

## Problem 1

$$1 - 2 \tan^2 \theta = -1 \tan^2 \theta$$

Strategy Group tangents

$$\begin{array}{r} 1 - 2 \tan^2 \theta = -1 \tan^2 \theta \\ + 2 \tan^2 \theta \quad + 2 \tan^2 \theta \\ \hline \end{array}$$

$$1 = \tan^2 \theta$$

Since  $\tan^2 \theta = 1$  implies

$$\begin{array}{l} \& \tan \theta = 1 \\ \& \tan \theta = -1 \end{array}$$

&  $\tan \theta = 1$  at  $\theta = \frac{\pi}{4}$  & period of

&  $\tan \theta = -1$  at  $\theta = \frac{3\pi}{4}$   $\tan \theta = \pi$

All solutions

$$\theta_1 = \frac{\pi}{4} + \pi n \quad \text{with } n \in \text{integers}$$

$$\theta_2 = \frac{3\pi}{4} + \pi n$$

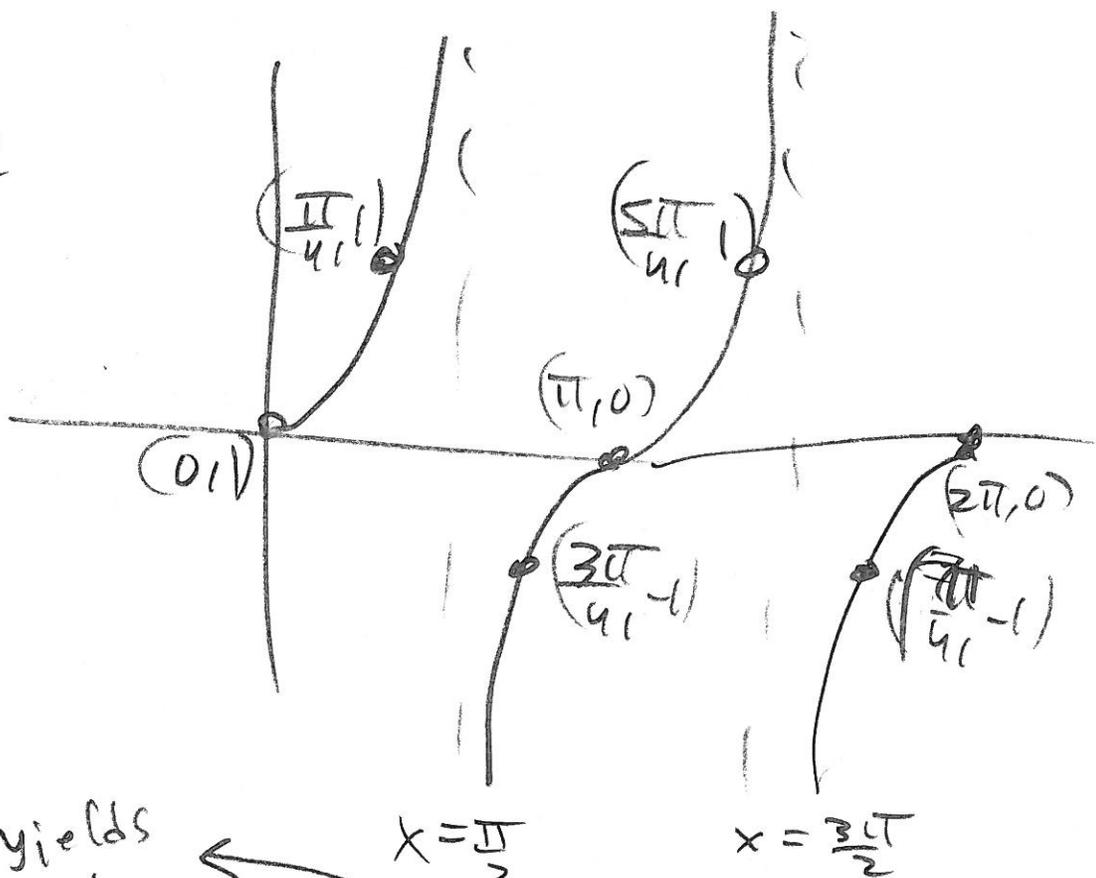
$\theta_1$  = set of solutions to  $\tan \theta = 1$

