

SOLVING TRIGONOMETRIC EQUATIONS

Directions: Solve each trigonometric function for ALL POSSIBLE VALUES IN DEGREES. Use the hints provided.

HINT COLLECT LIKE TERMS

1.) $\cos x + \sqrt{3} = -\cos x$

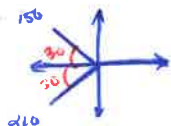
$2\cos x + \sqrt{3} = 0$

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = 150^\circ, 210^\circ$

$x = 150 + 360k$
 $x = 210 + 360k$



HINT EXTRACT SQUARE ROOTS

2.) $4\sin^2 \theta - 3 = 0$

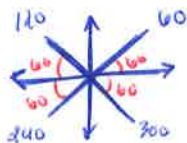
$4\sin^2 \theta = 3$

$\sqrt{\sin^2 \theta} = \sqrt{\frac{3}{4}}$

$\sin \theta = \pm \frac{\sqrt{3}}{2}$

$\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

$\theta = 60^\circ + 180k$
 $\theta = 240^\circ + 180k$



Directions: Solve each trigonometric function for ALL POSSIBLE VALUES IN RADIANS. Use the hints provided.

HINT FACTOR GCF

3.) $2\cos \theta \sin \theta = \cos \theta$

$2\cos \theta \sin \theta - \cos \theta = 0$

$\cos \theta (2\sin \theta - 1) = 0$

$\cos \theta = 0$ $2\sin \theta - 1 = 0$

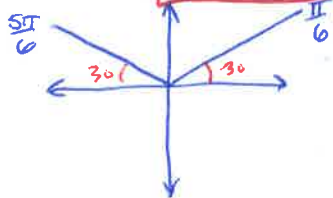
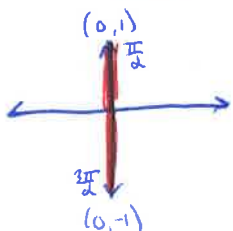
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin \theta = \frac{1}{2}$

$\theta = \frac{\pi}{2} + \pi k$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{6} + 2\pi k$
 $\theta = \frac{5\pi}{6} + 2\pi k$



HINT FACTOR EQUATION AS QUADRATIC TYPE

4.) $2\sin^2 x - 3\sin x + 1 = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$

$\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$\sin x = 1$

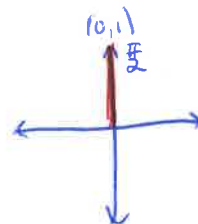
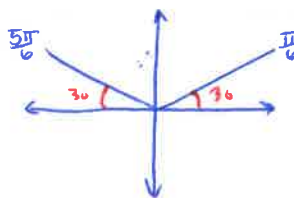
$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{2}$

$x = \frac{\pi}{6} + 2\pi k$

$x = \frac{\pi}{2} + 2\pi k$

$x = \frac{5\pi}{6} + 2\pi k$



Directions: Solve each trigonometric function *IN THE INTERVAL* $[0, 2\pi)$. Use the hints provided.

HINT REWRITE WITH SINGLE TRIG FUNCTION

5.) $3 \sec^2 x - 2 \tan^2 x - 4 = 0$

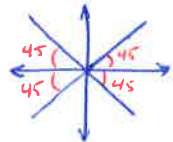
$3(\tan^2 x + 1) - 2 \tan^2 x - 4 = 0$

$3 \tan^2 x + 3 - 2 \tan^2 x - 4 = 0$

$\tan^2 x - 1 = 0$

$\tan^2 x = 1$

$\tan x = \pm 1$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

HINT SQUARE & CONVERT TO QUADRATIC TYPE

6.) $(\sin \theta + 1)^2 = (\cos \theta)^2$ CHECK FOR EXTRANEOUS SOLUTIONS!

