GUIDED NOTES

$$f(x) = 100(0.88)^x$$

$b = 1 - r$

$r = 1 - b$

$r = 1 - 0.88$

$r = 0.12$

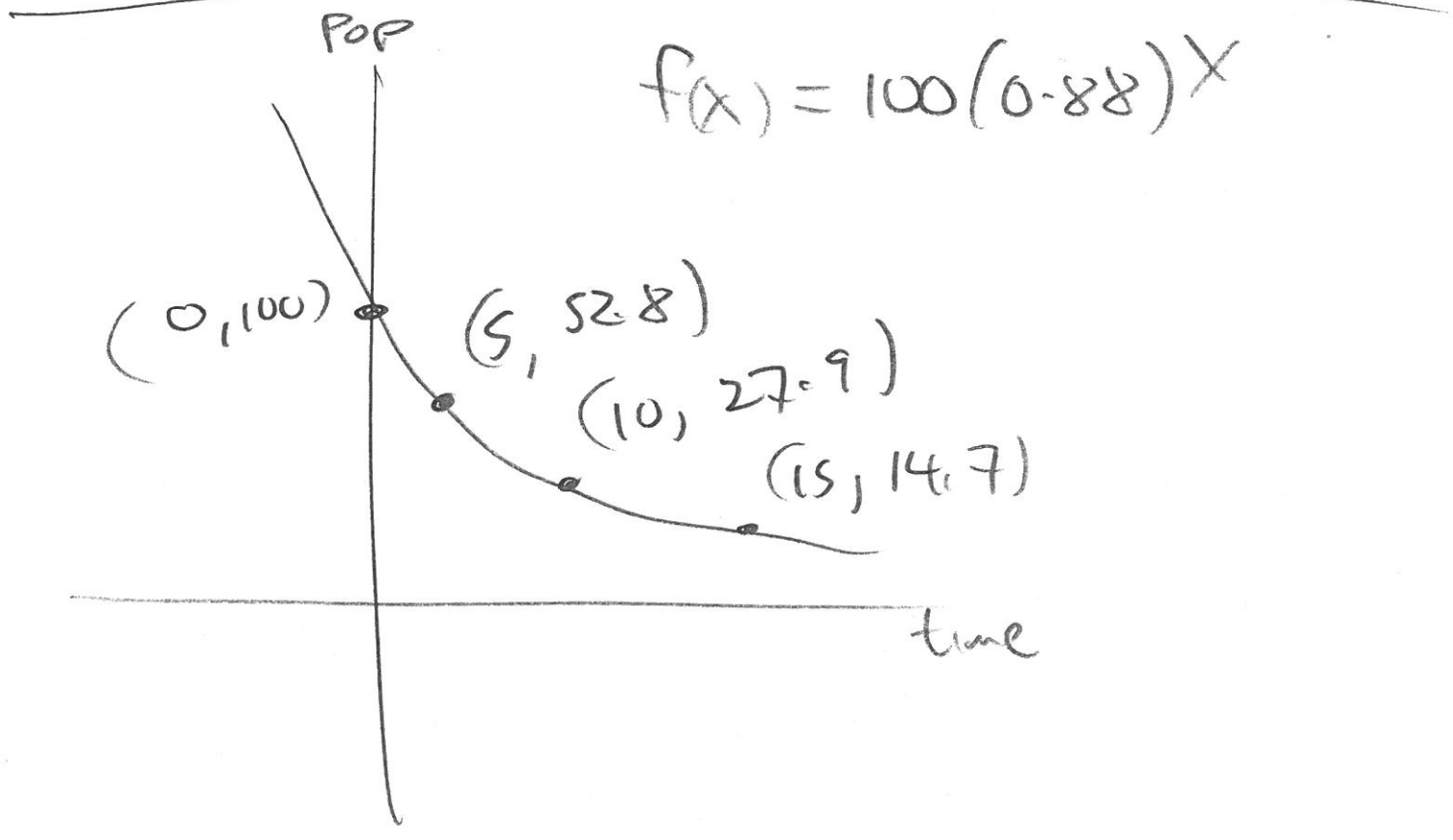
$r\% = 12\%$

P = 100

Initial amount

Decay Factor

$b = 0.88$



$$f(x) = 100(0.88)^x$$

Guided Notes

$$f(x) = 100(0.88)^x$$

(15, 14.7)

time — pop at time x = 15

x	f(x)	
0	100	
1	88	
2	77.44	← reach pop of 75 $x \in (2, 3)$
3	68.1472	
4	59.9700	
5	52.8	← reach pop of 50 $x \in (5, 6)$
6	46.440	
10	27.9	← reach pop of 16 $x \in (10, 15)$
15	14.7	

