LOGARITHMS AND THEIR PROPERTIES

Definition of a logarithm: If $x > 0$ and b is a constant ($b \neq 1$), then $y = \log_b x$ if and only if $b^y = x$.
In the equation $y = \log_b x$, y is referred to as the **logarithm,** b is the **base,** and x is the **argument.**

The notation $\log_b x$ is read "the logarithm (or log) base b of x." The definition of a logarithm indicates that *a logarithm is an exponent.*

$y = \log_b x$ is the logarithmic form of $b^y = x$

$b^y = x$ is the exponential form of $y = \log_b x$

Examples of changes between logarithmic and exponential forms:

Write each equation in its exponential form.

a. $2 = \log_7 x$ **b.** $3 = \log_{10}(x + 8)$ **c.** $\log_5 125 = x$

Solution:

Use the definition $y = \log_b x$ if and only if $b^y = x$.

a.

Logarithms are exponents

$2 = \log_7 x$ if and only if $7^2 = x$

Base

b. $3 = \log_{10}(x + 8)$ if and only if $10^3 = (x + 8)$.

c. $\log_5 125 = x$ if and only if $5^x = 125$.

Write the following in its logarithmic form: $x = 25^{1/2}$

Solution:

Use $x = b^y$ if and only if $y = \log_b x$.

Exponent

$x = 25^{1/2}$ *if and only if* $\frac{1}{2} = \log_{25} x$

Base

Equality of Exponents Theorem: If b is positive real number ($b \neq 1$) such that $b^x = b^y$, then $x = y$.

Example of Evaluating a Logarithmic Equation:

Evaluate: $\log_2 32 = x$

Solution:

$\log_2 32 = x$ if and only if $2^x = 32$

Since $32 = 2^5$, we have $2^x = 2^5$

Thus, by Equality of Exponents, $x = 5$

PROPERTIES OF LOGARITHMS:

If b, a, and c are positive real numbers, $b \neq 1$, and n is a real number, then:

1. Product: $\log_b(a \cdot c) = \log_b a + \log_b c$
2. Quotient: $\log_b \frac{a}{c} = \log_b a - \log_b c$
3. Power: $\log_b a^n = n \cdot \log_b a$
4. $\log_b 1 = 0$
5. $\log_b b = 1$
6. Inverse 1: $\log_b b^n = n$
7. Inverse 2: $b^{\log_b n} = n,\ n > 0$
8. One-to-One: $\log_b a = \log_b c$ if and only if $a = c$
9. **Change of Base:** $\log_b a = \frac{\log_c a}{\log_c b} = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$

Examples – Rewriting Logarithmic Expressions Using Logarithmic Properties:

Use the properties of logarithms to rewrite each expression as a single logarithm:

a. $2\log_b x + \frac{1}{2}\log_b(x+4)$

b. $4\log_b(x+2) - 3\log_b(x-5)$

Solution:

a. $2\log_b x + \frac{1}{2}\log_b(x+4)$

$= \log_b x^2 + \log_b(x+4)^{1/2}$ **Power Property**

$= \log_b\left[x^2(x+4)^{1/2}\right]$ **Product Property**

b. $4\log_b(x+2) - 3\log_b(x-5)$

$= \log_b(x+2)^4 - \log_b(x-5)^3$ **Power Property**

$= \log_b \frac{(x+2)^4}{(x-5)^3}$ **Quotient Property**

Use the properties of logarithms to express the following logarithms in terms of logarithms of $x, y,$ and z.

a. $\log_b(xy^2)$

b. $\log_b \frac{x^2\sqrt{y}}{z^5}$

Solution:

a. $\log_b(xy^2) = \log_b x + \log_b y^2$ **Product Property**

$= \log_b x + 2\log_b y$ **Power Property**

b. $\log_b \frac{x^2\sqrt{y}}{z^5}$

$= \log_b(x^2\sqrt{y}) - \log_b z^5$ **Quotient Property**

$= \log_b(x^2\sqrt{y}) - \log_b z^5$ **Quotient Property**

$= \log_b x^2 + \log_b \sqrt{y} - \log_b z^5$ **Product Property**

$= 2\log_b x + \frac{1}{2}\log_b y - 5\log_b z$ **Power Property**

Other Logarithmic Definitions:

- Definition of **Common Logarithm**:

Logarithms with a base of 10 are called **common logarithms.** It is customary to write $\log_{10} x$ as $\log x$.

- Definition of **Natural Logarithm:**

Logarithms with the base of e are called **natural logarithms.** It is customary to write $\log_e x$ as $\ln x$.

PRACTICE PROBLEMS

Evaluate:

1. $0.6^{\sqrt{3}}$ 2. $e^{3.2}$ 3. $(1.005)^{400}$ 4. $\log_4 64$ 5. $\ln 1$ 6. $\ln\sqrt{7}$

Rewrite into logarithms:

7. $2^4 = 16$ 8. $\sqrt{64} = 8$ 9. $e^4 = 54.60$

Evaluate without a calculator:

10. $\log_5 25$ 11. $\log_3 \frac{1}{81}$ 12. $\ln e^{-2}$

Use the change of base formula to evaluate the logarithms: (Round to 3 decimal places.)

13. $\log_7 3$ 14. $\log_2 \frac{1}{2}$ 15. $\log_{15} 42$

Use the properties of logarithms to rewrite each expression into lowest terms (i.e. expand the logarithms to a sum or a difference):

16. $\log 10x$ 19. $\log_4 4x^2$ 22. $\ln\frac{\sqrt{3x}}{7}$

17. $\ln\frac{xy}{z}$ 20. $\log_3 \sqrt{x-2}$

18. $\log_b \frac{x^4}{z^2}$ 21. $\ln\frac{x^5z^2}{y^3}$

Write each expression as a single logarithmic quantity:

23. $\log 7 - \log x$ 26. $\log_2 5 + \log_2 x - \log_2 3$ 29. $\frac{1}{2}\log_5 7 - 2\log_5 x$

24. $3\ln x + 2\ln y - 4\ln z$ 27. $1 + 3\log_4 x$

25. $\frac{3}{2}\ln x^6 - \frac{3}{4}\ln x^8$ 28. $2\ln 8 + 5\ln x$

Using properties of logarithms find the following values if:

$\log_b 3 = 0.562$ $\log_b 2 = 0.356$ $\log_b 7 = 0.872$

30. $\log_b 18$ 31. $\log_b \sqrt{28}$ 32. $\log_b \frac{1}{21}$ 33. $\log_b 3b^2$ 34. $\log_b 1$

Write the exponential equation in logarithmic form:

35. $4^3 = 64$ 36. $25^{3/2} = 125$

Write the logarithmic equation in exponential form:

37. $\ln e = 1$ 38. $\log_3 \frac{1}{9} = -2$

Evaluate the following logarithms without a calculator:

39. $\log 1000$ 42. $\log_4 \frac{1}{16}$ 45. $\ln 1$

40. $\log_9 3$ 43. $\ln e^7$ 46. $\ln e^{-3}$

41. $\log_3 \frac{1}{9}$ 44. $\log_a \frac{1}{a}$

Evaluate the following logarithms for the given values of x:

47. $f(x) = \log_3 x$

a. $x = 1$ b. $x = 27$ c. $x = 0.5$

48. $g(x) = \log x$

a. $x = 0.01$ b. $x = 0.1$ c. $x = 30$

49. $f(x) = \ln x$

a. $x = e$ b. $x = \frac{1}{3}$ c. $x = 10$

50. $h(x) = \ln x$

a. $x = e^2$ b. $x = \frac{5}{4}$ c. $x = 1200$

51. $g(x) = \ln e^{3x}$

a. $x = -2$ b. $x = 0$ c. $x = 7.5$

52. $f(x) = \log_2 \sqrt{x}$

a. $x = 4$ b. $x = 64$ c. $x = 5.2$

Use the change of base formula to evaluate the following logarithms: (Round to 3 decimal places.)

