Name____Natural Growth and Decay Models Period ______

Assume all models and scenarios follow the natural growth or natural decay model

$Natural\ Growth\ Model\ A(x) = Pe^{rx}\ with\ r > 0$ $\quad$ $Natural\ Decay\ Model\ A(x) = Pe^{-rx}\ with\ r > 0$

A(x) = population or amount after time x   P = initial population or initial amount   r = rate of change as a decimal

x = time (this changes dependent on the model and the wording of the model)

1. The half-life of a radioactive isotope is 4 days. If 3.2 kg are present now, how much will be present after:
   a. 8 days?
   b. 20 days?
   c. Write a model to show how much is present after t-days.

2. A bacteria colony triples every 6 days. The initial population of bacteria is 250. Write a function p(t) to represent the number of bacteria present t-days later. Then use your model to determine the number of bacterial after 20 days.

3. When a certain medicine enters the bloodstream, it gradually dilutes, decreasing exponentially with a half-life of 3 days. The initial amount of medicine in the bloodstream is 500 mg. What will the amount be 20 days later? If a patient needs to maintain a level of 275 mg of medication in the bloodstream, after how many days should the patient take the next dose of medication?

4. The population P (in thousands) of Reno, Nevada can be modeled by $P(t)=134e^{kt}$ where t is the year, with t = 0 corresponding to the year 1990. In 2000, the population was 180,000.

a. Find the value of k for the model. Round your result to four decimal places.

b. Use the model you find in part a to predict the population in 2020.

BE CAREFUL 1990 is NOT an x value for this model 1990 is x = 0 TIME IS WHERE PEOPLE mess this type of problem up!

5. The certain breed of rabbit was introduced onto a small island about 8 years ago. The current rabbit population on the island is estimated to be 4100 with a relative growth rate of 55%.

   a. Find a function that models the number of bacteria after t-years.

   b. What is the estimated count after 12 years? rabbits

   c. What was the initial rabbit population 8 years ago?

   d. When will the rabbit population reach 40,000?

6. A culture starts with 10,000 bacteria and the number doubles every 40 minutes.

   a. Find the number of bacteria after 1 hour.

   b. After how many minutes will there be 50,000 bacteria.

Practice Natural Growth & Decay Models

$P = 3.2$

① $\frac{1}{2}$ life is 4 days

$\frac{1}{2}$ of $3.2 = 1.6$

$A = \frac{1}{2}P = Pe^{r(4)}$ Or $1.6 = 3.2e^{4r}$

$\frac{\frac{1}{2}P}{P} = \frac{Pe^{4r}}{P}$ $\qquad \frac{1.6}{3.2} = \frac{3.2e^{4r}}{3.2}$

$\frac{1}{2} = e^{4r}$ $\qquad 0.5 = e^{4r}$

$\ln(\frac{1}{2}) = 4r$ $\qquad \ln 0.5 = 4r$

$$\boxed{r = \frac{\ln(\frac{1}{2})}{4} \quad r \approx -0.1733}$$

$$\boxed{r = \frac{\ln 0.5}{4} \quad r = -0.1733}$$

c)

$$\boxed{A = 3.2e^{\frac{\ln\frac{1}{2}}{4}x} \quad \text{or} \quad A = 3.2e^{-0.1733x}}$$ model

a) $$\boxed{A(8) = 3.2e^{\frac{\ln\frac{1}{2}}{4}(8)} \approx 0.8}$$

a) $$\boxed{A(8) = 3.2e^{-0.1733(8)} \approx 0.7999}$$

b) $$\boxed{A(20) = 3.2e^{\frac{\ln\frac{1}{2}(20)}{4}} \approx 0.1}$$

b) $$\boxed{A(20) = 3.2^{-0.1733(20)} \approx 0.09997}$$

② $P = 250$

$3P = 750$ in 6 days

$$\boxed{A(x) = 250e^{rx}}$$

$3P = Pe^{6r}$

$\frac{3P}{P} = \frac{Pe^{6r}}{P}$

$3 = e^{6r}$

$\ln 3 = 6r$

$$\boxed{r = \frac{\ln 3}{6} \quad r \approx 0.1831}$$

OR

$750 = 250e^{6r}$

$\frac{750}{250} = \frac{250e^{6r}}{250}$

$3 = e^{6r}$

$\ln 3 = 6r$

$$\boxed{r = \frac{\ln 3}{6} \quad r \approx 0.1831}$$

Model $\boxed{A(x) = 250e^{\frac{\ln 3}{6}x} \text{ or } A(x) = 250e^{0.1831x}}$

2I)

2II

$A(20) = 250e^{\frac{\ln 3}{6} \cdot 20}$

$= 9735.185$

≈ 9735 bacteria

or $A(20) = 250e^{0.1831(20)}$

≈ 9734.7858

≈ 9735 bacteria

#3 $A = 500e^{rx}$ $\frac{1}{2}$ life = 3 days

$\frac{1}{2}P = Pe^{r(3)}$ OR $250 = 500e^{r3}$

$\frac{\frac{1}{2}P}{P} = \frac{Pe^{3r}}{P}$ $\frac{250}{500} = \frac{500e^{3r}}{500}$

