

Solutions version (S)

$$\sqrt[4]{5xy^8} = 5^{\frac{1}{4}} x^{\frac{1}{4}} y^{\frac{8}{4}} \\ = \boxed{5^{\frac{1}{4}} x^{\frac{1}{4}} y^2}$$

$$\sqrt{44y^1w^1} = \sqrt{44^{\frac{1}{2}} y^{\frac{1}{2}} w^{\frac{1}{2}}} \\ \boxed{2 \cdot 11^{\frac{1}{2}} \cdot y^{\frac{1}{2}} w^{\frac{1}{2}}} \quad \swarrow \text{better} \\ \boxed{21 \cdot 11^{\frac{1}{2}} y^{\frac{1}{2}} w^{\frac{1}{2}}}$$

$$36^{\frac{1}{4}} x^{\frac{3}{4}} y^{\frac{5}{4}} \quad \text{Since } 36 = 2^2 \cdot 3^2$$

$$\boxed{\sqrt[4]{36x^3y^5}} = \boxed{y^{\frac{5}{4}} \sqrt[4]{36x^3y}}$$

$$21^{\frac{1}{3}} x^{\frac{4}{3}} y^1 \quad \text{Since } 21 = 3^1 \cdot 7^1 \\ = \boxed{y \sqrt[3]{21x^4}} = \boxed{xy \sqrt[3]{21x}}$$

$$\sqrt{256x^6y^4} \quad 256 = 2^8 \\ = \sqrt{256} \sqrt{x^6} \sqrt{y^4} = 16x^3y^2$$

Error Analysis ① doubled 256 instead of $\sqrt{\quad}$

② $x^2 \rightarrow x^3$

$$\frac{1}{\sqrt[3]{16}} \sqrt[3]{2^2} = \frac{\sqrt[3]{4}}{\sqrt[3]{64}} = \frac{\sqrt[3]{4}}{\sqrt[3]{4^3}} = \frac{\sqrt[3]{4}}{4}$$

$$\frac{1}{\sqrt[3]{16}} \frac{\sqrt[3]{16^2}}{\sqrt[3]{16^2}} = \frac{\sqrt[3]{16^2}}{\sqrt[3]{16^3}} = \frac{\sqrt[3]{256}}{16} = \frac{\sqrt[3]{64} \sqrt[3]{4}}{16}$$

$$= \frac{4 \sqrt[3]{4}}{16} = \frac{1}{4} \frac{\sqrt[3]{4}}{1} = \frac{\sqrt[3]{4}}{4}$$

Error Analysis should have multiplied by

$$\frac{\sqrt[3]{16^2}}{\sqrt[3]{16^2}} \text{ or } \frac{\sqrt[3]{4}}{\sqrt[3]{4}}$$

$$\sqrt[3]{12x^8y^9} = \sqrt[3]{12} \sqrt[3]{x^8} \sqrt[3]{y^9} = \sqrt[3]{12} x^{8/3} y^{9/3}$$

since $2^2 \cdot 3^1 = 12$

$$= \sqrt[3]{12} x^{27/3} y^3 = x^9 y^3 \sqrt[3]{12}$$

Error Analysis $\div 12$ by 3 leave alone

$$\begin{aligned}
\sqrt[3]{-216 x^6 y^4} &= \sqrt[3]{-216} \sqrt[3]{x^6} \sqrt[3]{y^4} \\
&= \sqrt[3]{(-6)^3} x^{6/3} y^{4/3} \\
&= -6 x^2 y^{1\frac{1}{3}} \\
&= -6 x^2 y^1 y^{1/3} \\
&= \boxed{-6 x^2 y \sqrt[3]{y}}
\end{aligned}$$

$$\begin{aligned}
\text{OR } \sqrt[3]{-216 x^6 y^4} &= \sqrt[3]{-216 x^6 y^3 \cdot y^1} \\
&= \sqrt[3]{-216} \sqrt[3]{x^6} \sqrt[3]{y^3} \sqrt[3]{y^1} \\
&= \sqrt[3]{(-6)^3} \sqrt[3]{x^6} \sqrt[3]{y^3} \sqrt[3]{y^1} \\
&= -6^1 x^{6/3} y^{3/3} y^{1/3} \\
&= \boxed{-6 x^2 y \sqrt[3]{y}}
\end{aligned}$$

$$\sqrt[3]{\frac{64x^{14}}{125x^8y^6}} = \sqrt[3]{\frac{64}{125}} \sqrt[3]{\frac{x^{14}}{x^8}} \sqrt[3]{\frac{1}{y^6}}$$

$$= \frac{\sqrt[3]{64}}{\sqrt[3]{125}} \sqrt[3]{\frac{x^6}{1}} \sqrt[3]{\frac{1}{y^6}}$$

$$= \frac{4}{5} \frac{x^{6/3}}{1} \frac{1}{y^{6/3}}$$

$$= \frac{4x^2}{5y^2}$$

$$\frac{\sqrt[3]{64x^{14}}}{\sqrt[3]{125x^8y^6}} = \frac{\sqrt[3]{64} \sqrt[3]{x^{12}} \sqrt[3]{x^2}}{\sqrt[3]{125} \sqrt[3]{x^6} \sqrt[3]{x^2} \sqrt[3]{y^6}}$$