53. $\log_4 9$ 54. $\log_{1/2} 5$ 55. $\log_{12} 200$ 56. $\log_3 0.28$

Approximate the following logarithms given that $\log_5 2 \approx 0.43068$ and $\log_5 3 \approx 0.68261$:

57. $\log_5 18$

58. $\log_5 \sqrt{6}$

59. $\log_5 \frac{1}{2}$

60. $\log_5 \frac{2}{3}$

61. $\log_5 (12)^{2/3}$

62. $\log_5 (5^2 \cdot 6)$

Use the properties of logarithms to expand the expression:

63. $\log_4 6x^4$

64. $\log 2x^{-3}$

65. $\log_5 \sqrt{x+2}$

66. $\ln \sqrt[3]{\frac{x}{5}}$

67. $\ln \frac{x+2}{x-2}$

68. $\ln x(x-3)^2$

69. $\ln\left[\sqrt{2x}(x+3)^5\right]$

70. $\log_3 \frac{a^2\sqrt{b}}{cd^5}$

Use the properties of logarithms to condense the expression:

71. $-\frac{2}{3}\ln 3y$

72. $5\log_2 y$

73. $\log_8 16x + \log_8 2x^2$

74. $\log_4 6x - \log_4 10$

75. $-2(\ln 2x - \ln 3)$

76. $4(1 + \ln x + \ln x)$

77. $4[\log_2 k - \log_2(k-t)]$

78. $\frac{1}{3}(\log_8 a + 2\log_8 b)$

79. $3\ln x + 4\ln y + \ln z$

80. $\ln(x+4) - 3\ln x - \ln y$

True or False? Use the properties of logarithms to determine whether the equation is true or false. If false, state why or give an example to show that it is false.

81. $\log_2 4x = 2\log_2 x$

82. $\frac{\ln 5x}{\ln 10x} = \ln\frac{1}{2}$

83. $\log 10^{2x} = 2x$

84. $e^{\ln t} = t$

85. $\log_4 \frac{16}{x} = 2 - \log_4 x$

86. $6\ln x + 6\ln y = \ln(xy)^6$

Practice Problems

(1) $0.6^{\sqrt{3}} \approx$

(2) $e^{3.2} \approx$

(3) $1.005^{400} \approx$

(4) $\log_4 64 = \cancel{\log_4 4^3} = 3$

$= \log_4 2^6 = 6\log_4 2$

$= 6\left(\frac{1}{2}\right) = 3$

(5) $\ln 1 = 0$

(6) $\ln\sqrt{7} = \ln 7^{1/2}$

$= \frac{1}{2}\ln 7$

$\approx$

(7) $2^4 = 16 \Longleftrightarrow \log_2 16 = 4$

(8) $\sqrt{64} = 8 \rightarrow 64^{\frac{1}{2}} = 8 \Longleftrightarrow \log_{64} 8 = \frac{1}{2}$

(9) $e^4 = 54.60 \Longleftrightarrow \ln 54.60 = 4$

(10) $\log_5 25 = \log_5 5^2 = 2\log_5 5 = 2(1) = 2$

(11) $\log_3 \frac{1}{81} = \log_3 1 - \log_3 81$

$= 0 - \log_3 3^4$

$= 0 - 4\log_3 3$

$= 0 - 4(1)$

$= -4$

$\log_3 \frac{1}{81} = \log_3 \frac{1}{3^4}$

$= \log_3 3^{-4}$

$= -4\log_3 3$

$= -4(1)$

$= -4$

$\ln e^{-2} = \cancel{\log_e e^{-2}} = -2$

(13) $\log_7 3 = \dfrac{\log 3}{\log 7} \approx$

(14) $\log_2 \frac{1}{2} = \dfrac{\log \frac{1}{2}}{\log 2} =$

$\log_2 2^{-1} = -1 \log_2 2 = -1$

(15) $\log_{15} 42 = \dfrac{\log 42}{\log 15} \approx$

(16) $\log_{10} 10x = \log_{10} 10 + \log_{10} x$

$= 1 + \log_{10} x$

(17) $\ln\left(\dfrac{xy}{z}\right) = \ln xy - \ln z$

$= \ln x + \ln y - \ln z$

(18) $\log_b \dfrac{x^4}{z^2} = \log_b x^4 - \log_b z^2$

$= 4 \log_b x - 2 \log_b z$

(19) $\log_4 4x^2 = \log_4 4 + \log_4 x^2$

$= 1 + 2 \log_4 x$

(20) $\log_3 \sqrt{x-2} = \log_3 (x-2)^{1/2} = \frac{1}{2} \log_3 (x-2)$

$\ln \frac{x^5 z^2}{y^3} = \ln x^5 z^2 - \ln y^3$

$= \ln x^5 + \ln z^2 - \ln y^3$

$= 5\ln x + 2\ln z - 3\ln y$

(22) $\ln \frac{\sqrt{3x}}{7} = \ln\left(\frac{(3x)^{1/2}}{7}\right)$

$= \ln(3x)^{1/2} - \ln 7$

$= \frac{1}{2}\ln(3x) - \ln 7$

$= \frac{1}{2}[\ln 3 + \ln x] - \ln 7$

$= \frac{1}{2}\ln 3 + \frac{1}{2}\ln x - \ln 7$

Also $\ln\left(\frac{\sqrt{3x}}{7}\right) = \ln \frac{\sqrt{3}\sqrt{x}}{7} = \ln\left(\frac{3^{1/2}x^{1/2}}{7}\right)$

$= \ln 3^{\frac{1}{2}} x^{\frac{1}{2}} - \ln 7$

$= \ln 3^{1/2} + \ln x^{\frac{1}{2}} - \ln 7$

$= \frac{1}{2}\ln 3 + \frac{1}{2}\ln x - \ln 7$

(23) $\log 7 - \log x = \log\left(\frac{7}{x}\right)$

(24) $3\ln x + 2\ln y - 4\ln z = \ln x^3 + \ln y^2 - \ln z^4$

$= \ln x^3 y^2 - \ln z^4 = \ln\left(\frac{x^3 y^2}{z^4}\right)$

(25) $\frac{3}{2}\ln x^6 - \frac{3}{4}\ln x^8$

$\ln(x^6)^{3/2} - \ln(x^8)^{3/4}$

$\ln x^{18/2} - \ln x^{\frac{24}{4}}$

$\ln x^9 - \ln x^6$

$\ln \frac{x^9}{x^6} = \ln\left(x^{9-6}\right) = \boxed{\ln x^3}$

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(26) $\log_2 5 + \log_2 x - \log_2 3$

$\log_2 5x - \log_2 3$

$\log_2\left(\frac{5x}{3}\right)$

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(27) $1 + 3\log_4 x$

$1 + \log_4 x^3$

Note $1 = \log_b b$ or $\log_4 4$

$\log_4 4 + \log_4 x^3$

$\boxed{\log_4 4x^3}$

(28) $2\ln 8 + 5\ln x$

$\ln 8^2 + \ln x^5$

$\ln 64 + \ln x^5$

$\ln 64x^5$ also ok $\ln 8^2x^5$

$\ln 2^6x^5$

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(29) $\frac{1}{2}\log_5 7 - 2\log_5 x$

$\log_5 7^{1/2} - \log_5 x^2$

$$\boxed{\begin{array}{l} \log_5 \dfrac{7^{1/2}}{x^2} \\ \\ \log_5 \dfrac{\sqrt{7}}{x^2} \end{array}}$$

Provided $\log_b 3 = 0.562$ $\log_b 2 = 0.356$

$\log_b 7 = 0.872$

(#30) $\log_b 18 = \log_b 2 \cdot 3^2 = \log_b 2 + \log_b 3^2$

$= \log_b 2 + 2\log_b 3$

$= (0.356) + 2(0.562)$

≈ 1.48

18

2 9

2 3 3

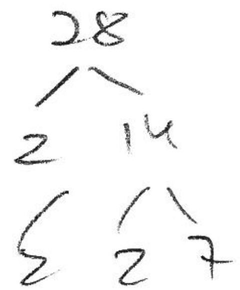
$18 = 2 \cdot 3^2$

Provided that $\log_b 2 = 0.356$

$\log_b 3 = 0.562$

$\log_b 7 = 0.872$

#31 $\log_b \sqrt{28} = \log_b \sqrt{2^2 \cdot 7} = \log_b 2^1 7^{1/2}$



$28 = 2^2 \cdot 7$

$= \log_b 2 + \log_b 7^{1/2}$

$= \log_b 2 + \frac{1}{2} \log_b 7$

$= 0.356 + \frac{1}{2}(0.872)$

≈ 0.792

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32 $\log_b \frac{1}{21} = \log_b 1 - \log_b 21$

