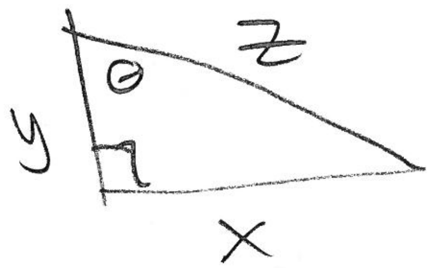
Dock Problem

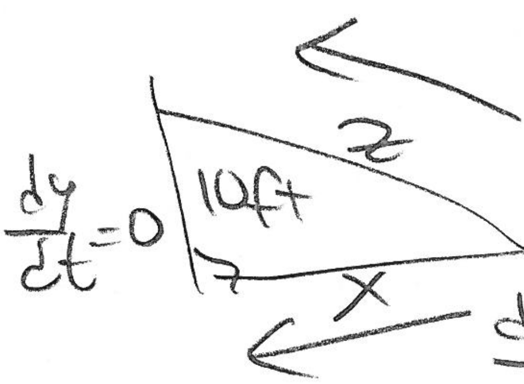


$$\left(\tan\theta = \frac{x}{y}\right)\frac{d}{dt}$$

$$\boxed{\sec^2\theta \frac{d\theta}{dt} = \frac{1y\frac{dx}{dt} - 1x\frac{dy}{dt}}{y^2}}$$

$$\left(x^2+y^2=z^2\right)\frac{d}{dt}$$

$$\boxed{x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}}$$



$\frac{dz}{dt}$ decreasing length $= -9\frac{ft}{sec}$

$\frac{dx}{dt}$ decreasing distance < 0

So $\sec\theta = \frac{z}{10}$

$$\boxed{\sec^2\theta = \frac{z^2}{100}}$$

$x = ?$ $\quad \frac{dx}{dt} = ?$

$y = 10$ $\quad \frac{dy}{dt} = 0$

$z = ?$ $\quad \frac{dz}{dt} = -9$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}$$

$$x\frac{dx}{dt}+10(0)=-9z$$

Dock Problem cont

So $\boxed{\sec^2\theta = \frac{z^2}{100}}$ for any scenario

$$\boxed{x\frac{dx}{dt} = -9z \quad \text{for any scenario}}$$

Now read given Question

$z = 52$ ft $\rightarrow$

$\sqrt{52^2-10^2} = \sqrt{2704-100}$

$= \sqrt{2604}$

So this $\boxed{\sec^2\theta = \frac{(52)^2}{100} = \frac{2704}{100}}$

& this $\boxed{x\frac{dx}{dt} = -9z} \rightarrow \sqrt{2604}\,\frac{dx}{dt} = -9(52)$

$\sqrt{2604}\,\frac{dx}{dt} = -468$

$$\boxed{\frac{dx}{dt} = \frac{-468}{\sqrt{2604}}}$$

Dock Problem con't

So $\sec^2\theta \dfrac{d\theta}{dt} = \dfrac{y\dfrac{dx}{dt} - 1x\dfrac{dy}{dt}}{y^2}$

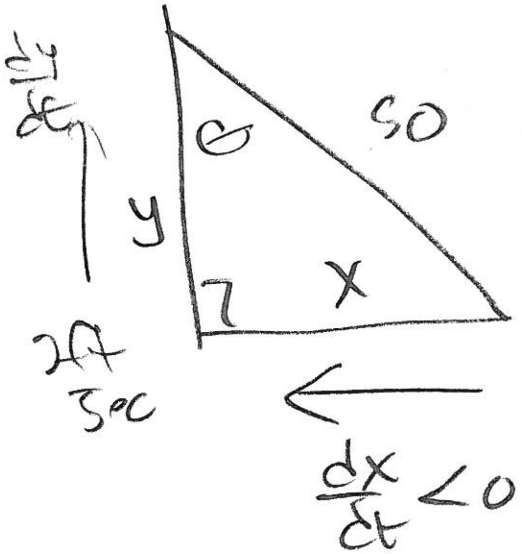
yields $\dfrac{2704}{100}\dfrac{d\theta}{dt} = \dfrac{10\left(\dfrac{-468}{\sqrt{2604}}\right) - \sqrt{2604}(0)}{(10)^2}$

or $\dfrac{d\theta}{dt} = \dfrac{1\cancel{00}}{2704} \cdot \dfrac{\dfrac{-4680}{\sqrt{2604}}}{1\cancel{00}}$

$$\boxed{\frac{d\theta}{dt} = \frac{-4680}{2704\sqrt{2604}}} \approx -0.034\,\frac{rad}{sec}$$

or $\approx -1.943°/sec$

Ladder Problem



$\frac{dz}{dt}=0$

$(x^2+y^2=z^2)\frac{d}{dt} \to \boxed{x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}}$

