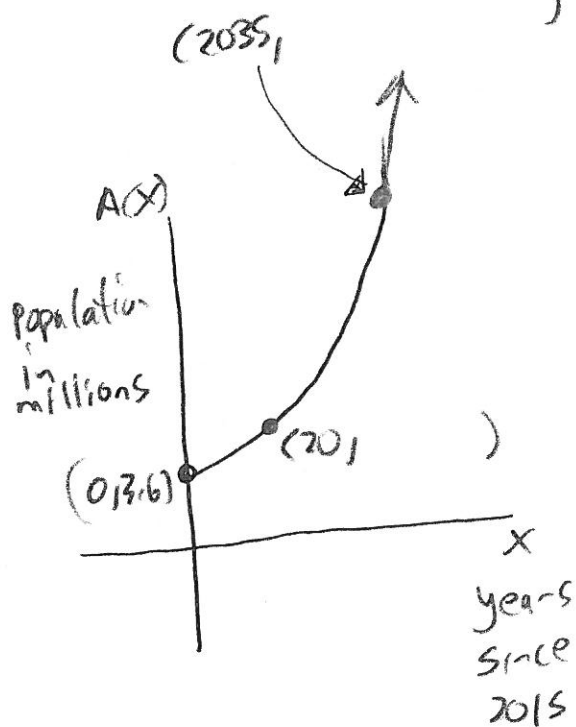
Why is  $4,500,000 = 3.6 e^{0.0078x}$

WRONG?

$4,500,000 = A(x)$

means  $4,500,000 \cdot 1,000,000$

$4,500,000,000,000$



$$A(x) = 3.6 e^{0.0078x}$$

Population in millions  
 x years after 2035  
 3.6 million Initial Population  
 $r = 0.0078$   
 $r\% = 0.78\%$   
 years since 2015

(13)  $A(2035) =$  Population after 2035 years  
 or Population during 2015 + 2035 = 4050  
 Year 4050

$$\begin{aligned}
 &= 3.6 e^{0.0078(2035)} \\
 &= 3.6 e \\
 &\approx
 \end{aligned}$$

This is the population in the year 4050

(14)  $A(20) =$  Population after 20 years  
 of Population during 2015 + 20 = 2035

$$\begin{aligned}
 &= 3.6 e^{0.0078(20)} \\
 &= 3.6 e^0 \\
 &\approx
 \end{aligned}$$

This is the population in the year 2035

(15) In the year 2035 Population growing and  $\approx$  million