$0.5 = e^{3r}$

$\frac{1}{2} = e^{3r}$

$\ln(0.5) = 3r$

$\ln(\frac{1}{2}) = 3r$

$$\boxed{r = \frac{\ln\frac{1}{2}}{3} \quad r \approx -0.2310}$$

$$\boxed{r = \frac{\ln 0.5}{3} \quad r \approx -0.2310}$$

Model $A(x) = 500e^{\frac{\ln\frac{1}{2}}{3}x}$ or $A(x) = 500e^{-0.2310x}$

3I $A(20) = 500e^{\frac{\ln\frac{1}{2}}{3}(20)} \approx 4.9216$

or $A(20) = 500e^{-0.2310(20)} \approx 4.9264$

3II $A(x) = 275$

$275 = 500e^{\frac{\ln\frac{1}{2}}{3}x}$ or $275 = 500e^{-0.2310x}$

$\frac{275}{500} = e^{\frac{\ln\frac{1}{2}}{3}x}$ OR $\frac{275}{500} = e^{-0.2310x}$

$0.55 = e^{-0.2310x}$

$\ln\left(\frac{275}{500}\right) = \frac{\ln\frac{1}{2}}{3}x$

$\ln 0.55 = -0.2310$

$x = \frac{\ln\left(\frac{275}{500}\right)}{\ln\left(\frac{1}{2}\right)} \approx 2.587$ $x = \frac{\ln 0.55}{-0.2310} \approx 2.58$

after about 2.587 days

#4 $A(x) = 134e^{kx}$ → years since 1990

↙ pop in 1000's

Let $x=0$ be 1990 so

$2000 \Rightarrow x=10$

$2010 \Rightarrow x=20$

$2020 \Rightarrow x=30$

If $A(10) = \underset{\text{in 2000}}{\overset{\text{Reno's pop}}{180{,}000}} \rightarrow A(10) = 180$

$180 = 134e^{k10}$

$\frac{180}{134} = \frac{134e^{10k}}{134}$

$\frac{180}{134} = e^{10k}$

$\ln\left(\frac{180}{134}\right) = 10k$

4a) $k = r = \frac{\ln\left(\frac{180}{134}\right)}{10}$

$r \approx 0.02951$

$r\% = 2.951\%$

model $A(x) = 134e^{0.02951x}$

$A(x) = 134e^{\frac{\ln\left(\frac{180}{134}\right)}{10}x}$

4b) $A(30) = 134e^{0.02951(30)}$

≈ 324.7773

$A(30) = 134e^{\ln\left(\frac{180}{134}\right)\cdot 3}$

≈ 324.7939

Reno pop 2020

≈ 325000

5) $P = ?$

$A(8) = 4100$

growth rate 55%

$\boxed{r = 0.55}$

$A(8) = Pe^{0.55(8)}$

$4100 = Pe^{4.4}$

$P = \frac{4100}{e^{4.4}}$

$\boxed{P \simeq 50.3371}$

model

$$\boxed{\begin{aligned} A(x) &= \frac{4100}{e^{4.4}} e^{0.55x} \\ A(x) &\approx 50.3371 e^{0.55x} \end{aligned}}$$

5a

$\boxed{\text{vv } A(8) = 4100.0005}$

5b

$$\boxed{\begin{aligned} A(12) &= 50.3371 e^{0.55(12)} \simeq 37002.560 \\ &\simeq 37003 \end{aligned}}$$

5c) $P \approx 50$ rabbits

5d) $A(x) = 40000$

$40000 = 50.3371 e^{0.55x}$

$\frac{40000}{50.3371} = \frac{50.3371 e^{0.55x}}{50.3371}$

$e^{0.55x} = \frac{40000}{50.3371}$

$0.55x = \ln\left(\frac{40000}{\ldots}\right)$

$$\boxed{\begin{aligned} x &= \frac{\ln\left(\frac{40000}{50.3371}\right)}{0.55} \\ x &\simeq 12.142 \text{ years} \end{aligned}}$$

6 $P = 10000$ double time = 40 minutes

$A(40) = 20000$

$20000 = 10000e^{r(40)}$

$\frac{20000}{10000} = \frac{10000e^{40r}}{10000}$

$2 = e^{40r}$

$\ln 2 = 40r$

$\boxed{r = \frac{\ln 2}{40} \approx 0.0173}$

model $\boxed{A(x)} = 10000\, e^{\frac{\ln 2}{40} x}$ minutes

minutes

Population after x minutes

$\boxed{A(x) = 10000\, e^{0.0173x}}$

a) $A(x) = ?$ in 1 hr note 1hr = 60 minutes

$A(60) = 10000e^{\frac{\ln 2}{40}(60)} \approx 28284.2712$

$\approx 10000e^{0.0173(60)} \approx 28235.6$

$$\boxed{\begin{array}{l} \text{By Approx model} \approx 28236 \\ \text{By Exact model} \approx 28284 \end{array}}$$

6b

$$A(x) = 10000\, e^{\frac{\ln 2}{40} x}$$

$$A(x) = 10000\, e^{0.0173x}$$

$$50000 = 10000\, e^{\frac{\ln 2}{40} x}$$

$$\frac{50000}{10000} = \frac{10000\, e^{\frac{\ln 2}{40} x}}{10000}$$

$$5 = e^{\frac{\ln 2}{40} x}$$

$$\ln 5 = \frac{\ln 2}{40} x$$

$$\boxed{x = \frac{\ln 5}{\left(\frac{\ln 2}{40}\right)} \approx 92.877}$$

OR

$$50000 = 10000\, e^{0.0173x}$$

$$\frac{50000}{10000} = \frac{10000\, e^{0.0173x}}{10000}$$

$$5 = e^{0.0173x}$$

$$\ln 5 = 0.0173x$$

$$\boxed{x = \frac{\ln 5}{0.0173} \approx 93.0311}$$