Guided Notes $f(x) = 100(0.88)^x$

(2a) $\frac{1}{2}$ pop $= 100\left(\frac{1}{2}\right) = 50$

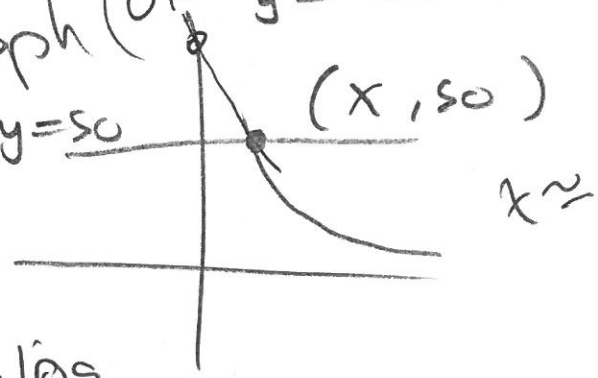
$50 = 100(0.88)^x$

$\frac{50}{100} = 0.88^x$

Method 1 Defn: $\log_{0.88}\left(\frac{1}{2}\right) = x$

$x \approx$

Method 2 Graph $y = 100(0.88)^x$



$x \approx$

Method 3 Apply log

$\log \frac{1}{2} = \log(0.88^x)$

$\log \frac{1}{2} = x \log(0.88)$

$x = \frac{\log \frac{1}{2}}{\log 0.88} = \log_{0.88}\left(\frac{1}{2}\right)$

$x \approx$

Guided Notes $f(x) = 100(0.88^x)$

Method (35) Apply Natural Log

$$\ln\left(\frac{1}{2}\right) = \ln(0.88^x)$$

$$\ln\left(\frac{1}{2}\right) = x\ln(0.88)$$

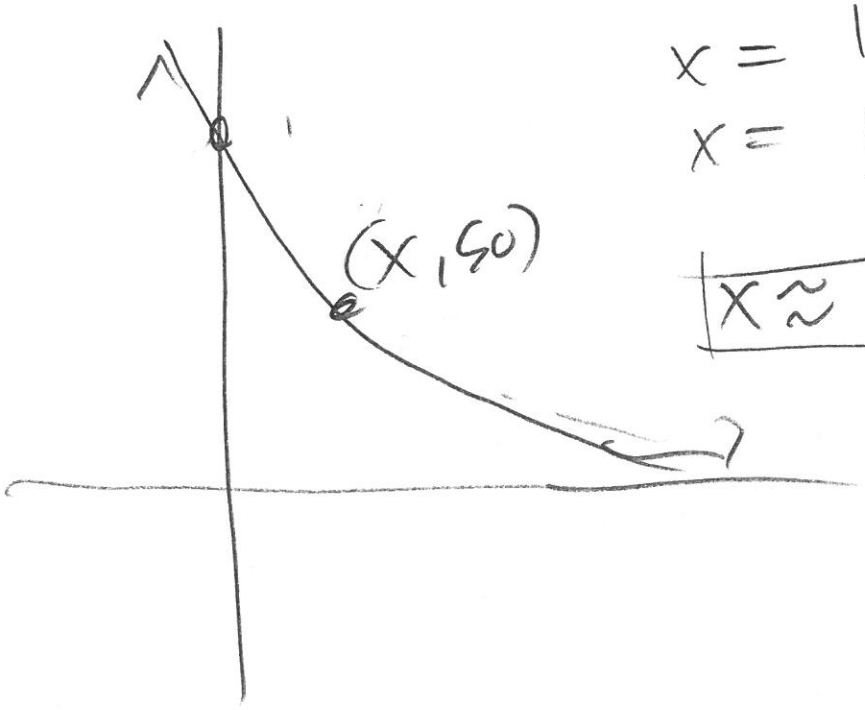
$$\frac{\ln\left(\frac{1}{2}\right)}{\ln(0.88)} = \frac{x\ln(0.88)}{\ln(0.88)}$$

$$x = \frac{\ln\frac{1}{2}}{\ln 0.88} \approx$$

$$x = \log_{0.88}\left(\frac{1}{2}\right)$$

$$x = \frac{\ln\left(\frac{1}{2}\right)}{\ln(0.88)}$$

$$\boxed{x \approx 5.472}$$



(3a) $f(x) = 100(0.88)^x$

$75 = 100(0.88)^x$

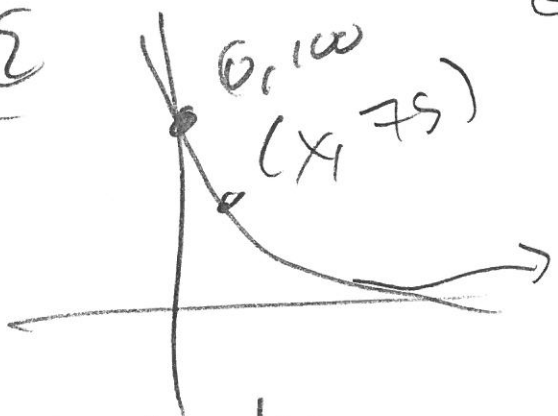
$\frac{75}{100} = \frac{100(0.88)^x}{100}$

