Name____HWK Natural Growth and Decay Models Period ______

Assume all models and scenarios follow the natural growth or natural decay model

$Natural\ Growth\ Model\ A(x) = Pe^{rx}\ with\ r > 0$ $\qquad$ $Natural\ Decay\ Model\ A(x) = Pe^{-rx}\ with\ r > 0$

A(x) = population or amount after time x P = initial population or initial amount r = rate of change as a decimal

x = time (this changes dependent on the model and the wording of the model)

1. **The half-life of radium is about 1600 years. If 1 kg is present now, how much will be present after:**
   a. **3200 years?**
   b. **800 years?**
   c. **Write a model to represent the amount present after t-years.**

2. **An amount A_0 of a radioactive iodine has a half-life of 8.1 days. If the initial amount is 40 grams, how much is present after 5 days? How many days until the amount left is 15 grams?**

3. **The population P (in thousands) of Las Vegas, Nevada can be modeled by $P(t) = 258e^{kt}$ where t is the year, with t = 0 corresponding to the year 1990. In 2000, the population was 478,000.**
   a. **Find the value of k for the model. Round your result to four decimal places.**
   b. **Use the model you find in part a to predict the population in 2017.**

4. The initial bacterium count in a culture is 500. A biologist later makes a sample count of bacteria in the culture and finds that the relative rate of growth is 40% per hour.

   a. Find a function that models the number of bacteria after t-hours.
   b. What is the estimated count after 10 hours?
   c. How many hours before the population reaches half a million bacteria?

5. A culture starts with 1500 bacteria and the number doubles every 30 minutes.

   a. Find the number of bacteria after 2 hours.
   b. After how many minutes will there be 4000 bacteria?

6. The population of the world in 2000 was 6.1 billion, and the estimated relative growth rate was 1.4 % per year. If the population continues to grow at this rate, when will it reach 122 billion?

Hwk Natural Growth & Decay Models

(#1) $\frac{1}{2}$ life = 1600 years

$P = 1\text{kg}$

$\frac{1}{2}P = Pe^{r(1600)}$ OR $\frac{1}{2}(1) = 1e^{1600r}$

$\frac{\frac{1}{2}P}{P} = \frac{Pe^{1600r}}{P}$

$\frac{1}{2} = e^{1600r}$

$\frac{1}{2} = e^{1600r}$

$\ln\frac{1}{2} = 1600r$

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$r = \frac{\ln\frac{1}{2}}{1600}$

$r \approx -0.0004332$

$r = \frac{\ln\frac{1}{2}}{1600}$

$r \approx -0.0004332$

(1c) Model $A(x) = 1\left(e^{\frac{\ln\frac{1}{2}}{1600}x}\right)$ OR $A(x) = e^{-0.0004332(x)}$

1a) $A(3200) = e^{\frac{\ln 1/2}{1600}(3200)}$ OR $A(3200) = e^{-0.0004332(3200)}$

$= 0.25$ ≈ 0.2500135

1b) $A(800) = e^{\frac{\ln\frac{1}{2}}{1600}(800)}$ or $A(800) = e^{-0.0004332(800)}$