$= 0 - \log_b (3 \cdot 7)$

$= -[\log_b 3 + \log_b 7]$

$= -\log_b 3 - \log_b 7$

$= -0.562 - 0.872$

≈ -1.434

Provided that $\log_b 2 = 0.356$

$\log_b 3 = 0.562$

$\log_b 7 = 0.872$

#33 $\log_b 3b^2 = \log_b 3 + \log_b b^2$

$= \log_b 3 + 2\log_b b$

$= \log_b 3 + 2(1)$

$= \log_b 3 + 2$

$= 0.562 + 2$

$= 2.562$

#34 $\log_b 1 = 0$

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35 $4^3 = 64 \Leftrightarrow \log_4 64 = 3$

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36 $25^{3/2} = 125 \Leftrightarrow \log_{25} 125 = 3/2$

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37 $\ln e = 1$

$\log_e e^1 = 1 \Leftrightarrow e^1 = e^1$

(38) $\log_3 \frac{1}{9} = -2 \iff 3^{-2} = \frac{1}{9}$

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(39) $\log 1000 = \log_{10} 10^3 = 3\log_{10} 10 = 3(1)$

$= 3$

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(42) $\log_4 \frac{1}{16} = \log_4 1 - \log_4 16$

$= 0 - \log_4 4^2$

$= 0 - 2\log_4 4$

$= 0 - 2(1)$

$= -2$

OR $\log_4 \frac{1}{16} = \log_4 16^{-1} = -1\log_4 16$

$= -1\log_4 16 = -1\log_4 4^2$

$= -2\log_4 4$

$= -2(1)$

$= -2$

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(40) $\log_9 3 = n \iff 9^n = 3^1 \iff 3^{2n} = 3^1$

$\iff 2n = 1$

$n = 1/2$

one to one prop.

so $\log_9 3 = \frac{1}{2}$

(41) $\log_3 \frac{1}{9} = \log_3 1 - \log_3 9$

$= 0 - \log_3 3^2$

$= -2 \log_3 3$

$= -2(1) = -2$

$\log_3 \frac{1}{9} = \log_3 3^{-2} = -2 \log_3 3$

$= -2(1) = -2$

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(43) $\ln e^7 = 7 \ln e = 7(1) = 7$

$\log_e e^7 = 7 \log_e e = 7(1) = 7$

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(44) $\log_9 \frac{1}{9} = \log_9 9^{-1} = -1 \log_9 9$

$= -1(1) = -1$

or $\log_9 \frac{1}{9} = \log_9 1 - \log_9 9$

$= 0 - 1$

$= -1$

(45) $\ln 1 = 0$

$\log_e 1 = 0 \Leftrightarrow e^0 = 1$

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(46) $\ln e^{-3} = -3 \ln e = -3(1) = -3$

or $\log_e e^{-3} = -3 \log_e e = -3(1) = -3$

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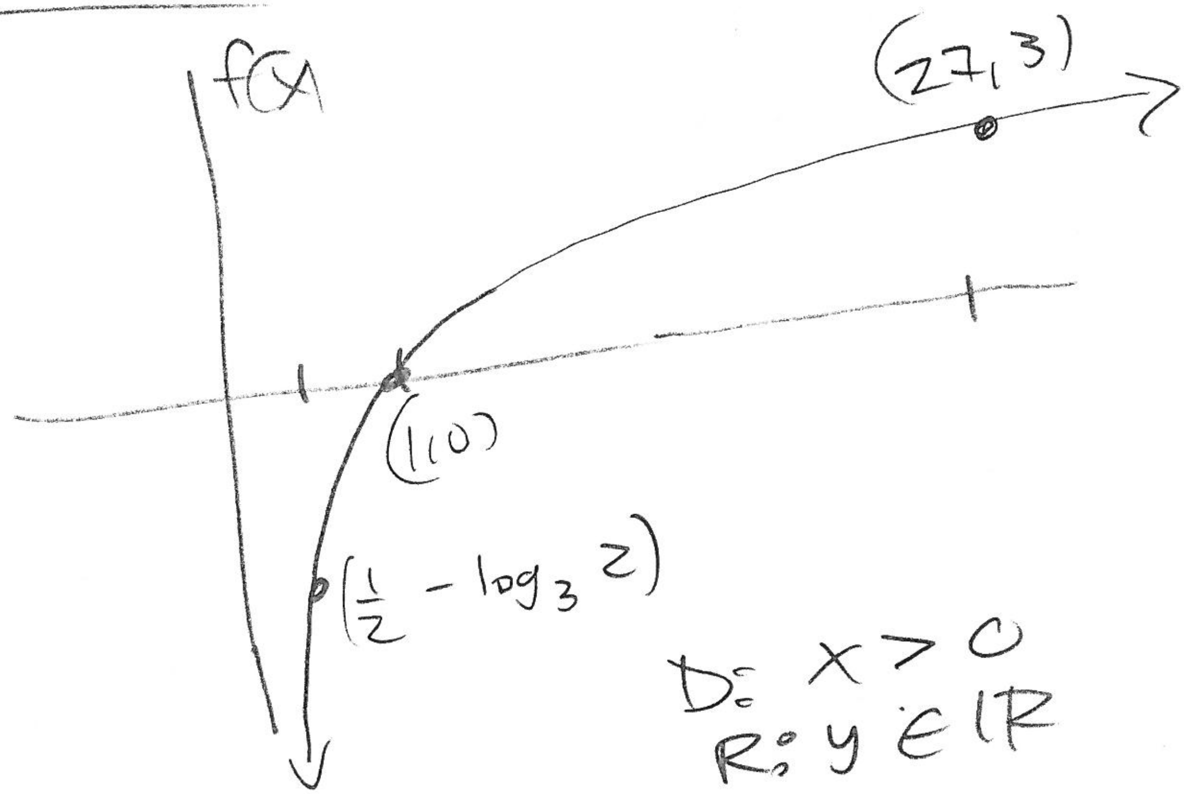
(47) $f(x) = \log_3 x$

a) $(1, f(1)) = (1, \log_3 1) = (1, 0)$

b) $(27, f(27)) = (27, \log_3 27)$
$= (27, \log_3 3^3)$
$= (27, 3 \log_3 3)$
$= (27, 3(1))$
$= (27, 3)$

c) $(\frac{1}{2}, f(\frac{1}{2})) = (\frac{1}{2}, \log_3 \frac{1}{2})$
$= (\frac{1}{2} \log_3 1 - \log_3 2)$
$= (\frac{1}{2}, 0 - \log_3 2)$
$= (\frac{1}{2}, -\log_3 2)$