$\theta_2$  = set of solutions to  $\tan \theta = -1$

But we were given that

$$0 \leq \theta \leq 2\pi$$

So since



$$\theta \in \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$$

↑ yields -1

↓ yields 1

$$(3) \quad 3 \csc^2 \theta = 1 + 2 \csc^2 \theta$$

Strategy: Group  $\csc^2 \theta$

$$\begin{array}{r} 3 \csc^2 \theta = 1 + 2 \csc^2 \theta \\ -2 \csc^2 \theta \quad \quad -2 \csc^2 \theta \\ \hline \end{array}$$

$$\csc^2 \theta = 1$$

$$\text{Now, } \csc^2 \theta = 1 \rightarrow \sin^2 \theta = 1$$

$$\text{So } \sin \theta = 1 \quad \text{or} \quad \sin \theta = -1$$

$$\theta_1 = \frac{\pi}{2}$$

$$\theta_2 = \frac{3\pi}{2}$$

Since period of  $\sin \theta / \csc \theta = 2\pi$

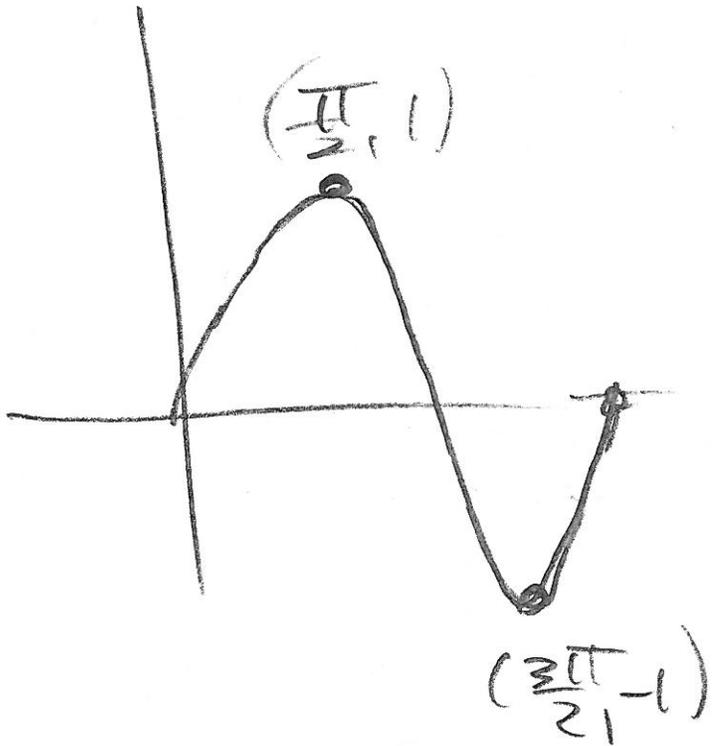
All solutions

$$\theta_1 = \frac{\pi}{2} + 2\pi n$$

with  $n \in \text{Integers}$

$$\theta_2 = \frac{3\pi}{2} + 2\pi n$$

Graphically



for  $0 \leq \theta \leq 2\pi$

Solutions are  $\theta = \left\{ \frac{\pi}{2}, \frac{3\pi}{2} \right\}$

# ⑤ Trouble 2 trig Functions

$$2\sqrt{3} \cos \theta \sin \theta - \cos \theta = 2 \cos \theta$$

If we isolate  $2\sqrt{3} \cos \theta \sin \theta$   
then eventually we will get "some"  
but not all solutions

Group all trig functions on left side

$$2\sqrt{3} \cos \theta \sin \theta - \cos \theta = 2 \cos \theta$$
$$-2 \cos \theta \quad -2 \cos \theta$$

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$$2\sqrt{3} \cos \theta \sin \theta - 3 \cos \theta = 0$$

Now factor!

$$(2\sqrt{3} \sin \theta - 3)(\cos \theta) = 0$$

$$\downarrow$$
$$2\sqrt{3} \sin \theta - 3 = 0$$

$$2\sqrt{3} \sin \theta = 3$$

$$\sin \theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\cos \theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\textcircled{7} \sqrt{2} \cot \theta - 3 \sec \theta = \cot \theta \sec \theta - 3 \sec \theta$$

