

SINUSOIDAL APPLICATION PROBLEMS from Paul Foerster

FERRIS WHEEL

- 1) As you ride the Ferris wheel, your distance from the ground varies sinusoidally with time. You are the last seat filled and the ferris wheel starts immediately. Let t be the number of seconds that have elapsed since the ferris wheel started. You find that it takes you 3s to reach the top, 43 ft. above the ground, and that the wheel makes a revolution once every 8s. The diameter of the wheel is 40 ft.
- Sketch a graph.
 - What is the lowest you go as the ferris wheel turns, and why is this number greater than zero?
 - Write an equation.
 - Predict your height above the ground when: 1) $t = 6$ 2) $t = 13/3$ 3) $t = 0$
 - What is the value of t the second time you are 18 ft above the ground?

TARZAN PROBLEM

- 2) Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of meters Tarzan is from the riverbank. Assume that y varies sinusoidally with t , and that y is positive when Tarzan is over water and negative when he is over land. Jane finds that when $t = 2$, Tarzan is at one end of his swing, where $y = -23$. She finds when $t = 5$ he reaches the other end of his swing and $y = 17$.
- Sketch a graph.
 - Write an equation expressing Tarzan's distance from the riverbank in terms of t .
 - Predict y when: $t = 2.8$ $t = 15$
 - Where was Tarzan when Jane started the stopwatch?

OIL WELL PROBLEM

- 3) The jack on an oil well goes up and down, pumping oil out of the ground. As it does so, the distance varies sinusoidally with time. At time = 1 sec, the distance is at its maximum, 3.7 meters. At time = 4 sec, distance is at its minimum, 1.5 m.
- Sketch a graph.
 - Write an equation.
 - Find the distance when time = 5.5 sec.
 - Find the first time when distance = 1.78m.

EXTRATERRESTIAL BEING PROBLEM

- 4) Researchers find a creature from an alien planet. Its body temperature varies sinusoidally with time. 35 minutes after they start timing, it reaches a high of 120°F . 20 minutes after that it reaches its next low, 104°F .
- Sketch a graph.
 - Write an equation expressing temperature in terms of minutes since they started timing.
 - What was its temperature when they first started timing?
 - Find the first three times after they started timing at which the temperature was 114°F .

STEAMBOAT PROBLEM

5) Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, he noticed a dead fish caught on one of the paddles. As the wheel turned, the distance, d , that the fish was from the water's surface was a sinusoidal function of time. When his stopwatch read 4 seconds, the fish was at its highest, 16 feet above the water's surface. It took another 10 seconds before the fish reached that height again. The diameter of the wheel was 18 feet.

- Sketch a graph.
- Write an equation.
- How far above the surface was the point when Mark's stopwatch read: 1) 5s 2) 17s
- What is the first positive value of time at which the point was at the water's surface? At that time, was it going into or coming out of the water? Explain.

BOATING PROBLEM

6) If the equilibrium point is $y = 0$, then $y = -5 \cos \frac{\pi}{6} x$ models a buoy bobbing up and down in the water.

- Where is the buoy at $t = 0$? at $t = 7$?
- What is the maximum height of the buoy? the minimum?
- What is the period?

SPRING PROBLEM

7) A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. (Ignore gravity and assume the spring keeps moving.) You start a stopwatch. When the stopwatch reads 0.3s, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8s.

- Sketch a graph.
- Write an equation expressing distance from the floor in terms of the number of seconds the stopwatch reads.
- Predict the distance from the floor when the stopwatch reads 17.2 s.
- What was the distance from the floor when you started the stopwatch?
- Predict the first positive value of time at which the weight is 59 cm above the floor.

FOX POPULATION PROBLEM

8) Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time $t = 0$ years. A minimum number, 200 foxes, existed when $t = 2.9$ years. The next maximum, 800 foxes, occurred at $t = 5.1$ years.

- Sketch a graph.
- Write an equation expressing the number of foxes as a function of time t .
- Predict the population when $t = 7$.
- Foxes are declared to be an endangered species when their population drops below 300. Between what two nonnegative values of t were foxes first endangered?

MUSIC PROBLEM

9) The initial behavior of the vibrations of the note E above middle C can be modeled by $y = 0.5 \cos 660\pi t$.

- What is the amplitude in this problem?
- What is the period?
- What is the range? What does it mean in the real world?

ENTERTAINMENT PROBLEM

10) A rodeo performer spins a lasso in a circle perpendicular to the ground. The height of the knot from the ground is modeled by

$$y = -3 \cos \frac{5\pi}{3} t + 3.5, \text{ where } t \text{ is time measured in seconds.}$$

- What is the highest point reached by the knot?
- What is the lowest point reached by the knot?
- What is the period?
- Find the height of the knot after 25 seconds. (find $f(25)$).