Graph of 47 $f(x) = \log_3 x$



48 $g(x) = \log x$

a) $x = 0.01 = \frac{1}{100} = 100^{-1} = 10^{-2}$

$g(0.01) = \log_{10} \frac{1}{100} = \log_{10} 10^{-2} = -2 \log_{10} 10$

$= -2(1) = -2$ $\quad (0.01, -2)$

b) $x = \frac{1}{10} = 0.1 = 10^{-1}$

$g(\frac{1}{10}) = \log_{10} \frac{1}{10} = \log_{10} 1 - \log_{10} 10$

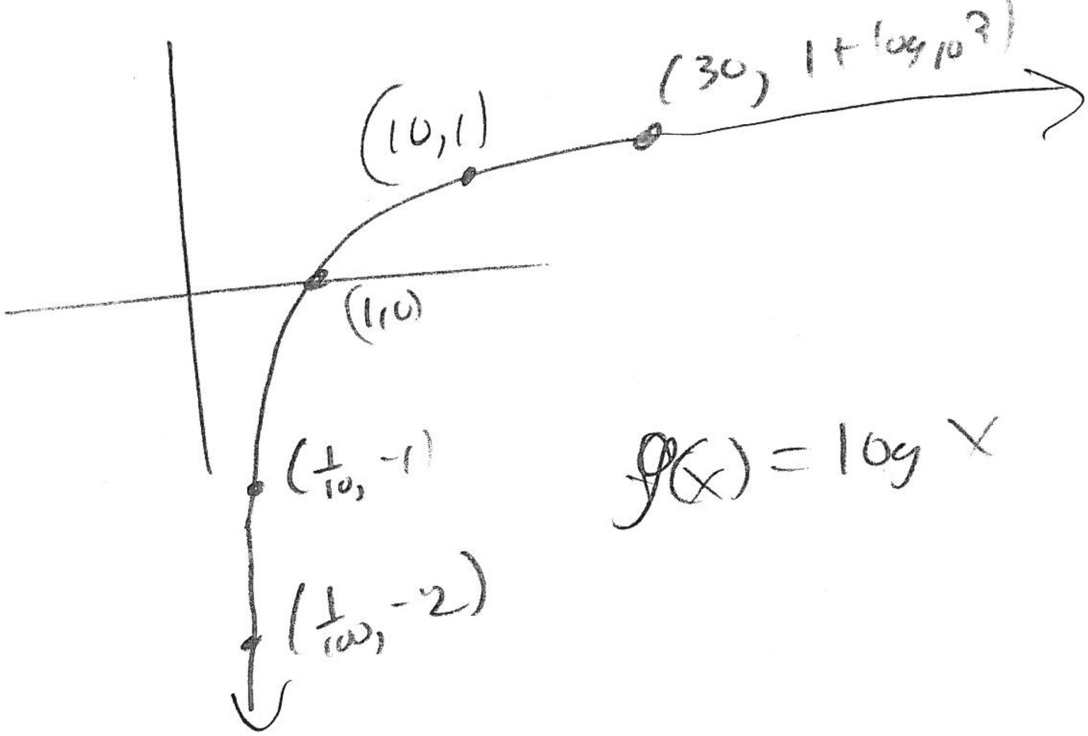
$= 0 - 1 = -1$

$(0.1, -1)$

(48) $g(x) = \log x$

c) $x = 30 = 3 \cdot 10$

$$\log 30 = \log 3 + \log 10$$
$$= \log 3 + \log_{10} 10$$
$$= 1 + \log_{10} 3$$



(49) $f(x) = \ln x$

a) $x = e$ $\quad f(e) = \ln e = \log_e e^1 = 1$

$(e, 1)$

b) $x = \frac{1}{3}$ $\quad f(\frac{1}{3}) = \ln \frac{1}{3} = \ln 1 - \ln 3$

$= 0 - \ln 3$

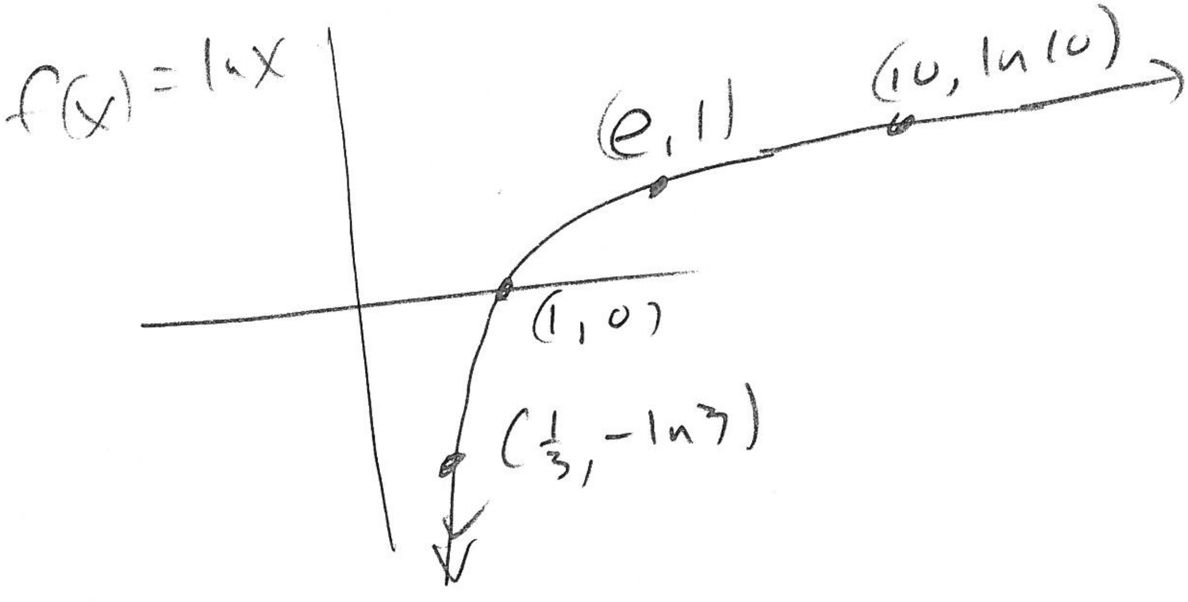
$= -\ln 3$

$(\frac{1}{3}, -\ln 3)$

c) $x = 10$ $\quad f(10) = \ln 10$

$(10, \ln 10)$ $\quad 10 > e^2$

$(10, \ln 10) > (10, 2)$



54 $h(x) = \ln x$

a) $x = e^2$ $\quad h(e^2) = \ln e^2$

$= \log_e e^2$

$= 2\log_e e$

$= 2(1)$

$= 2$

$(e^2, 2)$

b) $x = \frac{5}{4}$ $\quad h\left(\frac{5}{4}\right) = \ln \frac{5}{4}$

$= \ln 5 - \ln 4$

$\left(\frac{5}{4}, \ln \frac{5}{4}\right)$

c) $x = 1200$ $\quad h(1200) = \ln(1200)$

1200
400 3
20 20 3
4 5 4 5 3
2 2 5 2 2 5 3

