

## Section 3-10 : Implicit Differentiation

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For problems 1 – 6 do each of the following.

(a) Find  $y'$  by solving the equation for  $y$  and differentiating directly.

(b) Find  $y'$  by implicit differentiation.

(c) Check that the derivatives in (a) and (b) are the same.

1.  $x^2y^9 = 2$

2.  $\frac{6x}{y^7} = 4$

3.  $1 = x^4 + 5y^3$

4.  $8x - y^2 = 3$

5.  $4x - 6y^2 = xy^2$

6.  $\ln(xy) = x$

For problems 7 – 21 find  $y'$  by implicit differentiation.

7.  $y^2 - 12x^3 = 8y$

8.  $3y^7 + x^{10} = y^{-2} - 6x^3 + 2$

9.  $y^{-3} + 4x^{-1} = 8y^{-1}$

10.  $10x^4 - y^{-6} = 7y^3 + 4x^{-3}$

11.  $\sin(x) + \cos(y) = e^{4y}$

12.  $x + \ln(y) = \sec(y)$

13.  $y^2(4 - x^2) = y^7 + 9x$

14.  $6x^{-2} - x^3y^2 + 4x = 0$

$$15. 8xy + 2x^4 y^{-3} = x^3$$

$$16. yx^3 - \cos(x)\sin(y) = 7x$$

$$17. e^x \cos(y) + \sin(xy) = 9$$

$$18. x^2 + \sqrt{x^3 + 2y} = y^2$$

$$19. \tan(3x + 7y) = 6 - 4x^{-1}$$

$$20. e^{x^2+y^2} = e^{x^2y^2} + 1$$

$$21. \sin\left(\frac{x}{y}\right) + x^3 = 2 - y^4$$

For problems 22 - 24 find the equation of the tangent line at the given point.

$$22. 3x + y^2 = x^2 - 19 \text{ at } (-4, 3)$$

$$23. x^2 y = y^2 - 6x \text{ at } (2, 6)$$

$$24. 2\sin(x)\cos(y) = 1 \text{ at } \left(\frac{\pi}{4}, -\frac{\pi}{4}\right)$$

For problems 25 - 27 determine if the function is increasing, decreasing or not changing at the given point.

$$25. x^2 - y^3 = 4y + 9 \text{ at } (2, -1)$$

$$26. e^{1-x} e^{y^2} = x^3 + y \text{ at } (1, 0)$$

$$27. \sin(\pi - x) + y^2 \cos(x) = y \text{ at } \left(\frac{\pi}{2}, 1\right)$$

For problems 28 - 31 assume that  $x = x(t)$ ,  $y = y(t)$  and  $z = z(t)$  and differentiate the given equation with respect to  $t$ .

$$28. x^4 - 6z = 3 - y^2$$

$$29. x y^4 = y^2 z^3$$

$$30. z^7 e^{6y} = (y^2 - 8x)^{10} + z^{-4}$$

$$31. \cos(z^2 x^3) + \sqrt{y^2 + x^2} = 0$$