SPACE SHIP PROBLEM

11) When a spaceship is fired into orbit, it goes into an orbit that takes it alternately north and south of the equator. Its distance from the equator is approximately a sinusoidal function of time. Suppose that a spaceship is fired into orbit. After ten minutes it reaches its farthest distance north of the equator, 4000km. Half a cycle later it reaches its farthest distance south of the equator, also 4000 km. The spaceship completes an orbit once every 90 min. Let y be the number of kilometers the spaceship is north of the equator. Let t be the number of minutes that have elapsed since liftoff.

- Sketch a complete cycle of the graph. Write an equation expressing y in terms of t .
- Use your equation to predict the distance of the spaceship from the equator when:
 $t = 25$ $t = 163$
- What is the smallest positive value of t at which the spaceship is 1600 km south of the equator?

TIDE PROBLEM

12) Suppose that you are on the beach at Port Aransas, Texas. At 2:00 pm on March 19, the tide is in (i.e., the water is at its deepest). At that time you find that the depth of the water at the end of the pier is 1.5 m. At 8:00 pm the same day when the tide is out, you find that the depth of the water is 1.1 m. Assume the depth of the water varies sinusoidally with time.

- Derive the sinusoidal model for this problem expressing the depth of the water in terms of the number of hours since noon on March 19th.
- Predict the depth of the water at 4 pm on March 19th.
- What is the earliest time on March 20th that the water will be 1.27m deep?

HEALTH PROBLEM

13) An average seated adult breathes in and out every 4 seconds. The average minimum amount of air in the lungs is 0.08 liter, and average maximum amount of air in the lungs is 0.82 liter. Suppose the lungs have a minimum amount of air at $t = 0$, where t is the time in seconds.

- Sketch a graph.
- Write an equation that models the amount of air in the lungs.
- Determine the amount of air in the lungs at 5.5 seconds.

TIDE PROBLEM #2

14) Burntcoat Head in Nova Scotia, Canada, is known for its extreme fluctuations in tides. One day in April, the first high tide rose to 13.25 ft. at 4:30 am. The first low tide at 1.88 ft. occurred at 10:15 am. The second high tide was recorded at 4:45 pm.

- Sketch a graph.
- Write an equation modeling the tides in terms of x , the number of hours since midnight.
- How high was the tide at 8 am.?
- What is the vertical shift and what does it represent?
- What is the amplitude and what does it represent?

Additional SINUSOIDAL application problems

15. A portion of a rollercoaster track is to be built in the shape of a sinusoid. You have been hired to calculate the lengths of the horizontal and vertical timber supports to be used. The high and low points on the track are separated by 50 m horizontally and by 30 m vertically. The low point is 3 m below the ground.

- Letting y be the number of meters the track is above the ground and x the number of meters horizontally from the high point, write an equation expressing y in terms of x .
- How long is the vertical timber at the high point.? At $x = 4$ m? At $x = 32$ m?
- How long is the horizontal timber that is 25 meters from the ground?
- Where does the track first go below the ground?

16. For several hundred years, astronomers have kept track of the number of solar flares, or “sunspots” which occur on the surface of the sun. The number of sunspots counted varies periodically from a minimum of about 10 per year to a maximum of about 110 per year. Between the maximums that occurred in the years 1750 and 1948, there were 18 completed cycles.

- What is the period of the sunspot cycle?
- Assume that the number of sunspots varies sinusoidally within the first year. Sketch a graph of two sunspot cycles, starting in 1948.
- Write an equation expressing the number of sunspots per year in terms of the year.
- What is the first year after 2000 in which the number of sunspots will be about 35? A maximum?

17. A tsunami is a fast moving ocean wave caused by an underwater earthquake. The water first goes down from its normal level and then rises an equal distance above its normal level, and finally returns to its normal level. The period is about 15 minutes. Suppose that a tsunami with an amplitude of 10 meters approaches the pier at Honolulu, where the normal depth of the water is 9 meters.

- Assuming that the depth of the water varies sinusoidally with time as the tsunami passes, predict the depth of the water at the following times after the tsunamis first reaches the pier.
1. 2 min. 2. 4 min 3. 12 min.
- According to your model what will be the minimum depth of the water be?
- Between what two times is there no water at the pier?

18. As you stop at a traffic light, a pebble becomes stuck in the tire treads. When you start off, the distance of the pebble from the pavement varies sinusoidally with the distance you have traveled. Assume that the diameter of the tire is 24 inches.

- Sketch a graph of this function.
- Write an equation of this function.
- Predict the distance from the pavement when you have gone 15 inches.
- What are the first two distances when the pebble is 11 inches from the pavement?