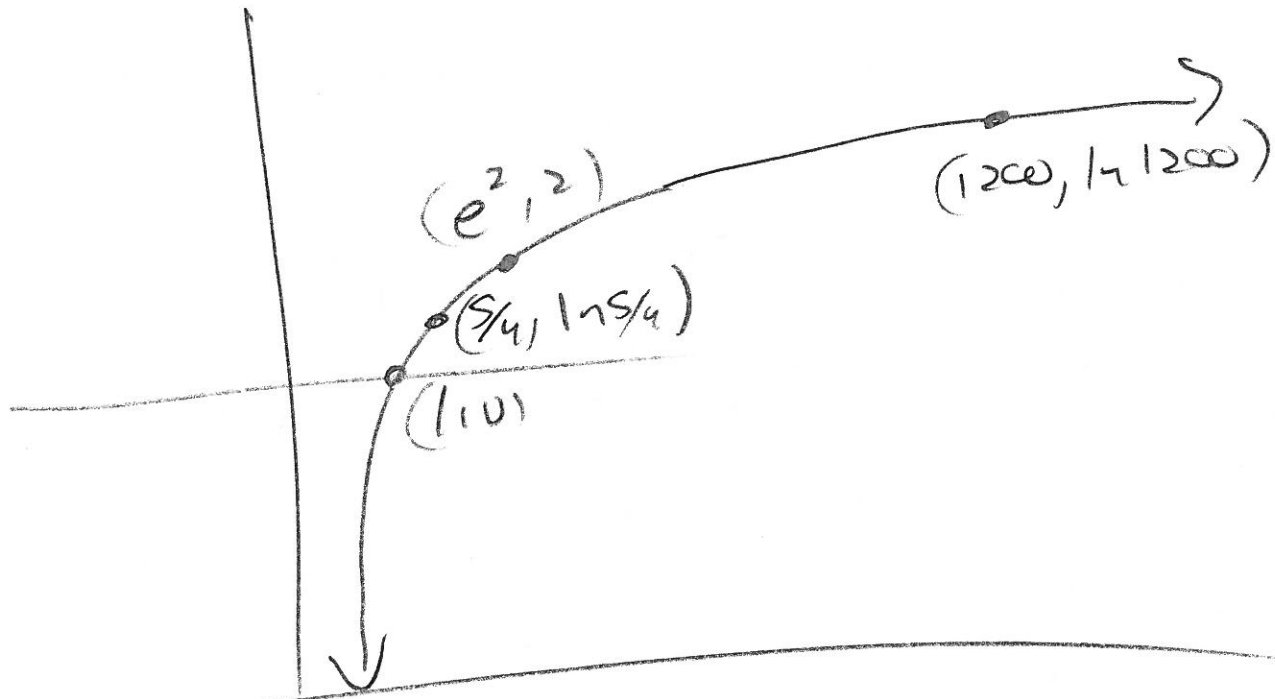
$1200 = 2^4 5^2 3^1$

$\ln(1200) = \ln(2^4 5^2 3^1)$

$= \ln 2^4 + \ln 5^2 + \ln 3$

$= 4\ln 2 + 2\ln 5 + \ln 3$

50 $h(x) = \ln x$



51 $g(x) = \ln e^{3x} = 3x \ln e = 3x \cdot 1$

$g(x) = 3x$

a) $\cancel{x} = -2$ $\quad g(-2) = 3(-2) = -6$

✓✓ $\ln e^{3(-2)} = \ln e^{-6} = \log_e e^{-6} = -6 \log_e e$

$= -6$ $\quad (-2, -6)$

b) $x = 0$ $\quad g(0) = 3(0) = 0$

✓✓ $\ln e^{3(0)} = \ln e^{0} = 0 \ln e = 0$

$= \ln 1 = 0$ $\quad (0, 0)$

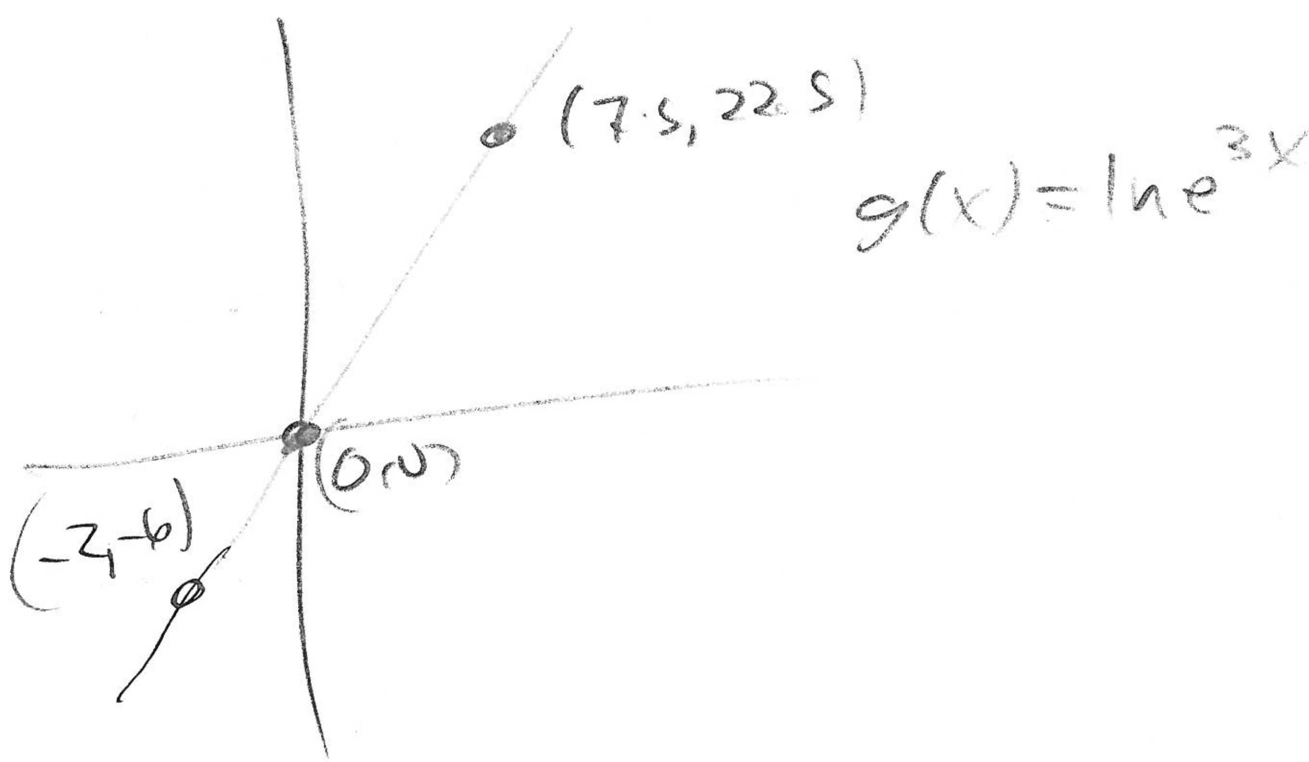
(51) $g(x) = \ln e^{3x} = 3x$

$g(7.5) = \ln e^{3(7.5)} = \ln e^{22.5}$

$g(7.5) = 22.5$

or $g(7.5) = 3(7.5) = 22.5$

$(7.5, 22.5)$



(52) $f(x) = \log_2 \sqrt{x}$

$= \log_2 x^{1/2}$

$= \frac{1}{2} \log_2 x$

a) $x = 4$ $\quad f(4) = \log_2 \sqrt{4} = \log_2 2 = 1$

$f(4) = \frac{1}{2} \log_2 4$

$= \frac{1}{2} \log_2 2^2 = \frac{1}{2}(2) \log_2 2 = 1 \cdot 1$

$= 1$

$(4, 1)$

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b) $x = 64$ $\quad f(x) = \log_2 \sqrt{64} = \log_2 8$

$= \log_2 2^3 = 3 \log_2 2$

$= 3(1) = 3$

$(64, 3)$

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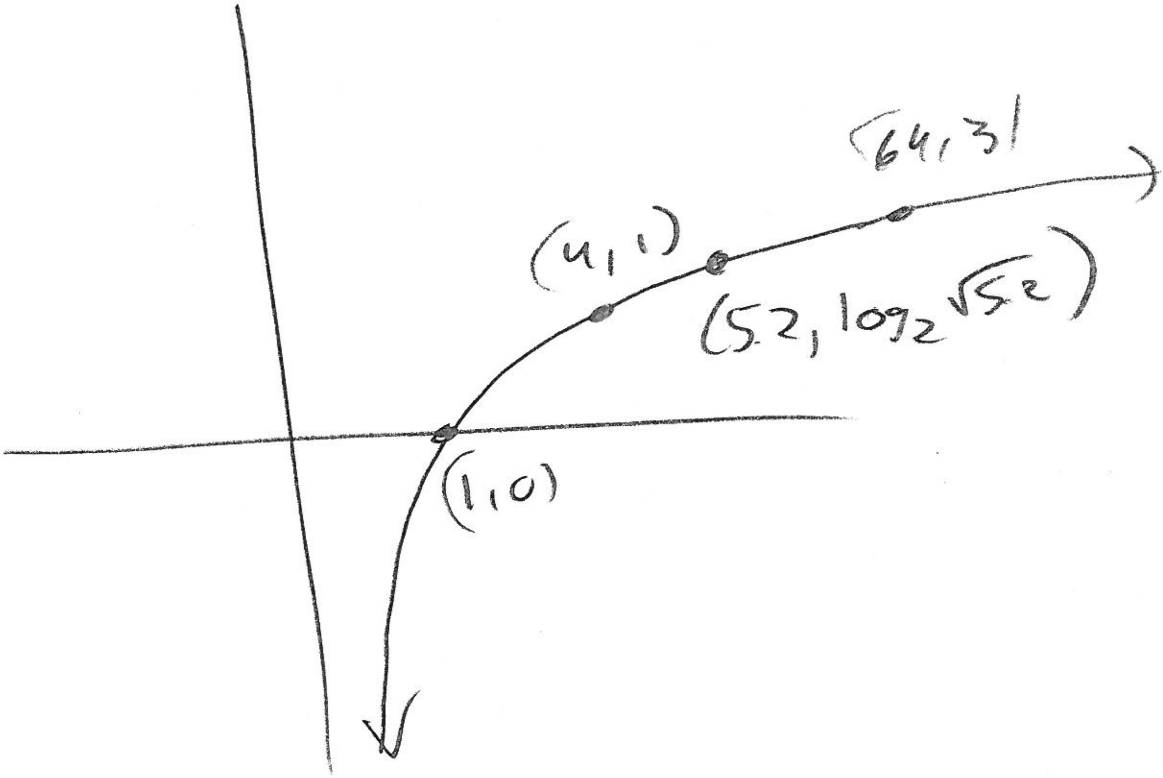
c) $x = 5.2$ $\quad f(5.2) = \log_2 \sqrt{5.2}$

