WORKED OUT SOLUTIONS STUDY GUIDE

Model ①

$A(x) = 2000\, e^{0.025x}$

"e" implies continuously compounded

$R = 0.025$ $\qquad P = 2000$

$R\% = 100R = 2.5\%$

(#2) When has model ① EARNED \$120

$$\boxed{A(x) = 2000 + 120 = 2120}$$

So $\quad 2120 = 2000\, e^{0.025x}$

$$\frac{2120}{2000} = \frac{2000\, e^{0.025x}}{2000}$$

$$1.06 = e^{0.025x}$$

By Def^n $\longrightarrow$ $\ln 1.06 = 0.025x$

$$\boxed{x = \frac{\ln 1.06}{0.025} \approx 2.3308 \text{ yrs}}$$

Model ②

$$A(x) = 4000\left(1 + \frac{0.036}{12}\right)^{12x}$$

(P: 4000; R: 0.036; monthly)

$P = 4000$

$R = 0.036$

$R\% = 100R = 3.6\%$

$n = 12$ (monthly)

#1) When will balance of account DOUBLE?

$A(x) = (4000)(2) = 8000$

$$8000 = 4000\left(1 + \frac{0.036}{12}\right)^{12x}$$

$$8000 = 4000\,(1.003)^{12x}$$

$$\frac{8000}{4000} = (1.003)^{12x}$$

$$2 = 1.003^{12x}$$

By Def. $\log_{1.003}(2) = 12x$

$$x = \frac{\log_{1.003} 2}{12} = 19.283$$

Problem 3/4/5

$P = 10000$

$r\% = 4.8\%$

$r = 0.048$

$n =$ cts compoundings

$A = Pe^{rx}$

(#3) $A(x) = 10000\, e^{0.048x}$ → model

(#4) Balance after 3 yrs

$A(3) = 10000(e^{0.048(3)})$

$A(3) = 11548.84$

(#5) When is $A(x) = 12000$

$12000 = 10000(e^{.048x})$

$\frac{12000}{10000} = \frac{10000\, e^{0.048x}}{10000}$

$1.2 = e^{0.048x}$

By ln

$\ln 1.2 = 0.048x$

$\frac{\ln 1.2}{6.048} = x$

$x \approx 3.798$ years

#6/7/8

$P = 5000$

$r = ?$

$n = 365$ (daily)

$A(5) = 6200$

$$A(x) = P\left(1 + \frac{r}{n}\right)^{nx}$$

$$A(5) = 5000\left(1 + \frac{r}{365}\right)^{365(5)}$$

$$A(5) = 5000\left(1 + \frac{r}{365}\right)^{365x}$$

$$6200 = 5000(b)^{365(5)}$$

(6200 → known; 365(5) → known)

$$\frac{6200}{5000} = \frac{5000(b)^{1825}}{5000}$$

$$1.24 = b^{1825}$$

$$b = \sqrt[1825]{1.24}$$

Model #6

$$A(x) = 5000\left(\sqrt[1825]{1.24}\right)^{365x}$$

#7 Balance after 3 years

$$A(3) = 5000\left(\sqrt[1825]{1.24}\right)^{365(3)}$$

$$= \$688.83$$

#8 $A(x) = 5000 \sqrt[1825]{1.24}^{365x}$

$A(x) = 8000$ = balance

$8000 = 5000 \sqrt[1825]{1.24}^{365x}$

$\frac{8000}{5000} = \frac{5000 \sqrt[1825]{1.24}^{365x}}{5000}$

$1.6 = \left(\sqrt[1825]{1.24}\right)^{365x}$

$\log_{\sqrt[1825]{1.24}} 1.6 = 365x$

$x = \frac{\log_{\sqrt[1825]{1.24}} (1.6)}{365} \approx 10.925$

9/10/11/12

$P = 100$

Radioactive decay

$r = ?$

$\frac{1}{2}$ life $= 250$ years

$\frac{1}{2}P = 50$

$50 = 100e^{250r}$

$\frac{50}{100} = \frac{100e^{250r}}{100}$

$0.5 = e^{250r}$

$\ln 0.5 = 250r$

$r = \frac{\ln 0.5}{250}$

$r \approx -0.002773$

#9

Model $A(x) = 100e^{\frac{\ln 0.5}{250}x}$

$\approx A(x) = 100e^{-0.002773x}$

#10 $r = \frac{\ln 0.5}{250} \approx -0.002773$

$r\% = 100r = -0.2773\%$

#11 $A(x) = 90 \rightarrow$

$90 = 100e^{\frac{\ln 0.5}{250}x}$

$\frac{90}{100} = 100e^{\frac{\ln 0.5}{250}x}$

$0.9 = e^{\frac{\ln 0.5}{250}x}$

$\ln 0.9 = \frac{\ln 0.5}{250}x$

$\frac{\ln 0.9}{\frac{\ln(0.5)}{250}} = x \approx 38.008$

#12 $A(100) = ?$

$A(100) = 100e^{\frac{\ln 0.5}{250}(100)}$

$= 75.786$

#13/14/15

$A(x) = Pe^{0.015x}$ — whole Bacteria / hours

#13 $P = 18 \quad A(x) = 2P = 36$

$36 = 18e^{0.015x}$

$\frac{36}{18} = \frac{18e^{0.015x}}{18}$

$2 = e^{0.015x}$

$\ln 2 = 0.015x$

$x = \frac{\ln 2}{0.015} \simeq 46.2098$

#14 $A(3\text{ day}) = A(72\text{ hours}) \simeq 53$ Bacteria

$A(72) = 18e^{0.015(72)} = 53.0042$

#15 $A(3\text{hrs}) = 18e^{0.015(3)} \simeq 18.8$

$A(3\text{hrs}) \simeq 19$ Bacteria

#16/17/18/19/20

Let $t = 0$ at 2020
$t = 2$ at 2022
$t = 10$ at 2030

$A(x) = Pe^{rx}$

$A(0) =$ ~~Pop~~ in 2020

$A(0) = P = 1250$

$A(2) = 1850 =$ ~~Pop~~ in 2022

$A(2) = A(0)e^{r(2)}$

$1850 = 1250e^{2r}$

$\frac{1850}{1250} = \frac{1250e^{2r}}{1250}$

$1.48 = e^{2r}$

$\ln 1.48 = 2r$

$r = \frac{\ln 1.48}{2}$

#16 exact R

#17 $r \approx 0.196021$

$r\% = 100r = 19.6021\%$

#18 $A(x) \approx 1250e^{0.196021x}$ $A(x) = 1250e^{\frac{\ln 1.48}{2}x}$

(19) $A(x) = 1250\, e^{\frac{\ln 1.48}{2} x}$

$A(x) = 2500$

$2500 = 1250\, e^{\frac{\ln 1.48}{2} x}$

$\frac{2500}{1250} = \frac{1250\, e^{\frac{\ln 1.48}{2} x}}{1250}$

$2 = e^{\frac{\ln 1.48}{2} x}$

$\ln 2 = \frac{\ln 1.48}{2} x$

$$x = \frac{\ln 2}{\left(\frac{\ln 1.48}{2}\right)} \approx 3.53609$$

Note $A(x) = 1250\, e^{0.196021 x}$

$2500 = 1250\, e^{0.196021 x}$

$2 = e^{0.196021 x}$

$\ln 2 = 0.196021 x$

$$x = \frac{\ln 2}{0.196021} \approx 3.53609$$

(20) A in 2030 $= A(10) = 1250\, e^{\frac{\ln 1.48}{2} \cdot 10}$

$\approx 887603 \approx 8876$