19. Suppose that one day 200 million people climbed up on tables. At time $t = 0$, we all jumped off. The resulting shock wave that hit the earth’s surface started the entire earth vibrating in such a way that its surface first moved down from its normal position and then moved up an equal distance above its normal position. The displacement y of the surface is a sinusoidal function of time with a period of about 54 minutes. Assume that the amplitude is 50 meters.

- At what time will the first maximum (the greatest distance above the normal) occur?
- Write an equation expressing displacement in terms of time since the people jumped.
- Predict the displacement when $t = 21$ minutes.
- What are the first three times at which the displacement is -37 meters?

20. A variable star is one whose brightness alternately increases and decreases. For the most visible star, Delta Cephei, the time between periods of maximum brightness is about 6 days. The average brightness (magnitude) of the star is 4.0 and its brightness varies from a low of 3.5 to a high of 4.5 magnitudes. Express the brightness as a function of time.

21. A Ferris wheel has a radius of 10 meters, and the bottom of the wheel passes 1 meter above the ground. The Ferris wheel makes one complete revolution every 20 seconds, find an equation that gives the height above the ground as a function of time?

22. A mass is suspended on a spring. The spring is compressed so that the mass is located 5 cm above its rest position. The mass is released at time $t = 0$ and allowed to oscillate. It is observed that the mass reaches its lowest point 0.5 seconds after it’s released. Find an equation that describes the motion of the mass.

Notes**SINUSOIDAL Regression**

Professor Barney and his assistant Otis are trying to decide where to spend their spring break. Barney favors Chicago while Otis would like to go to Honolulu. Using the information given in the table below, help them make a decision.

The table gives the normal daily high temperatures for Honolulu H and Chicago C (in degrees Fahrenheit) for month t , with $t = 1$ corresponding to January.

Year	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Unemp	8.539	8.312	8.237	7.425	6.701	6.528	7.047	8.628	9.613	8.940	7.996	7.404

- 1) Find a trigonometric model for Honolulu.
- 2) Find a trigonometric model for Chicago.

- 3) Use a graphing calculator to graph the data points and the model for the temperatures in Honolulu. How well does the model fit?

- 4) Use a graphing calculator to graph the data points and the model for the temperatures in Chicago. How well does the model fit?

- 5) Use the models to estimate the average annual temperature in each city. Which term of the models did you use? Explain.

- 6) What is the period of each model? Are they what you expected? Explain.

- 7) Which city has the greater variability in temperature throughout the year? Which factor of the models determines this variability? Explain.

- 8) Spring break is the last week in March. Which city would be best?

Homework: Sinusoidal Regression

1) The following table shows the number, in millions, of unemployed people in the labor force for 1984-1995.

Year	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Unemp	8.539	8.312	8.237	7.425	6.701	6.528	7.047	8.628	9.613	8.940	7.996	7.404

Enter the data in your calculator and get a scatterplot, with $x = 0$ corresponding to 1980.

Does the data appear to be periodic? If so, find a model using sine regression.

Do you think this model is likely to be accurate much beyond the year 1995? Why?

What is the period of the function?

What is the average number of unemployed people and what part of the model did you use to find it?

2) The table shows the average sales S (in millions of dollars) of an outerwear manufacturer for each month t , where $t = 1$ represents January.

T	1	2	3	4	5	6	7	8	9	10	11	12
S	13.46	14.15	8.00	4.85	2.54	1.70	2.54	4.85	8.00	11.15	13.46	14.3

a. Create a scatter plot of the data.

b. Find a trigonometric model that fits the data. Graph the model on your scatter plot. How well does the model fit?

c. What is the period of the model? Do you think it is reasonable given the context? Explain

d. Interpret the meaning of the model's amplitude in the context of the problem.

3) The table shows the average monthly temperature in Chicago, IL, based on data from 1961 to 1990.

Month	Jan	Feb	March	April	May	June	July	August	Sept	Oct	Nov	Dec
Temp(F)	21.2	25.7	36.7	48.6	59.0	68.4	73.0	71.8	64.2	52.5	39.7	27.3

Use the 12 data points (with $x = 1$ corresponding to January) to find a periodic model of the data.

What is the period of the function found in part a? Is this reasonable?

What is the average annual temperature in Chicago? Which constant in the model did you use?

What is the amplitude and what does it mean?

4) The table shows the average monthly precipitation in inches, in San Francisco, CA, based on data from 1961 to 1990.

Month	Jan	Feb	March	April	May	June	July	August	Sept	Oct	Nov	Dec
Precipitation	4.4	3.2	3.1	1.4	0.2	0.1	0	0.1	0.2	1.2	2.9	3.1

Make a scatter plot of the data.

Use the sine regression feature on a calculator to find another sinusoidal model for the data.

What is the period of the function?

By how much does the precipitation vary in this period?