$= 0.707107$ or $A(800) \approx 0.707116$

(1c) See Above

#2 HWK $A(x) = Pe^{rx}$

$\frac{1}{2}$ life $= 8.1$

$\frac{1}{2}P = Pe^{r(8.1)}$

$\frac{\frac{1}{2}P}{P} = \frac{Pe^{8.1r}}{P}$

$\frac{1}{2} = e^{8.1r}$

$\ln\left(\frac{1}{2}\right) = 8.1r$

$\frac{\ln\frac{1}{2}}{8.1} = r$

$r \approx -0.08557$

$P = 40$

$20 = 40e^{r(8.1)}$

$\frac{20}{40} = \frac{40e^{8.1r}}{40}$

$0.5 = e^{8.1r}$

$\ln 0.5 = 8.1r$

$r = \frac{\ln 0.5}{8.1}$

$r \approx -0.08557$

model

$A(x) \approx 40e^{-0.08557x}$

$A(x) = 40e^{\frac{\ln(\frac{1}{2})}{8.1}x}$

2a

$A(5) = 40e^{-0.08557(5)} \approx 26.0764$

$A(5) = 40^{\frac{\ln(\frac{1}{2})}{8.1}\cdot 5} \approx 26.0759$

2b

$15 = 40e^{-0.08557x}$

$\frac{15}{40} = e^{-0.08557x}$

$\ln\left(\frac{15}{40}\right) = -0.08557x$

$x = \frac{\ln\left(\frac{15}{40}\right)}{-0.08557}$

$x \approx 11.462$

#3 LV Pop 258000 in 1990

Model $A(x) = 258e^{kx}$ x = years since 1990

$A(x)$ → pop in 1000's

$t_{2000} = 10$

$t_{1990} = 0$

$t_{2010} = 20$

$t_{2020} = 30$

$t_{2017} = 27$

$A(10) = 479000$

↓

$478 = 258e^{k(10)}$

$\frac{478}{258} = \frac{258e^{10k}}{258}$

$\frac{478}{258} = e^{10k}$

$\ln\left(\frac{478}{258}\right) = 10k$

3a

$K = \frac{\ln\left(\frac{478}{258}\right)}{10} \approx$

≈ 0.061665

Model

$A(x) = 258e^{\frac{\ln\left(\frac{478}{258}\right)}{10}x}$

$A(x) = 258e^{0.061665x}$

$A(27) = 1363.6$

↓

LV pop in 2017

3b

So Las Vegas pop in 2017 $\approx$ 1,364,000

(4) $P = 500$ $r = 40\%$ per hour

$A(x) = 500 e^{rx}$ $r = 0.40$

$$A(x) = 500 e^{0.40x}$$ (4a)

$$A(10) = 500 e^{0.40(10)} = 27299.075$$ (4b)

$$\approx 27299$$

$$A(x) = 500000$$

$$500000 = 500 e^{0.40x}$$

$$\frac{500000}{500} = \frac{500 e^{0.40x}}{500}$$

$$1000 = e^{0.40x}$$

$$\ln 1000 = 0.40x$$

$$x = \frac{\ln 1000}{0.40} \approx 17.269$$

(5) $A(x) = 1500e^{rx}$

Double time 30 minutes

$A(30) = 3000$

$3000 = 1500e^{r(30)}$

$\frac{3000}{1500} = \frac{1500e^{30r}}{1500}$

$2 = e^{30r}$

$\ln 2 = 30r$

$r = \frac{\ln 2}{30} \approx 0.0231$

Model $A(x) = 1500e^{\frac{\ln 2}{30}x}$

$A(x) = 1500e^{0.0231x}$

(5a) $A(x) = ?$ after 2 hours = 120 minutes

$A(120) = 1500e^{\frac{\ln 2}{30}(120)} \approx 24000$

$A(120) = 1500e^{0.0231(120)} \approx 23985.875$

(5b) $A(x) = 4000 \rightarrow$

$4000 = 1500e^{\frac{\ln 2}{30}x}$

$\frac{4000}{1500} = \frac{1500e^{\frac{\ln 2}{30}x}}{1500}$

$\frac{8}{3} = e^{\frac{\ln 2}{30}x}$

$\ln 8/3 = \frac{\ln 2}{}x$

$x \approx 42.451$

$x = \left(\frac{\ln 8/3}{\ln 2}\right)$

OR

$\frac{8}{3} = e^{0.0231x}$

$\ln\left(\frac{8}{3}\right) = 0.0231x$

$x = \frac{\ln\left(\frac{8}{3}\right)}{} \approx 42.460$

(6) Let t=0 be 2000   x = years since 2000

$P = 6.1$

$r = 4.1\%$ per year

$r = 0.041$

$A(x) = 6.1 e^{0.041x}$

A(x): in billions of people

x: years since 2000

$A(x) = 12.2$ billion

$$12.2 = 6.1 e^{0.041x}$$

$$\frac{12.2}{6.1} = e^{0.041x}$$

$$\ln\left(\frac{12.2}{6.1}\right) = 0.041x$$

$$x = \frac{\ln\left(\frac{12.2}{6.1}\right)}{0.041} \approx 73.0666$$

In the year 2073 according to model