Trouble 2 trig functions  
move all to same side

$$\sqrt{2} \cot \theta - 3 \sec \theta + 3 \sec \theta = \cot \theta \sec \theta$$

$$\sqrt{2} \cot \theta = \cot \theta \sec \theta$$

$$-\sqrt{2} \cot \theta \qquad \qquad \qquad -\sqrt{2} \cot \theta$$

$$\boxed{0 = \cot \theta \sec \theta - \sqrt{2} \cot \theta}$$

factor  $\cot \theta$  out

$$0 = \cot \theta (\sec \theta - \sqrt{2})$$

Set each term equal to 0 & solve

$$\cot \theta = 0$$

↓

$$\tan \theta = \text{undefined}$$

$$\sec \theta - \sqrt{2} = 0$$

$$\sec \theta = \sqrt{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{\theta = \frac{\pi}{2} \text{ \& \ } \frac{3\pi}{2}}$$

$$\boxed{\theta = \frac{\pi}{4}, \frac{7\pi}{4}}$$

Problem 7 Has what we call  
extraneous solutions

$$\text{at } \theta = \frac{\pi}{2} \text{ \& } \frac{3\pi}{2}$$

because  $\sec\theta = \frac{1}{\cos\theta}$        $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$$\cos\theta = 0 \text{ at } \frac{\pi}{2} \text{ \& } \frac{3\pi}{2}$$

$\therefore \sec\theta$  is undefined at  $\theta = \frac{\pi}{2} \text{ \& } \frac{3\pi}{2}$

$$\text{and } \theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$$

Only solutions  $\theta = \frac{\pi}{4}, \frac{7\pi}{4}$

$$\textcircled{9} \sin^2 \theta = \sin \theta + 1 - \sin^2 \theta$$

Trouble "Quadratic & Linear"

→ isolate with positive  $\sin^2 \theta$   
on one side & 0 on other side

$$\begin{array}{r} \sin^2 \theta = \sin \theta - \sin^2 \theta + 1 \\ + \sin^2 \theta \end{array}$$

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$$2\sin^2 \theta = \sin \theta + 1$$

$$\boxed{2\sin^2 \theta - \sin \theta - 1 = 0}$$

$$\text{factor } (2\sin \theta + 1)(\sin \theta - 1) = 0$$

$\begin{array}{c} \underbrace{\hspace{10em}} \\ 1\sin \theta \\ -2\sin \theta \end{array}$   
✓ ✓

Set factors = 0 & solve

$$2\sin \theta + 1 = 0$$

$$2\sin \theta = -1$$

$$\sin \theta = -\frac{1}{2}$$

$$\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6}}$$

$$\begin{array}{r} \sin \theta - 1 = 0 \\ +1 \quad +1 \\ \hline \end{array}$$

$$\sin \theta = 1$$

$$\boxed{\theta = \frac{\pi}{2}}$$

$$\textcircled{1} \cos^2 \theta = 2 + 2 \sin \theta$$

trouble Two DIFFERENT TRIG Functions  
& Quadratic & Linear

Solution Strategy Use  $1 - \sin^2 \theta$   
for  $\cos^2 \theta$

$$1 - \sin^2 \theta = 2 + 2 \sin \theta$$

② Isolate  $\sin^2 \theta$  so it is positive  
& = 0

$$\begin{array}{r} 1 - \sin^2 \theta = 2 + 2 \sin \theta \\ -1 + \sin^2 \theta \quad -1 \quad + \sin \theta \\ \hline \end{array}$$

$$0 = \sin^2 \theta + 2 \sin \theta + 1$$

③ Factor & solve

$$0 = (\sin \theta + 1)(\sin \theta + 1)$$

$$0 = (\sin \theta + 1)^2$$

$$\sin \theta + 1 = 0$$

$$\sin \theta = -1$$

$$\theta = \frac{3\pi}{2}$$

(one  
repeating  
solution)

$$\textcircled{13} \quad -\cos\theta + \sin\theta = 1$$

this is a challenging one

$$\begin{array}{r} -\cos\theta + \sin\theta = 1 \\ +\cos\theta \qquad \qquad \qquad +\cos\theta \end{array}$$