$0.75 = 0.88^x$

Method C Defn $\log_{0.88} 0.75 = x$

$x \approx$

Method B Graph



Method (3) Apply log

$\log(0.75) = \log(0.88^x)$

$\log 0.75 = x \log 0.88$

$x = \frac{\log(0.75)}{\log 0.88} = \log_{0.88}(0.75) \approx$

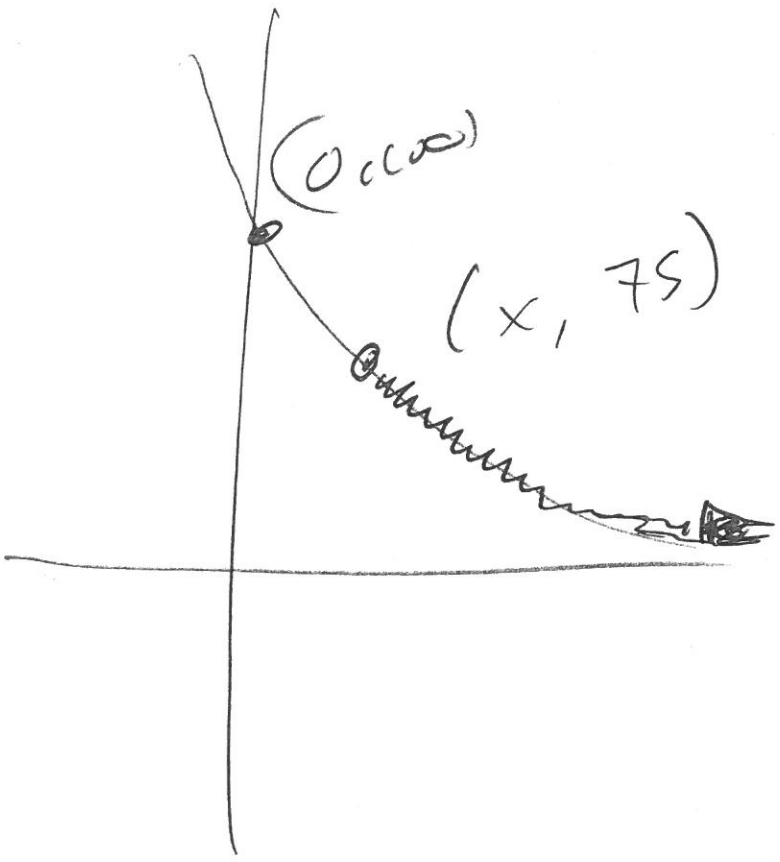
3a

method (3S) Apply ln

$$\ln(0.75) = \ln(0.88^x)$$

$$\ln(0.75) = x \ln(0.88)$$

$$x = \frac{\ln(0.75)}{\ln 0.88} \approx$$



$f(x) < 75$

when

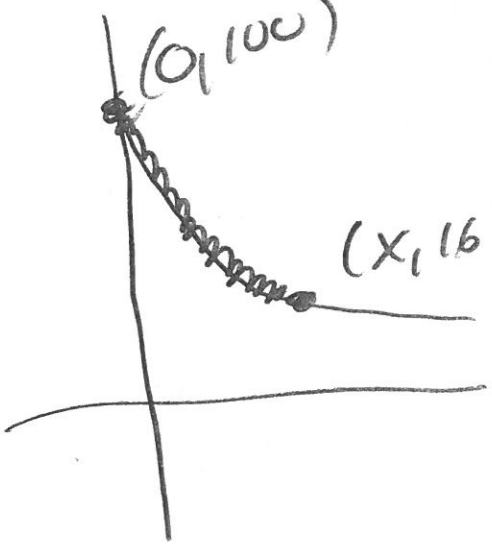
$$x > \log_{0.88} 0.75$$

$$x > \frac{\ln(0.75)}{\ln(0.88)}$$

$$x > 2.250$$

GUIDED NOTES

$f(x) = 100(0.88)^x$



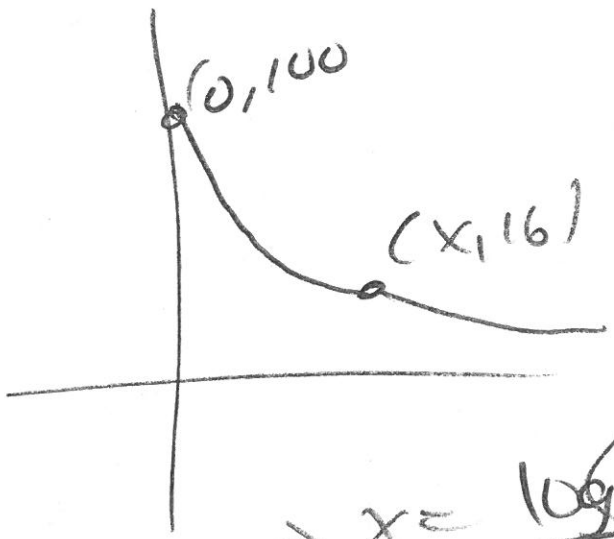
$16 = 100(0.88)^x$

$\frac{16}{100} = 0.88^x$

$0.16 = 0.88^x$

Method ① Defn $\log_{0.88} 0.16 = x$

Method ② Graph



Method ③ Apply log

$\log(0.16) = \log(0.88^x)$

$\log 0.16 = x \log(0.88)$

$x = \frac{\log(0.16)}{\log(0.88)}$

$x = \log_{0.88} 0.16$

$x \approx 14.336$

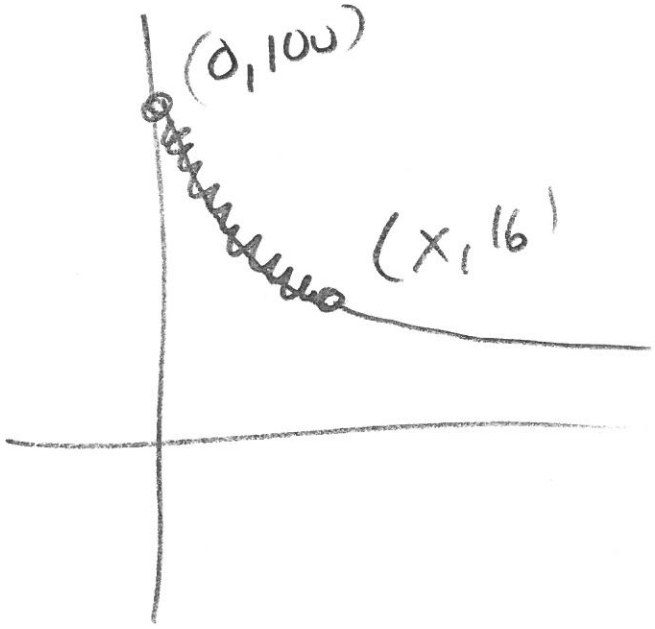
4 cont

Method (35) Apply Ln

$$\ln(0.16) = \ln(0.88^x)$$

$$\ln(0.16) = x \ln(0.88)$$

$$x = \frac{\ln(0.16)}{\ln 0.88} \approx 14.336$$



So (49) $f(x) \in [16, 100]$

$x \in [0, 14.336]$

$0 \leq x \leq 14.336$

GUIDED NOTES

$A(x) = 200(1.25)^x$ (#1)