$$= \frac{4}{5} \cdot \frac{x^{12/3}}{x^{6/3}} \frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}} \frac{1}{y^{6/3}} = \frac{4}{5} \frac{x^4}{x^2} \frac{1}{y^2}$$

$$= \frac{4}{5} \frac{x^2}{1} \frac{1}{y^2}$$

$$= \frac{4x^2}{5y^2}$$

$$\sqrt{\frac{4x^6y^5}{64x^1y^{21}}} = \frac{\sqrt{4x^6y^5}}{\sqrt{64x^1y^{21}}} = \frac{\sqrt{4} x^{6/2} \sqrt{xy^4y^1}}{\sqrt{64}\sqrt{x} \sqrt{y^{20}}\sqrt{y^1}}$$

$$= \frac{2x^3 y^{4/2} \sqrt{y^1}}{8\sqrt{x} y^{20} \sqrt{y^1}} = \frac{2x^3 y^2 \sqrt{y}}{8y^{10} \sqrt{xy}}$$

$$= \frac{2}{8} \frac{x^3}{1} \frac{y^2}{y^{10}} \frac{\sqrt{y}}{\sqrt{y}} \frac{1}{\sqrt{x}}$$

$$= \frac{1}{4} \frac{x^3}{1} \frac{1}{y^8} \frac{1}{\sqrt{x}} = \frac{x^3}{4y^8} \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}}$$

$$= \frac{x^3 \sqrt{x}}{4y^8 \sqrt{x^2}} = \frac{x^3 \sqrt{x}}{4y^8 x} = \left(\frac{x^2 \sqrt{x}}{4y^8} \right)$$

$$\sqrt{\frac{4x^6y^5}{64x^1y^{21}}}$$

$$= \sqrt{\frac{4}{64}} \sqrt{\frac{x^6}{x^1}} \sqrt{\frac{y^5}{y^{21}}}$$

$$= \sqrt{\frac{1}{16}} \sqrt{\frac{x^5}{1}} \sqrt{\frac{1}{y^{16}}}$$

$$= \sqrt{\left(\frac{1}{4}\right)^2} \frac{\sqrt{x^4} \sqrt{x}}{1} \frac{1}{\sqrt{y^{16}}}$$

$$= \frac{1}{4} \frac{x^{4/2} \sqrt{x}}{1} \frac{1}{y^{16/2}}$$

$$= \frac{1}{4} \frac{x^2 \sqrt{x}}{1} \frac{1}{y^8}$$

$$= \frac{x^2 \sqrt{x}}{4y^8}$$

$$\left[\frac{(25x^2y^6)^3}{27x^{24}y^{-11}} \right]^{-\frac{1}{3}}$$

$$\left[\frac{25^3 x^6 y^{18}}{27^1 x^{24} y^{-11}} \right]^{-\frac{1}{3}} = \left[\frac{25^3}{27^1} \frac{1}{x^{18}} \frac{y^{18}}{1} \right]^{-\frac{1}{3}}$$

$$= \left[\frac{25^3}{27^1} \frac{1}{x^{18}} \frac{y^{18}}{1} \right]^{-\frac{1}{3}}$$

$$= \frac{25^{-3/3}}{27^{-1/3}} \frac{1}{x^{-18/3}} \frac{y^{-18/3}}{1}$$

$$= \frac{25^{-1}}{27^{-1/3}} \frac{1}{x^{-6}} \frac{y^{-6}}{1}$$

$$= \frac{27^{1/3}}{25^1} \frac{x^6}{1} \frac{1}{y^{2/3}} = \frac{3x^6}{25y^{2/3}}$$

$$\frac{3x^6}{25 \sqrt[3]{y^{27} y^2}}$$

$$\frac{3x^6}{25 y^9 \sqrt[3]{y^2}} \frac{\sqrt[3]{y^1}}{\sqrt[3]{y^1}}$$

$$\frac{3x^6 \sqrt[3]{y^1}}{25 y^9 y^1}$$

Also

$$\frac{3x^6 \sqrt[3]{y^1}}{25 y^{10}}$$

$$\frac{3x^6}{25 y^{2/3}}$$

$$\left[\frac{(25x^2y^6)^3}{27x^{24}y^{-11}} \right]^{\frac{1}{3}} = \overset{\text{cont}}{=} \frac{27^{\frac{1}{3}}}{25^1} \frac{x^6}{1} \frac{1}{y^{9^{\frac{2}{3}}}}$$

$$= \frac{3}{25} \frac{x^6}{1} \frac{1}{y^0 y^{\frac{2}{3}}} = \frac{3x^6}{25y^9} \frac{1}{\sqrt[3]{y^2}}$$

$$= \frac{3x^6}{25y^9} \frac{1}{\sqrt[3]{y^2} \sqrt[3]{y^1}} = \frac{3x^6 \sqrt[3]{y^1}}{25y^9 \sqrt[3]{y^3}}$$

$$= \frac{3x^6 \sqrt[3]{y^1}}{25y^{10}}$$