$\sin^2 \theta + 2 \sin \theta + 1 = \cos^2 \theta$

$\sin^2 \theta + 2 \sin \theta + 1 = 1 - \sin^2 \theta$

$2 \sin^2 \theta + 2 \sin \theta = 0$

$2 \sin \theta (\sin \theta + 1) = 0$

$2 \sin \theta = 0$

$\sin \theta = 0$

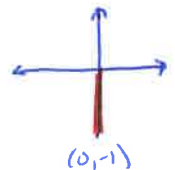
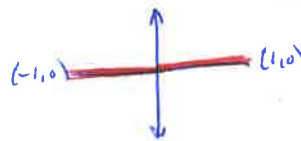
$\theta = 0, \pi$

$\sin \theta + 1 = 0$

$\sin \theta = -1$

$\theta = \frac{3\pi}{2}$

↑ EXTRANEOUS!



Directions: Solve each trigonometric function *IN THE INTERVAL* $[0, 360)$. Use the hints provided.

HINT FUNCTIONS OF MULTIPLE ANGLES

7.) $\sin 2x - \frac{\sqrt{3}}{2} = 0$

$\sin 2x = \frac{\sqrt{3}}{2}$

SET $2x = \theta$

$\sin \theta = \frac{\sqrt{3}}{2}$

$\theta = 60^\circ, 120^\circ$

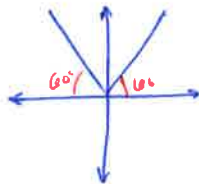
↓

$2x = 60$

$x = 30$

$2x = 120$

$x = 60$



HINT USING INVERSE FUNCTIONS (calculator)

8.) $4 \tan^2 \theta + 5 \tan \theta = 6$

$4 \tan^2 \theta + 5 \tan \theta - 6 = 0$

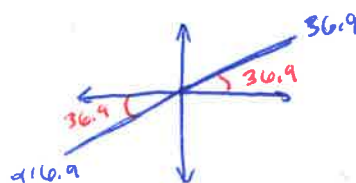
$(4 \tan \theta - 3)(\tan \theta + 2) = 0$

$4 \tan \theta - 3 = 0$

$\tan \theta = \frac{3}{4}$

$\theta = \tan^{-1}(\frac{3}{4})$

$\theta = 36.9^\circ$



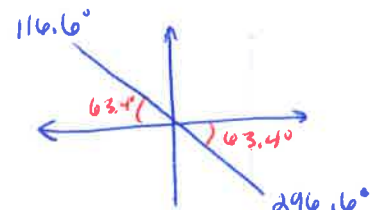
$\theta = 36.9^\circ, 216.9^\circ$

$\tan \theta + 2 = 0$

$\tan \theta = -2$

$\theta = \tan^{-1}(-2)$

$\theta = -63.4^\circ$



$\theta = 116.6^\circ, 296.6^\circ$

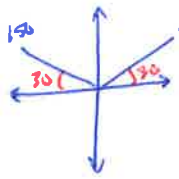
Directions: Solve each trigonometric function for ALL POSSIBLE VALUES IN DEGREES.

9.) $2 \sin^2 \theta + \sin \theta - 1 = 0$

$(2 \sin \theta - 1) (\sin \theta + 1) = 0$

$2 \sin \theta - 1 = 0$ $\sin \theta + 1 = 0$

$\sin \theta = \frac{1}{2}$ $\sin \theta = -1$



$\theta = 30^\circ, 150^\circ$

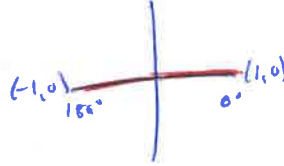
$\theta = 30 + 360k$
 $\theta = 150 + 360k$