Now we know $\to$ $x\frac{dx}{dt}+y(2)=50(0)$

$\to$ $x\frac{dx}{dt}+2y=0$

$\to$ $\boxed{\frac{dx}{dt}=\frac{-2y}{x}}$

we also know $(\tan\theta=\frac{x}{y})\frac{d}{dt}$

$\boxed{\sec^2\theta\frac{d\theta}{dt}=\frac{1y\frac{dx}{dt}-x\frac{dy}{dt}}{y^2}}$

Ladder Problem cont

Since $\sec\theta = \frac{z}{y}$

$$\sec^2\theta = \frac{z^2}{y^2}$$

or $\sec^2\theta = \frac{50^2}{y^2}$

$$\boxed{\sec^2\theta = \frac{2500}{y^2}}$$

& $\sec^2\theta \frac{d\theta}{dt} = \dfrac{1y\frac{dx}{dt} - 1x\frac{dy}{dt}}{y^2}$

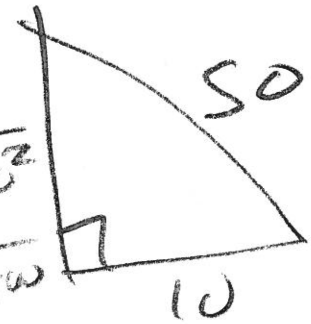
yields

$$\frac{2500}{y^2}\frac{d\theta}{dt} = \frac{y\frac{dx}{dt} - x(2)}{50^2}$$

$$\frac{2500}{y^2}\frac{d\theta}{dt} = \frac{y\frac{dx}{dt} - 2x}{2500}$$

Finally look at given in question

$x = 10$



$\sqrt{50^2 - 10^2}$

$\sqrt{2500 - 100}$

$\sqrt{2400}$

So $x = 10$

$y = \sqrt{2400}$

$z = 50$

$$\frac{dx}{dt} = \frac{-2y}{x} = \frac{-2\sqrt{2400}}{10} = -\frac{40\sqrt{6}}{10} = -4\sqrt{6}$$

$$\frac{dy}{dt} = 2$$

$$\frac{dz}{dt} = 0$$

$$\text{So } \frac{d\theta}{dt} \cdot \frac{2500}{2400} = \frac{\sqrt{2400}\left(-\frac{2\sqrt{2400}}{10}\right) - 2(10)}{(\sqrt{2400})^2}$$

y^2

$$\frac{d\theta}{dt} = \frac{\cancel{2400}}{2500} \cdot \left(\frac{\frac{-2 \cdot 2400}{10} - 20}{\cancel{2400}} \right)$$

$$\frac{d\theta}{dt} = \frac{-\frac{4800}{10} - 20}{2500}$$

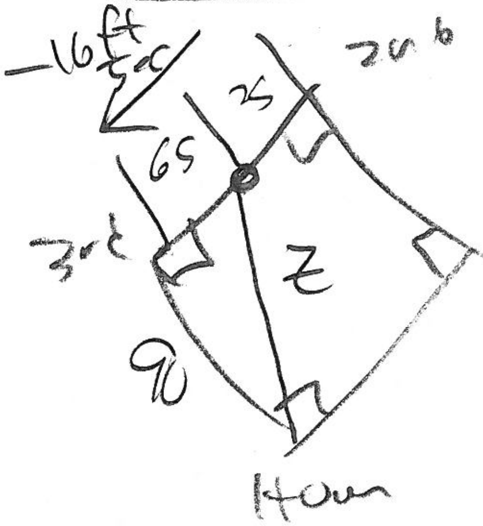
ladder problem

$$= \frac{-480 - 20}{2500} = \frac{-500}{2500}$$

$$\boxed{\frac{d\theta}{dt} = \frac{-1}{5} \frac{\text{rad}}{\text{sec}}}$$

$$\approx -11.459°/\text{sec}$$

Baseball Problem



$x =$ distance from runner to 3rd

$= 65$

$\frac{dx}{dt} < 0$ decreasing

$\frac{dx}{dt} = -16 \frac{ft}{sec}$

$y = 90$ (3rd to Home)