$= \frac{1}{2} \log_2 5.2$

$(5.2, \log_2 \sqrt{5.2})$

(52)



(53) $\log_4 9 = \dfrac{\log 9}{\log 4} \simeq 1.585$

✓✓ $4^{1.585} \simeq 9.0005$

(54) $\log_{1/2} 5 = \dfrac{\log 5}{\log \frac{1}{2}} \simeq -2.322$ ✓✓

$\frac{1}{2}^{-2.322} \simeq 5.00$

(55) $\log_{12} 200 = \dfrac{\log 200}{\log 12} \simeq 2.132$

✓✓ $12^{2.132} \simeq 199.9$

(56) $\log_3 0.28 = \dfrac{\log 0.28}{\log 3} \simeq -1.159$

✓✓ $3^{-1.159} \simeq 0.2799$

Provided that $\log_5 2 = 0.43068$

$\log_5 3 = 0.68261$

57) $\log_5 18 = \log_5 (3^2 \cdot 2)$

$= \log_5 3^2 + \log_5 2$

$18 = 3^2 \cdot 2$

$= 2 \log_5 3 + \log_5 2$

$= 2(0.68261) + 0.43068$

≈ 3.11329

---

58) $\log_5 \sqrt{6} = \log_5 6^{1/2} = \frac{1}{2} \log_5 6$

$= \frac{1}{2} [\log_5 (2 \cdot 3)]$

$= \frac{1}{2} [\log_5 2 + \log_5 3]$

$= \frac{1}{2} (0.43068 + 0.68261]$

≈ 0.556645

(59) $\log_5 \frac{1}{2} = \log_5 1 - \log_5 2$

$= 0 - \log_5 2$

$= -0.43068$

---

(60) $\log_5\left(\frac{2}{3}\right) = \log_5 2 - \log_5 3$

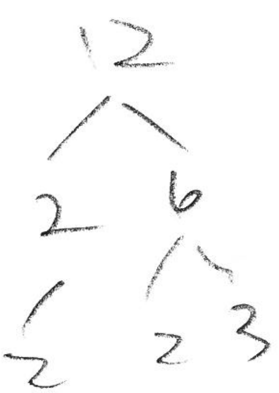
$= 0.43068 - 0.68261$

$= -0.25193$

---

(61) $\log_5 (12)^{2/3} = \frac{2}{3}\log_5 (12)$

$= \frac{2}{3}\left[\log_5 (2^2 3^1)\right]$



$= \frac{2}{3}\left[\log_5 2^2 + \log_5 3\right]$

$= \frac{2}{3}\left[2\log_5 2 + \log_5 3\right]$

$= \frac{4}{3}\log_5 2 + \frac{2}{3}\log_5 3$

$= \frac{4}{3}(0.43068) + \frac{2}{3}(0.68261)$

$\approx 1.02931\overline{33}$

(62) $\log_5 (5^2 \cdot 6) = \log_5 5^2 + \log_5 6$

$= 2\log_5 5 + \log_5 6$

$= 2(1) + \log_5 (3 \cdot 2)$

$= 2 + \log_5 2 + \log_5 3$

$= 2 + 0.43068 + 0.68261$

$= 3.11329$

---

(63) $\log_4 (6x^4) = \log_4 6 + \log_4 x^4$

$= \boxed{\log_4 6 + 4\log_4 x}$

---

(66) $\ln \sqrt[3]{\frac{x}{5}} = \ln \left(\frac{x}{5}\right)^{1/3} = \frac{1}{3} \ln\left(\frac{x}{5}\right)$

$= \frac{1}{3}\left[\ln x - \ln 5\right]$

$= \boxed{\frac{1}{3}\ln x - \frac{1}{3}\ln 5}$

(64) $\log 2x^{-3} = \log 2 + \log x^{-3}$

$= \log 2 + -3 \log x$

$= \log 2 - 3 \log x$

Note $\log 2x^{-3} = \log\left(\frac{2}{x^3}\right)$

---

(65) $\log_5 \sqrt{x+2} = \log_5 (x+2)^{1/2}$

$= \frac{1}{2} \log_5 (x+2)$

---

(67) $\ln\left(\frac{x+2}{x-2}\right) = \ln(x+2) - \ln(x-2)$

---

(68) $\ln(x(x-3)^2) = \ln x + \ln(x-3)^2$

$= \boxed{\ln x + 2\ln(x-3)}$

---

(69) $\ln\left[\sqrt{2x}\,(x+3)^5\right] = \ln\sqrt{2x} + \ln(x+3)^5$

$= \ln(2x)^{1/2} + 5\ln(x+3)$

$= \frac{1}{2}\ln(2x) + 5\ln(x+3)$

$= \frac{1}{2}\left[\ln 2 + \ln x\right] + 5\ln(x+3) = \frac{1}{2}\ln 2 + \frac{1}{2}\ln x + 5\ln x+3$

(70) $\log_3 \frac{a^2\sqrt{b}}{c\,d^5} = \log_3(a^2\sqrt{b}) - \log_3(c\,d^5)$

$= \log_3 a^2 + \log_3 \sqrt{b} - [\log_3 c + \log_3 d^5]$

$= 2\log_3 a + \log_3 b^{1/2} - [\log_3 c + 5\log_3 d]$

$= 2\log_3 a + \frac{1}{2}\log_3 b - \log_3 c - 5\log_3 d$

(71) $\frac{-2}{3}\ln 3y = \ln(3y)^{-2/3}$

$= \ln\left(\sqrt[3]{\frac{1}{(3y)^2}}\right)$

$= \ln\sqrt[3]{\frac{1}{9y^2}}$

(72) $5\log_2 y = \log_2 y^5$

(73) $\log_8 16x + \log_8 2x^2 = \log_8(16x \cdot 2x^2)$

$= \log_8(32x^3)$

(74) $\log_4 6x - \log_4 10$

$\log_4\left(\frac{6x}{10}\right) = \log_4\left(\frac{3x}{5}\right)$

---

(75) $-2(\ln 2x - \ln 3)$

$-2\left(\ln\left(\frac{2x}{3}\right)\right) = \ln\left(\frac{2x}{3}\right)^{-2}$