$b = 1.25$ growth factor

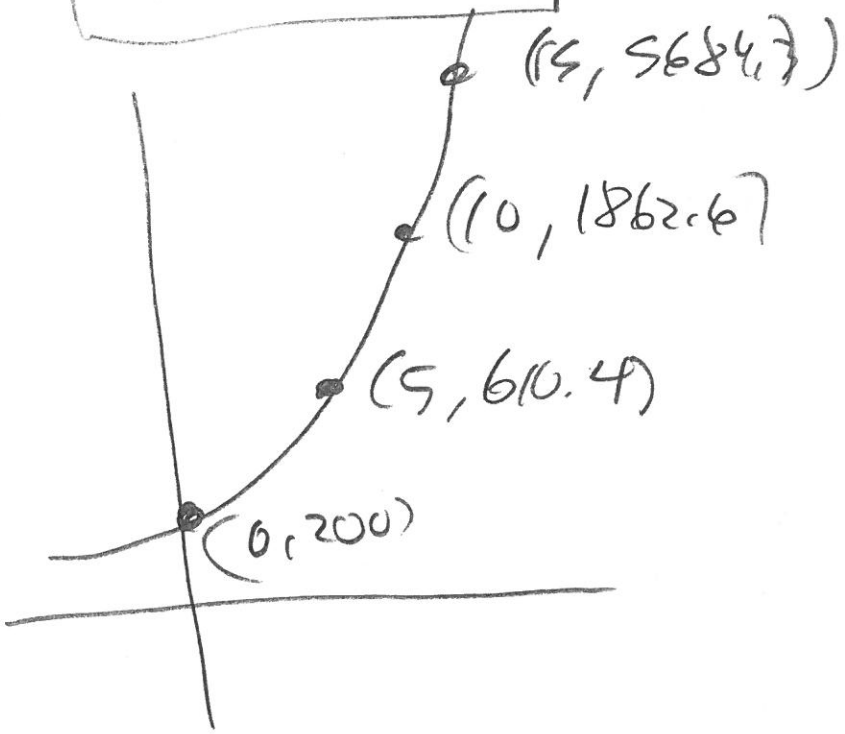
$r = b - 1$

$= 1.25 - 1$

$r = 0.25$

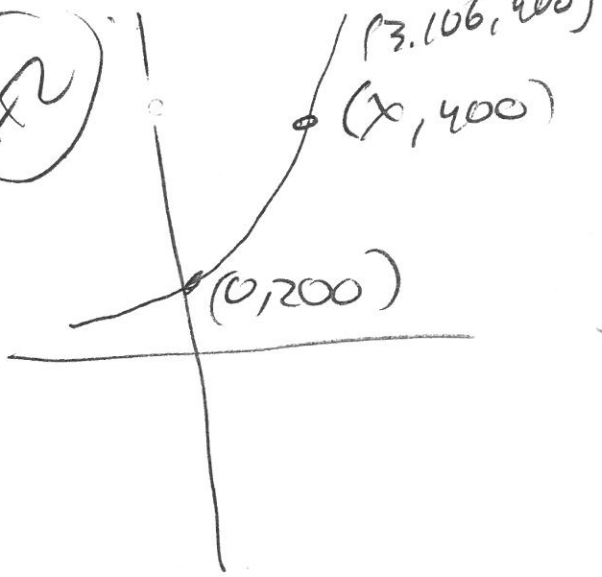
$r\% = 25\%$

$P = 200$ initial pop



x	A(x)
0	200
1	250
2	312.5
3	390.625 ← Doubles $x \in (3,4)$
4	488.28125
5	610.352 ← exceeds 525 $x \in (4,5)$
6	762.939

#2



$A(x) = 400$

$400 = 200(1.25)^x$

$2 = 1.25^x$

Defn $\log_{1.25} 2 = x$

$x \approx 3.106$

Apply log

$\log 2 = \log 1.25^x$

$\log 2 = x \log 1.25$

$x = \frac{\log 2}{\log 1.25} = \log_{1.25} 2$

$x \approx 3.106$

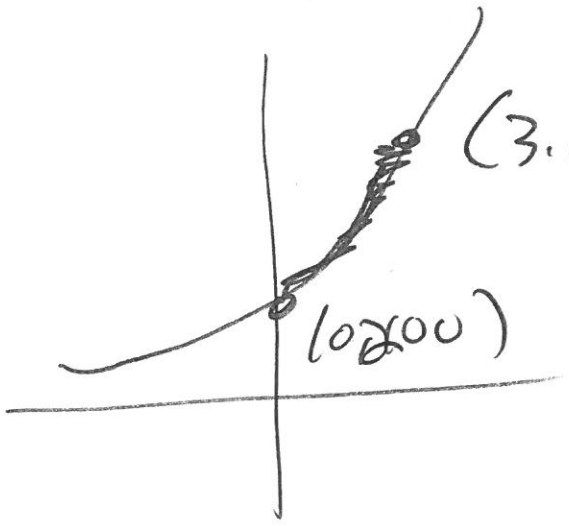
Apply Ln

$\ln 2 = \ln 1.25^x$

$\ln 2 = x \ln 1.25$

$x = \frac{\ln 2}{\ln 1.25} = 3.106$

#3 When will $f(x) = A(x)$ fall below 400?



$A(x) = 200(1.25)^x$

So $f(x) \in [200, 400)$

when $x \in [0, 3.106)$

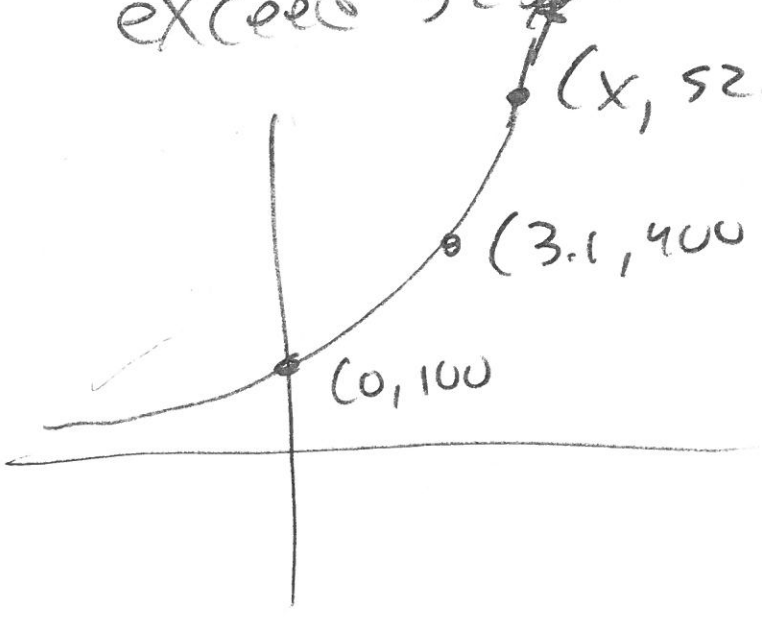
---

④ When will $f(x) = A(x)$ exceed 525?