$\frac{dy}{dt} = 0$

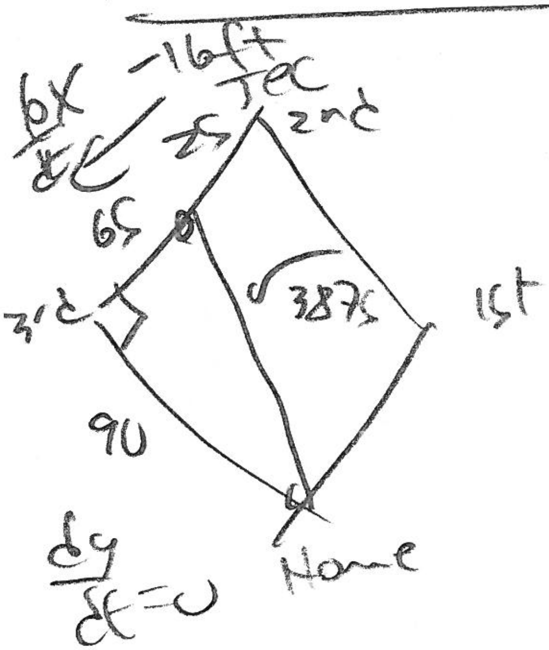
$\frac{dz}{dt} = ?$ $\frac{dz}{dt}$ decreasing so $\frac{dz}{dt} < 0$

$z = \sqrt{90^2 - 65^2}$

$= \sqrt{8100 - 4225}$

$z = \sqrt{3875}$

Baseball Problem (cont)



$$\left(x^2+y^2=z^2\right)\frac{d}{dt}$$

$$x\frac{dx}{dt}+y\frac{dy}{dt}=z\frac{dz}{dt}$$

$$65(-16)+90(0)=\sqrt{3875}\left(\frac{dz}{dt}\right)$$

$$-1040+0=\sqrt{3875}\frac{dz}{dt}$$

$$\boxed{\begin{aligned}\frac{dz}{dt}&=\frac{-1040}{\sqrt{3875}}\frac{ft}{sec}\\ \frac{dz}{dt}&\approx-16.707\frac{ft}{sec}\end{aligned}}$$

Kuta Software ②

#1 $y = \frac{1}{2} \cdot 5^x$

$p = \frac{1}{2}$ $\quad$ $b = 5$

rate of change 400%

rate of change as decimal 4

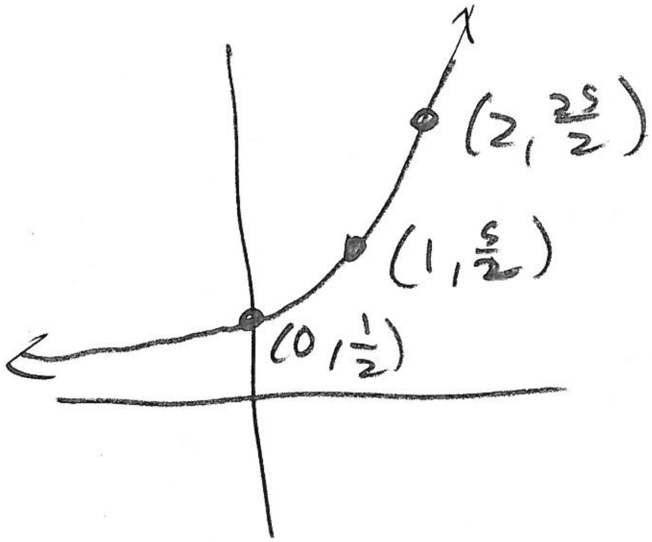
Note $b = 1 + r$ (growth)

$5 = 1 + r$

$5 - 1 = r$

$r = 4$ $\longrightarrow$ rate as decimal

$100r = 400$ $\longrightarrow$ rate as %



Horizontal Asymptote

$y = \frac{1}{2} \cdot 5^x + 0$

$c = 0$

$y = 0$ Horizontal Asymptote

#3 $y = 5\left(\frac{1}{2}\right)^x + 0$

$\swarrow$ P $\downarrow$ b $\downarrow$ C

P=5 (Initial Amount)   C = Horizontal Asymptote

$b = \frac{1}{2}$ (decay factor)