$= \ln\left(\frac{3}{2x}\right)^{2}$

$= \ln\left(\frac{3^2}{(2x)^2}\right)$

$= \ln\left(\frac{9}{4x^2}\right)$

---

(76) $4[1+\ln x+\ln x] = 4[1+2\ln x]$

$4[1+\ln x^2] = 4+4\ln x^2$

$= 4+\ln(x^2)^4$

$= 4+\ln x^8$

$4 = \ln e^4$

$= \ln e^4 + \ln x^8$

$= \ln(e^4 x^8)$

77. $4\left[\log_2 k - \log_2 (k-t)\right]$

$4\left[\log_2\left(\frac{k}{k-t}\right)\right]$

$\log_2\left(\frac{k}{k-t}\right)^4$

$\log_2\left(\frac{k^4}{(k-t)^4}\right)$

---

78. $\frac{1}{3}\log_8 a + 2\log_8 b$

$\log_8 a^{1/3} + \log_8 b^2$

$\log_8 a^{1/3} \cdot b^2$

$\log_8 \sqrt[3]{a}\ b^2$

$\log_8\left(b^2\sqrt[3]{a}\right)$

79 $3\ln x + 4\ln y + \ln z$

$\ln x^3 + \ln y^4 + \ln z$

$\ln x^3 y^4 + \ln z$

$\ln(x^3 y^4 z)$

---

80 $\ln(x-4) - 3\ln x - \ln y$

$\ln(x-4) - \ln x^3 - \ln y$

$\ln\left(\frac{x-4}{x^3}\right) - \ln y$

$\ln\left(\frac{x-4}{y x^3}\right)$

---

81 $\log_2 4x = \log_2 4 + \log_2 x$

$= \log_2 2^2 + \log_2 x$

$= 2\log_2 2 + \log_2 x$

$= 2(1) + \log_2 x$

$= \boxed{2 + \log_2 x}$

8 cont $2\log_2 x = \log_2 x^2$

Counter example

$\log_2 4(4) = \log_2 16 = \log_2 2^4 = 4\log_2 2$
$= 4(1) = 4$

$2\log_2 4 = 2\log_2 2^2 = 2(2\log_2 2)$
$= 2(2)(1)$
$= 4$

But $\log_2 4(1) = \log_2 4 = \log_2 2^2 = 2\log_2 2$
$= 2(1) = 2$

$\log_2 1^2 = \log_2 1 = 0$

$2 \neq 0$

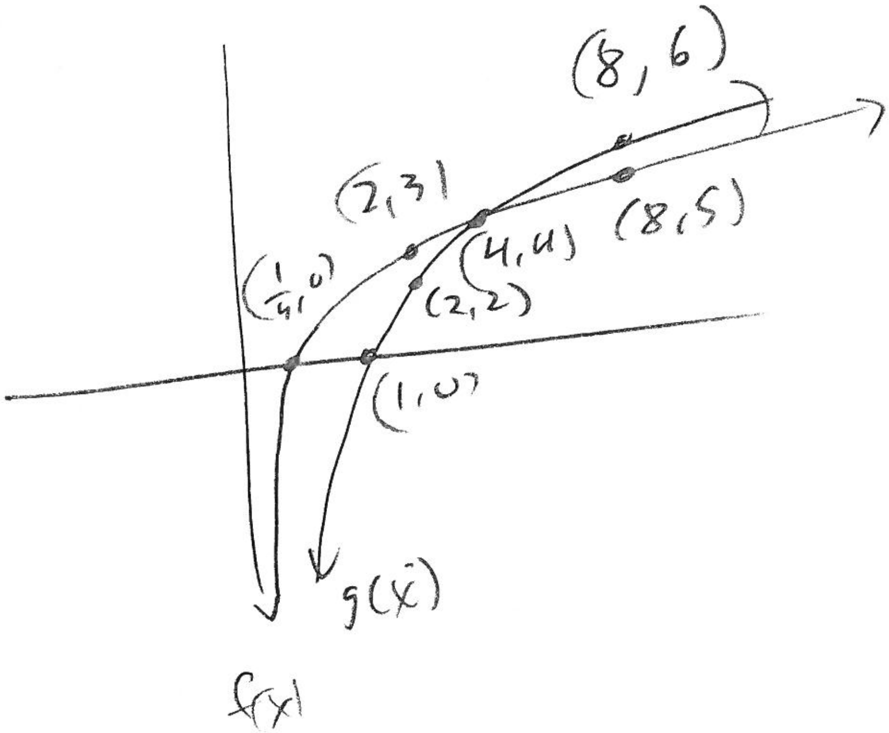
81 cont

$f(x) = 2\log_2 x$

x	y
$\frac{1}{2}$	-2
1	0
2	2
4	4
8	6

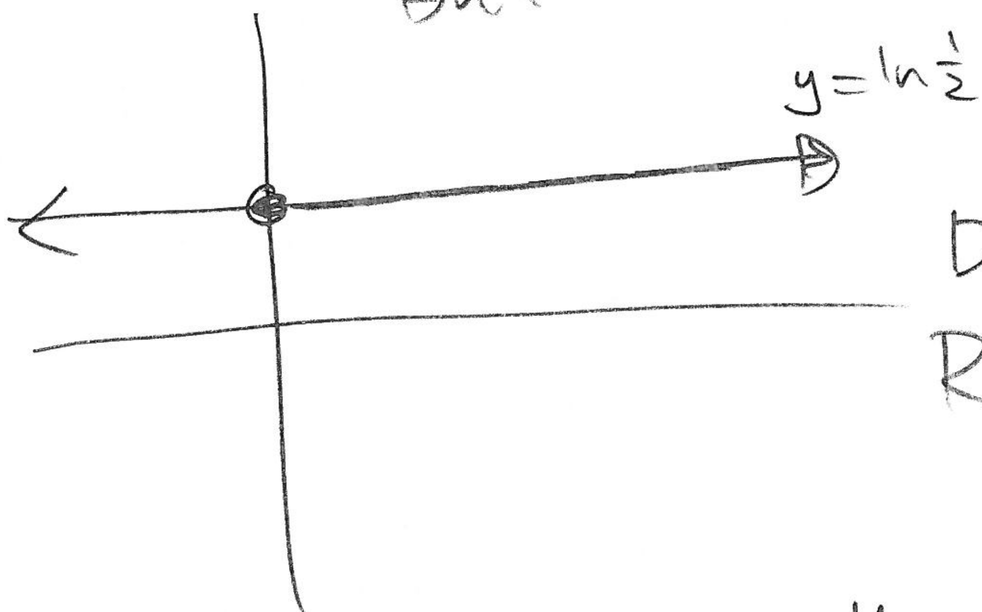
$g(x) = \log_2 4x$

x	y
$\frac{1}{2}$	1
1	2
2	3
4	4
8	5



(82) $\frac{\ln(5x)}{\ln(10x)} = \ln\left(\frac{5x}{10x}\right) = \ln\frac{1}{2}$