$525 = 200(1.25)^x$

$\frac{525}{200} = 1.25^x$

$2.625 = 1.25^x$

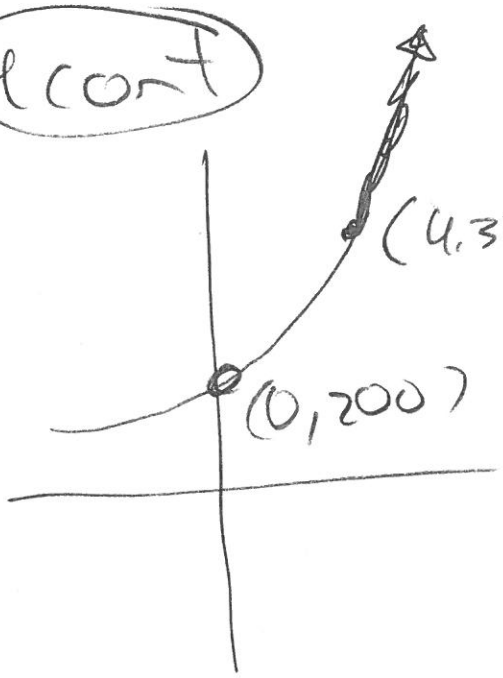


Defn

$x = \log_{1.25} 2.625$

$x \approx 4.325$

4 cont



$f(x) = A(x) > 525$

if $x > 4.325$

$A(x) = 200(1.25)^x$

method ② Apply log

$\log 2.625 = \log 1.25^x$

$\log 2.625 = x \log 1.25$

$x = \frac{\log 2.625}{\log 1.25}$

$x = \log_{1.25} 2.625$

$x \approx 4.325$

method ③ Apply LN

$\ln(2.625) = \ln(1.25)^x$

$\ln(2.625) = x \ln(1.25)$

$x = \frac{\ln(2.625)}{\ln 1.25} \approx 4.325$

(5) $A(x) = 50(4)^x$

$P = 50$ initial pop

$b = 4$ $4 > 1$ growth factor

$b = 1 + r$

$r = b - 1 = 4 - 1 = 3$

$r = 3.00$

$r\% = 300\%$ (this is QUADRUPLE)