10.) $5(\sin \theta + 1) = 5$

$\sin \theta + 1 = 1$

$\sin \theta = 0$

$\theta = 0, 180$



$\theta = 0 + 180k$

11.) $7 \tan \theta = 3\sqrt{3} + \tan \theta$

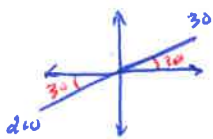
$6 \tan \theta = 3\sqrt{3}$

$\tan \theta = \frac{3\sqrt{3}}{6}$

$\tan \theta = \frac{\sqrt{3}}{3}$

$\theta = 30^\circ, 210^\circ$

$\theta = 30^\circ + 180k$



12.) $2 \sin \theta \cos \theta + \cos \theta = 0$

$\cos \theta (2 \sin \theta + 1) = 0$

$\cos \theta = 0$ $2 \sin \theta + 1 = 0$

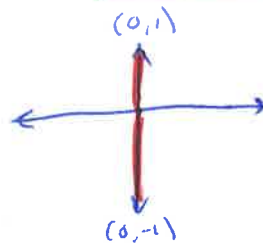
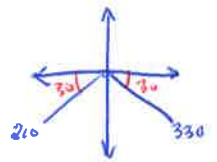
$\theta = 90^\circ, 270^\circ$

$\sin \theta = -\frac{1}{2}$

$\theta = 90^\circ + 180k$

$\theta = 210^\circ, 330^\circ$

$\theta = 210^\circ + 360k$
 $\theta = 330^\circ + 360k$



Directions: Solve each trigonometric function for ALL POSSIBLE VALUES IN RADIANS.

13.) $2 \cos \theta - 1 = 0$

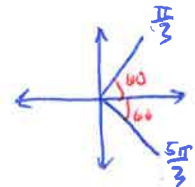
$2 \cos \theta = 1$

$\cos \theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$\theta = \frac{\pi}{3} + 2\pi k$

$\theta = \frac{5\pi}{3} + 2\pi k$



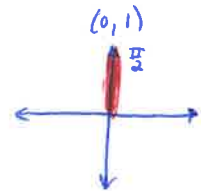
14.) $4 \sin \theta - 1 = 2 \sin \theta + 1$

$2 \sin \theta = 2$

$\sin \theta = 1$

$\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{2} + 2\pi k$



15.) $\sec \theta \csc \theta + \sqrt{2} \csc \theta = 0$

$\csc \theta (\sec \theta + \sqrt{2}) = 0$

$\csc \theta = 0$

$\frac{1}{\sin \theta} = 0$

NO SOLUTION

$\sec \theta + \sqrt{2} = 0$

$\sec \theta = -\sqrt{2}$

$\frac{1}{\cos \theta} = -\sqrt{2}$

$1 = -\sqrt{2} \cos \theta$

$\cos \theta = -\frac{1}{\sqrt{2}}$

$\cos \theta = -\frac{\sqrt{2}}{2}$

$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$

$\theta = \frac{3\pi}{4} + 2\pi k \quad \theta = \frac{5\pi}{4} + 2\pi k$

16.) $\cos^2 x + \sin x = 1$

$1 - \sin^2 x + \sin x = 1$

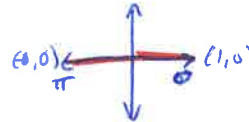
$\sin^2 x - \sin x = 0$

$\sin x (\sin x - 1) = 0$

$\sin x = 0$

$x = 0, \pi$

$x = 0 + \pi k$

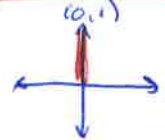


$\sin x - 1 = 0$

$\sin x = 1$

$x = \frac{\pi}{2}$

$x = \frac{\pi}{2} + 2\pi k$



Directions: Solve each trigonometric function IN THE INTERVAL [0, 360).

17.) $\sec x + \tan x = 1$

CHECK FOR EXTRANEU'S SOLUTIONS!