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$$\sin\theta = 1 + \cos\theta$$

Since  $\cos\theta \in [-1, 1]$

&  $\sin\theta \in [-1, 1]$

$$\sin\theta = 1 + \cos\theta$$

∴ this implies  $\cos\theta$  must be  $\in [-1, 0]$

$\cos\theta = 0$  at  $\frac{\pi}{2}, \frac{3\pi}{2}$

trial  $\sin\frac{\pi}{2} = 1 + \cos\frac{\pi}{2}$

$$1 = 1 + 0 \quad \checkmark$$

$\theta = \frac{\pi}{2}$   
worked

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trial  $\sin\frac{3\pi}{2} = 1 + \cos\frac{3\pi}{2}$

$$-1 \neq 1 + 0$$

but  $\theta = \frac{3\pi}{2}$

does not  
work

⑬ cont

$$\sin \theta = 1 + \cos \theta$$

can be written as

$$\sin \theta - 1 = \cos \theta$$

This implies  $\sin \theta \in [0, 1]$

if  $\sin \theta = 0$  then  $\sin \theta - 1 = -1$

$\sin \theta = 0$  at  $0, \pi, 2\pi$

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trial  $\sin(0) - 1 \stackrel{?}{=} \cos(0)$  fails

$$0 - 1 \neq 1$$

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trial  $\sin(\pi) - 1 \stackrel{\checkmark}{=} \cos \pi$

$$0 - 1 \stackrel{\checkmark}{=} -1$$

$$-1 \stackrel{\checkmark}{=} -1 \quad \checkmark \checkmark$$

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trial  $\sin 2\pi - 1 \stackrel{?}{=} \cos 2\pi$

$$0 - 1 \neq 1$$
$$-1 \neq 1$$

(13) So the two answers we found were

$$\boxed{\theta = -\frac{\pi}{2} \text{ \& } \theta = \pi}$$

\& we had to pretty much use some selective guessing

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Other attempts at a substitution or use of an identity are more worrisome

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Example  $-\cos\theta + \sin\theta = 1$

$$-\cos\theta + \frac{\cos(90-\theta)}{\text{cofunction ID}} = 1$$

$$-\cos\theta + \cos 90 \cos\theta + \sin\theta \sin 90 = 1$$

$$-\cos\theta + 0(\cos\theta) + \sin\theta(1) = 1$$

$$-\cos\theta + \sin\theta = 1 \quad \text{no closer}$$

Example  
Recall

$$-\cos\theta + \sin\theta = 1$$

$$-\frac{x}{r} + \frac{y}{r} = 1$$

$$-\frac{x+y}{r} = \frac{1}{1}$$

$$-x+y = r$$

$$\boxed{r^2 = x^2 + y^2}$$

Pyth Id  
circle radius  
 $r$

$$\text{So } r^2 = (-x+y)^2$$

$$r^2 = x^2 - 2xy + y^2$$

$$\text{So } r^2 = r^2$$

$$\begin{array}{r} x^2 + y^2 = x^2 - 2xy + y^2 \\ -x^2 - y^2 \quad -x^2 \quad -y^2 \\ \hline 0 = -2xy \end{array}$$

$$0 = -2xy$$

this mean  $x=0$

or

$$xy = 0$$

$$\frac{xy}{r} = 0$$

$\theta =$  worked  
at  $\left(\frac{\pi}{2}\right), \frac{3\pi}{2}$

$$\frac{y}{r} = \sin\theta = 0$$

at  $\theta = 0, \pi, 2\pi$   
↳ worked

$$\textcircled{2} \quad 4 \sin^2 \theta + 4 = 5$$

① isolate  $\sin^2 \theta$

$$4 \sin^2 \theta + 4 - 4 = 5 - 4$$

$$4 \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{1}{4}$$

$$\sin \theta = \pm \sqrt{\frac{1}{4}}$$

$$\sin \theta = \pm \frac{1}{2}$$

$$\sin \theta = \frac{1}{2}$$

$$\text{or } \sin \theta = -\frac{1}{2}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$