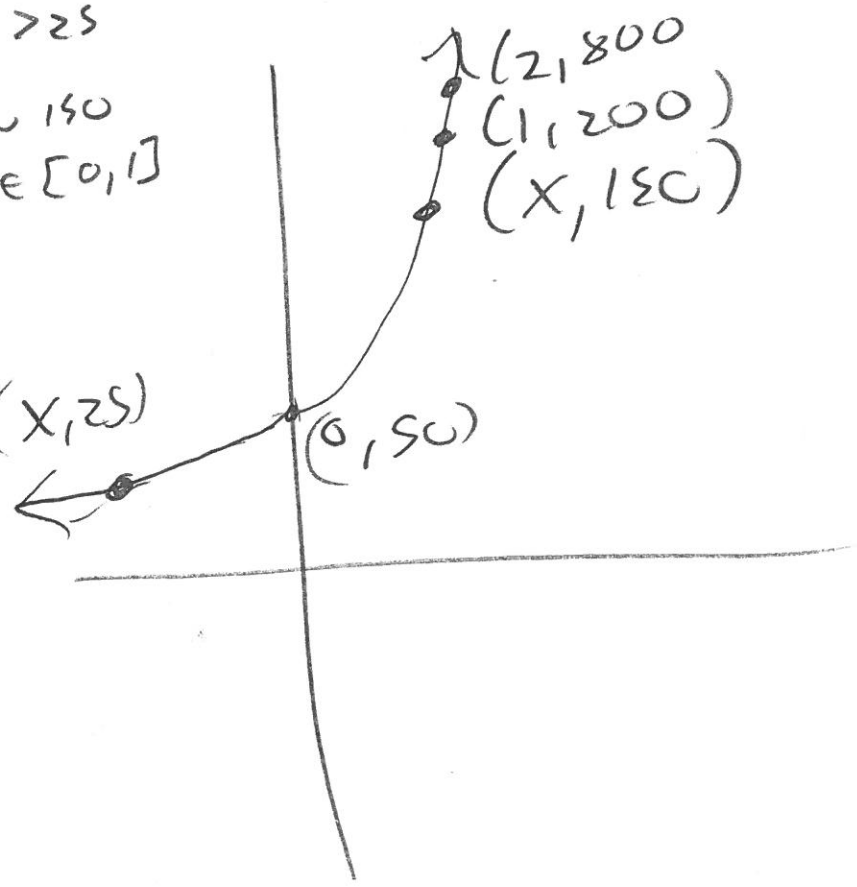
model

x	A(x)
0	50
1	200
2	800
3	3200
4	12800
5	51200
6	204800
10	52428800
	5.4×10^{10}

$x \in [-1, 0]$ >25

below 150

$x \in [0, 1]$



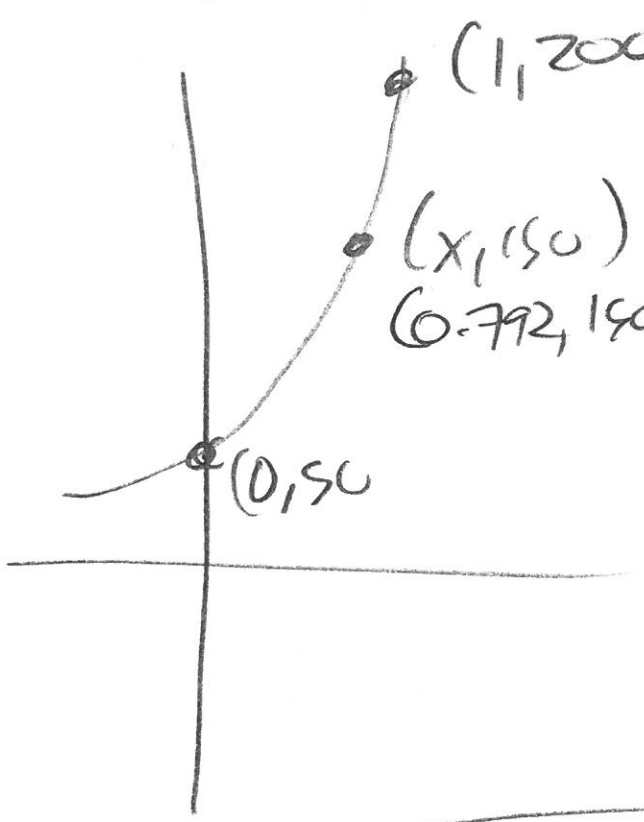
⑥ $A(x) = 3P$

$= 3(50)$

$A(x) = 150$

$150 = 50(4)^x$

$3 = 4^x$



Defn

$\log_4 3 = x$

$x \approx 0.792$

Method ③ Apply Ln

$\ln 3 = \ln 4^x$

$\ln 3 = x \ln 4$

$x = \frac{\ln 3}{\ln 4}$

$x \simeq 0.792$

Method ② Apply log

$\log 3 = \log 4^x$

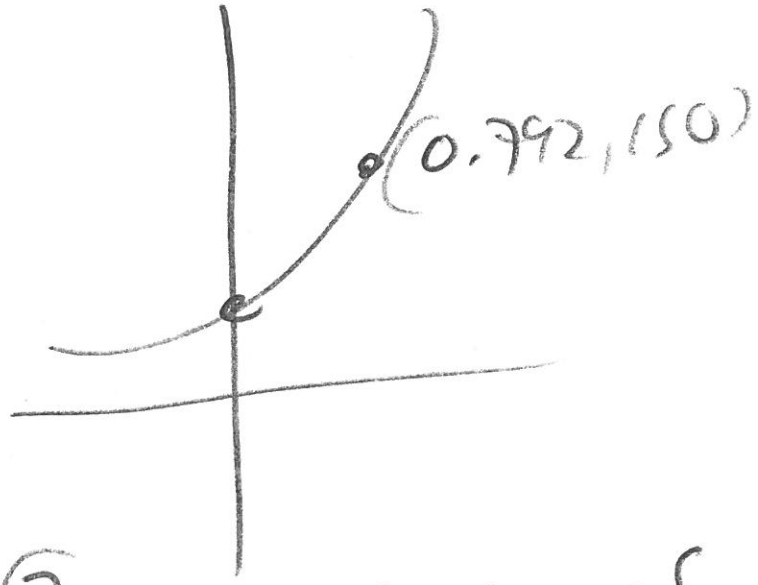
$\log 3 = x \log 4$

$x = \frac{\log 3}{\log 4} = \log_4 3$

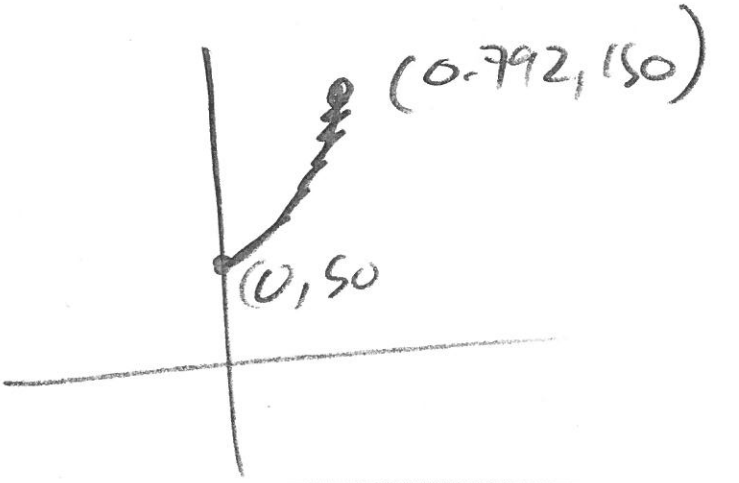
$x \simeq 0.792$

⑥ Pop reaches 150 → triple initial pop