when $0 < b < 1$

Exponential change = Decay

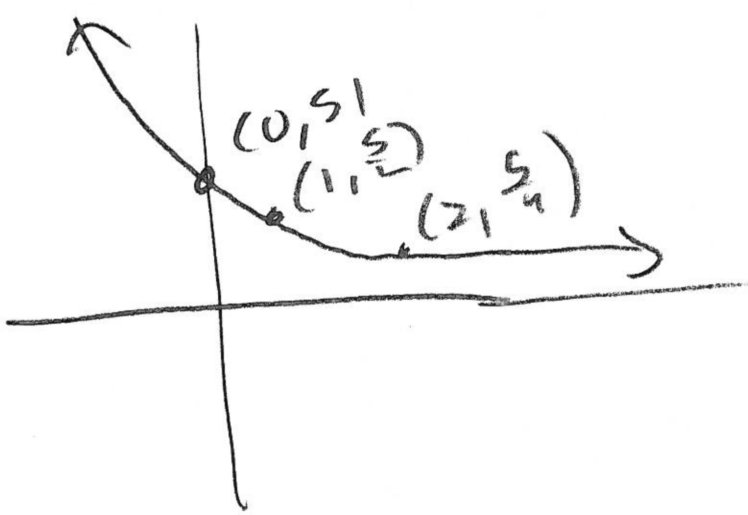
$b = 1 + r$

$\frac{1}{2} = 1 + r$

$\frac{1}{2} - 1 = r$

$-\frac{1}{2} = r = -0.5 \rightarrow -50\% \rightarrow$ 50% decay rate

when $r < 0$ decay rate



#5 $y = 4 \cdot 2^{x} - 2$

$\downarrow$ $\downarrow$ $\downarrow$

P b C

$b = 2$
"Doubling base"

$\boxed{P = \text{initial Amount} = 4}$

$\boxed{b = 2 \text{ since } 2 > 1 \text{ growthrate}}$

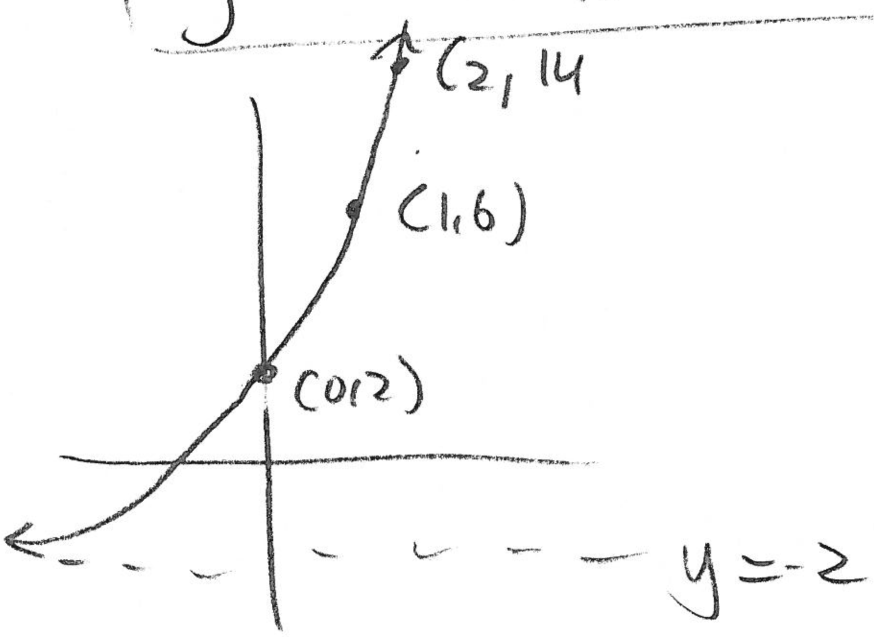
$b = 1 + r \longrightarrow 2 = 1 + r$

$-1 \quad -1$

$\boxed{1 = r}$ r as decimal 1.00

$100r = 1(100) = 100\%$ growth

$\boxed{y = -2 \text{ horizontal asymptote}}$



Problem 7

we know $(0,5)$ on graph
we know horizontal asymptote $\Rightarrow y=2$
we know $(1,8)$ on graph
we know $(2,14)$ on graph

$y = P \cdot b^x + C$

$5 = P b^0 + 2$

$5 = P \cdot 1 + 2$

$\boxed{3 = P}$

$y = P \cdot b^x + C$

$8 = P \cdot b^1 + 2$

$8-2 = P \cdot b^1$

$6 = P \cdot b^1$

$6 = 3b^1 \leftarrow$ we knew $P=3$

$\frac{6}{3} = \frac{3b^1}{3}$

$\boxed{b=2}$

so $\boxed{y = 3 \cdot 2^x + 2}$

test $y = 3 \cdot 2^0 + 2 = 3 \cdot 1 + 2 = 5$ ✓
$y = 3 \cdot 2^1 + 2 = 6 + 2 = 8$ ✓
$y = 3 \cdot 2^2 + 2 = 12 + 2 = 14$ ✓