But

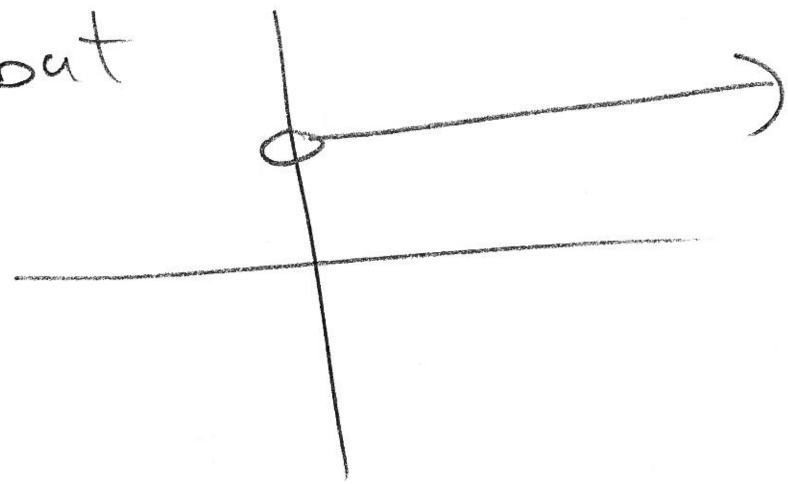


D: $x \in \mathbb{R}$

R: $y = \ln\frac{1}{2}$

$y = \frac{\ln 5x}{\ln 10x}$

but

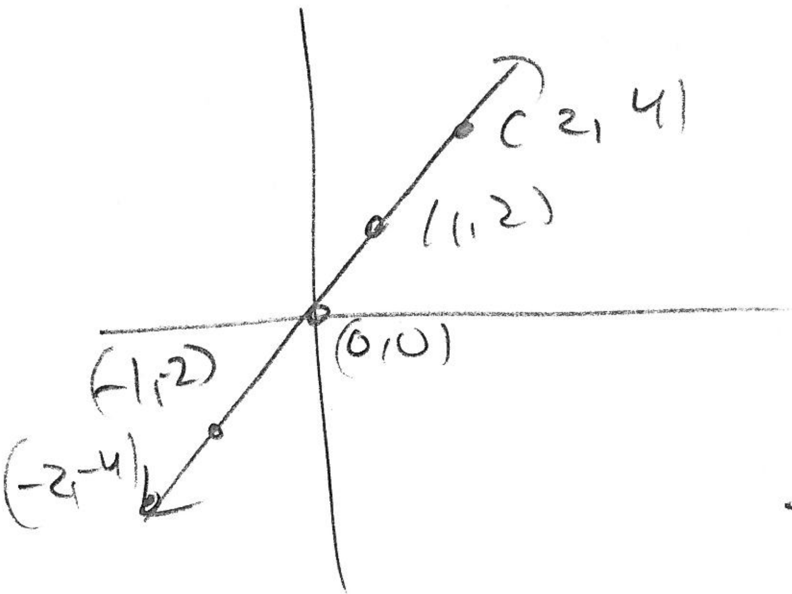


D: $x > 0$

R: $y = \ln\frac{1}{2}$

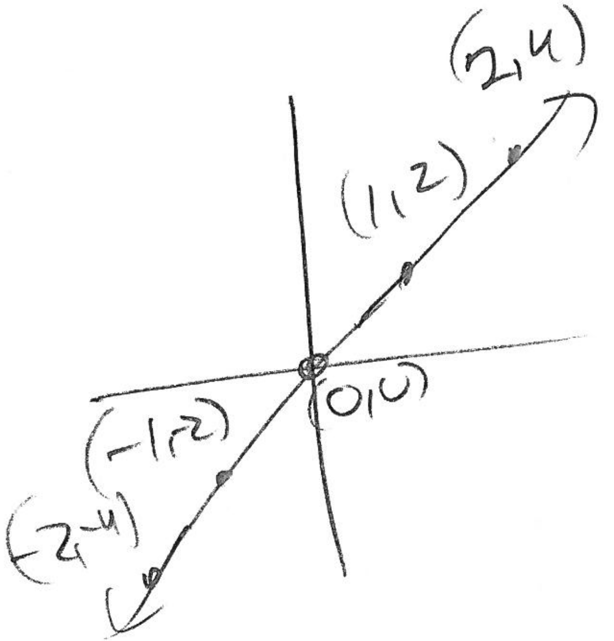
(83) $\log 10^{2x} = 2x \log 10 = 2x(1)$

$= 2x$ TRUE



$y = \log_{10} 10^{2x}$

x	y
-2	$\log \frac{1}{1000} = \log 10^{-4} = -4$
-1	$\log \frac{1}{100} = \log 10^{-2} = -2$
0	$\log 1 = 0$
1	$\log 100 = 2$
2	$\log 1000 = 4$



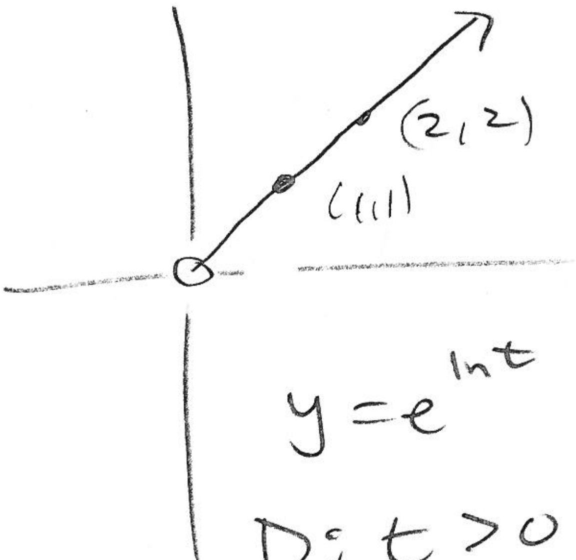
$y = 2x$

x	y
-2	-4
-1	-2
0	0
1	2
2	4

84 $e^{\ln t} = t$

False

$\cancel{e^{\log_e t}} = t$



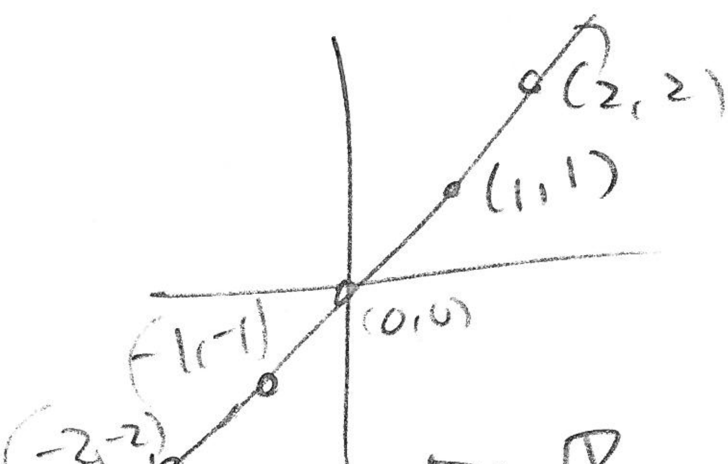
x	y
0	und
1	1
2	2
3	3

$y = e^{\ln t}$

D: $t > 0$

R: $y > 0$

x	y
0	0
1	1
2	2



D: $\mathbb{R}$

R: $\mathbb{R}$

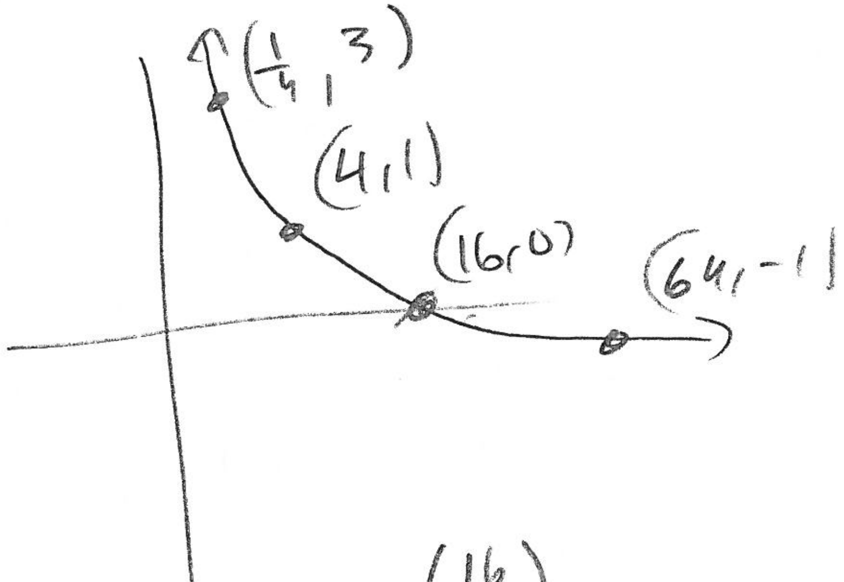
(85) $\log_4\left(\frac{16}{x}\right) = \log_4 16 - \log_4 x$

$= \log_4 4^2 - \log_4 x$

$= 2\log_4 4 - \log_4 x$

$= 2(1) - \log_4 x$

$= 2 - \log_4 x$



$y = \log_4\left(\frac{16}{x}\right)$

TRUE $\log_4\left(\frac{16}{x}\right) = 2 - \log_4 x$

$f(x) = \log_4\left(\frac{16}{x}\right)$

D: $x > 0$

R: $\mathbb{R}$

$g(x) = 2 - \log_4 x$

D: $x > 0$

R: $\mathbb{R}$

86)

$$6\ln x + 6\ln y = \ln(xy)^6$$

$$6(\ln x + \ln y) = 6(\ln(xy))$$

$$= \ln(xy)^6$$

TRUE