at $x = \log_4 3 \approx 0.792$

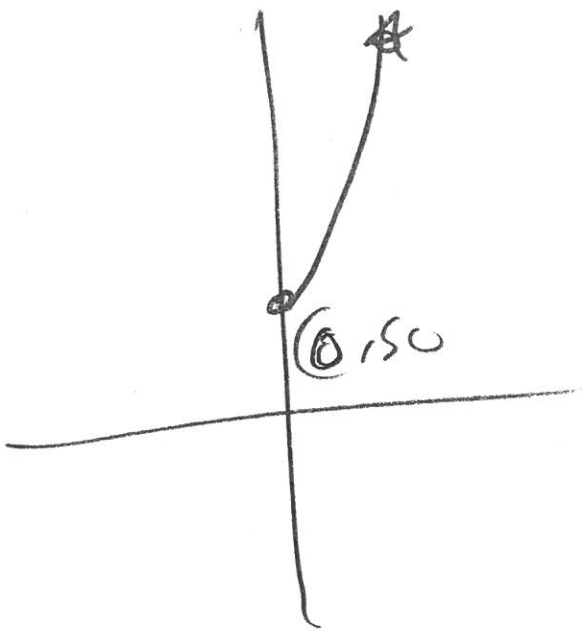


⑦ Pop < 150 if $0 \leq x < 0.792$



$$A(x) = 50(4)^x$$

(8) When does pop exceed 25



for $x \geq 0$

we can't comment about $x < 0$

But $P = 50$

$50 > 25$

So $x \geq 0$

mean $P(x) \geq 50$

(9) $A(x) = 600(0.45)^x$

$P = 600$ initial pop

$b = 0.45$ $0.45 < 1$ Decay Factor

$r = 1 - b$

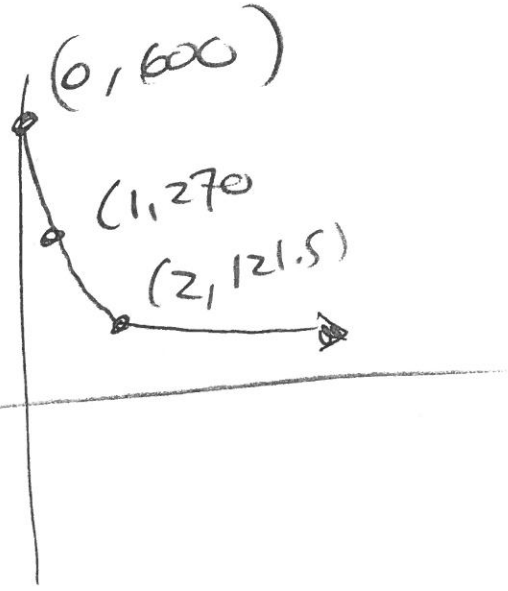
$= 1 - 0.45$

$r = 0.55$

$r\% = 55\%$

Note $\frac{1}{4}$ of 600 ↳ 150

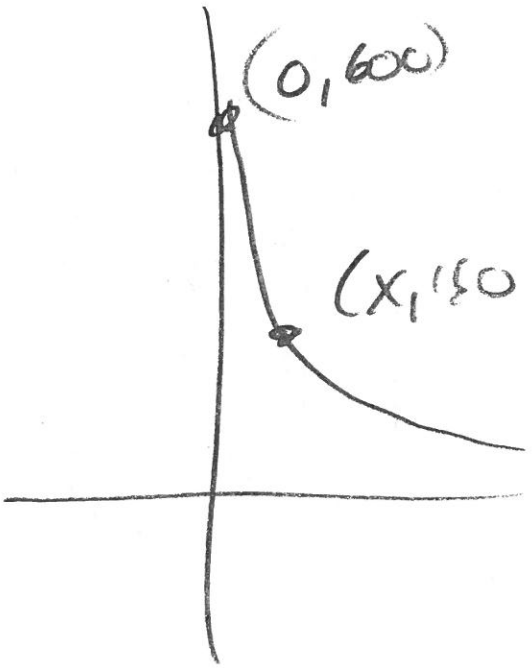
x	A(x)	
0	600	exceeds 375
1	270	falls below 200
2	121.5	falls below 150
3	54.675	$x \in [1, 2]$
4	24.60375	
5	11.072	
6	4.982	
10	0.204	
15	0.00377	



(10) - when is $A(x) = \frac{1}{4} P$

$= \frac{1}{4}(600)$

$A(x) = 150$



$150 = 600(0.45)^x$

$0.25 = 0.45^x$

Defn

$\log_{0.45} 0.25 = x$

$x \approx 1.736$

Apply log

$\log 0.25 = \log(0.45^x)$

$\log 0.25 = x \log(0.45)$

$x = \frac{\log 0.25}{\log 0.45}$

$x \approx 1.736$

Apply LN

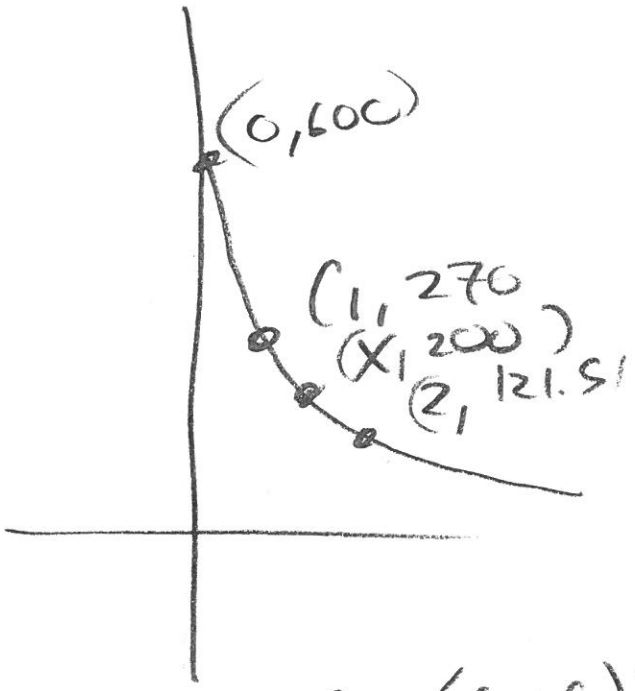
$\ln 0.25 = \ln 0.45^x$

$\ln 0.25 = x \ln 0.45$

$x = \frac{\ln 0.25}{\ln 0.45}$

$x \approx 1.736$

(b) when will population fall below 200



$A(x) = 600(0.45)^x$

$200 = 600(0.45)^x$

$\frac{200}{600} = 0.45^x$

$\frac{1}{3} = 0.45^x$

↗ do not round

Defn

$\log_{0.45}\left(\frac{1}{3}\right) = x$

$x \approx 1.376$

Apply log

$\frac{1}{3} = 0.45^x$

$\log \frac{1}{3} = \log 0.45^x$

$\log \frac{1}{3} = x \log 0.45$

$x = \frac{\log \frac{1}{3}}{\log 0.45} = \log_{0.45}\left(\frac{1}{3}\right)$

$x \approx 1.376$

Apply ln

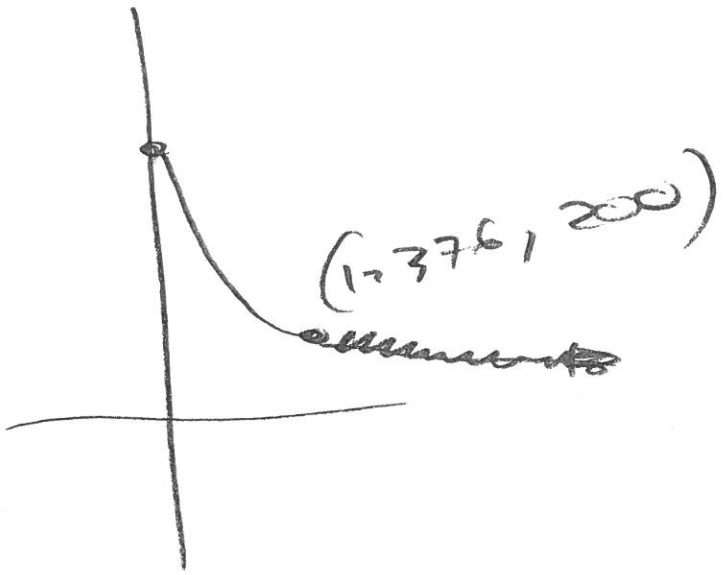
$\ln \frac{1}{3} = \ln (0.45^x)$

$\ln \frac{1}{3} = x \ln 0.45$

$x = \frac{\ln \frac{1}{3}}{\ln 0.45}$

$x \approx 1.376$

(11)



$f(x) = A(x) < 200$

when $x > 1.376$

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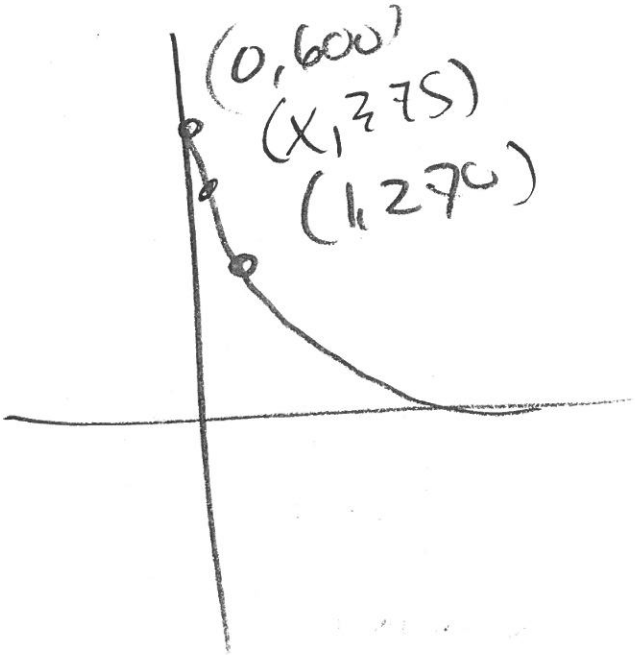
(12) when does population exceed 375?



$375 = 600(0.45)^x$

$\frac{375}{600} = 0.45^x$

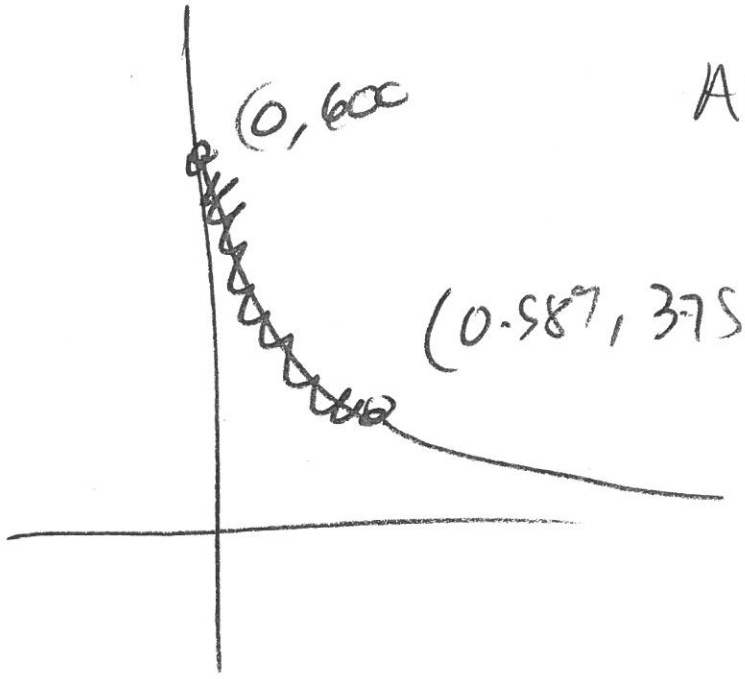
$0.625 = 0.45^x$

Defn $\log_{0.45} 0.625 = x$

$x \approx 0.589$

$A(x) = 600(0.45)^x$

(12) cont $A(x) = 600(0.45)^x$



$A(x) > 375$ if $x \in [0, 0.589)$

Apply Cos

$0.625 = 0.45^x$

$\log 0.625 = \log 0.45^x$

$\log 0.625 = x \log 0.45$

$x = \dfrac{\log 0.625}{\log 0.45}$

$x \approx 0.589$

Apply ln

$0.625 = 0.45^x$

$\ln(0.62) = \ln(0.45^x$

$\ln(0.625) = x \ln(0.45)$

$x = \dfrac{\ln(0.625)}{\ln(0.45)}$

$x \approx 0.589$