$(\sec x)^2 = (1 - \tan x)^2$

$\sec^2 x = 1 - 2\tan x + \tan^2 x$

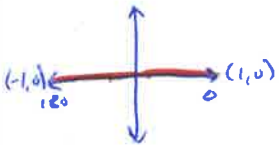
$1 + \tan^2 x = 1 - 2\tan x + \tan^2 x$

$0 = -2\tan x$

$\tan x = 0$

$x = 0^\circ, 180^\circ$

↑
EXTRANEU'S



18.) $\tan(3x) = 1$

SET $3x = \theta$

$\tan \theta = 1$

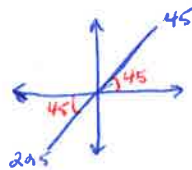
$\theta = 45^\circ, 225^\circ$

$3x = 45$

$x = 15^\circ$

$3x = 225$

$x = 75^\circ$



19.) $2 \sin x + 1 = \csc x$

$2 \sin x + 1 = \frac{1}{\sin x}$

$\sin x (2 \sin x + 1) = 1$

$2 \sin^2 x + \sin x = 1$

$2 \sin^2 x + \sin x - 1 = 0$

$(2 \sin x - 1)(\sin x + 1) = 0$

$2 \sin x - 1 = 0$

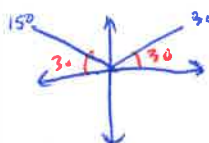
$\sin x = \frac{1}{2}$

$x = 30^\circ, 150^\circ$

$\sin x + 1 = 0$

$\sin x = -1$

$x = 270^\circ$



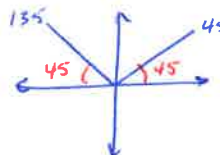
20.) $2 \sin^2 \theta - 1 = 0$

$\sqrt{\sin^2 \theta} = \sqrt{\frac{1}{2}}$

$\sin \theta = \frac{1}{\sqrt{2}}$

$\sin \theta = \frac{\sqrt{2}}{2}$

$\theta = 45^\circ, 135^\circ$



Directions: Solve each trigonometric function *IN THE INTERVAL* $[0, 2\pi)$.

21.) $2 \sin^2 \theta - \sin \theta = 3$

$$2 \sin^2 \theta - \sin \theta - 3 = 0$$

$$(2 \sin \theta - 3)(\sin \theta + 1) = 0$$

$$2 \sin \theta - 3 = 0$$

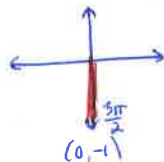
$$\sin \theta = \frac{3}{2}$$

NO SOLUTION!

$$\sin \theta + 1 = 0$$

$$\sin \theta = -1$$

$$\theta = \frac{3\pi}{2}$$



22.) $3 \tan^2 \theta = 1$

$$\tan^2 \theta = \frac{1}{3}$$

$$\tan \theta = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \frac{\sqrt{3}}{3}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

23.) $\csc x + \cot x = 1$

$$(\csc x)^2 = (1 - \cot x)^2$$

CHECK FOR
EXTRANEIOUS
SOLUTIONS!

$$\csc^2 x = 1 - 2\cot x + \cot^2 x$$

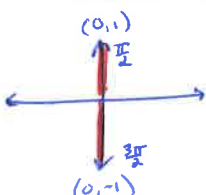
$$\cot^2 x + 1 = 1 - 2\cot x + \cot^2 x$$

$$0 = -2\cot x$$

$$0 = \cot x$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

↑ EXTRANEIOUS



24.) $2 \sin(2x) = -\sqrt{3}$

$$\sin(2x) = -\frac{\sqrt{3}}{2}$$

SET $2x = \theta$

$$\sin \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \frac{4\pi}{3}$$

$$\theta = \frac{5\pi}{3}$$

$$2x = \frac{4\pi}{3}$$

$$2x = \frac{5\pi}{3}$$

$$x = \frac{2\pi}{3}$$

$$x = \frac{5\pi}{6}$$

$$x = \frac{2\pi}{3}$$

Directions: Use inverse functions to solve each trigonometric function *IN THE INTERVAL* $[0, 360)$. Round all answers to the nearest tenth.

25.) $\tan^2 x - 6 \tan x + 5 = 0$

$$(\tan x - 5)(\tan x - 1) = 0$$

$$\tan x - 5 = 0$$

$$\tan x = 5$$

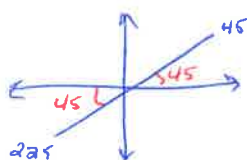
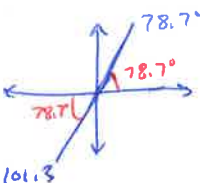
$$x = \tan^{-1}(5)$$

$$x = 78.7^\circ, 101.3^\circ$$

$$\tan x - 1 = 0$$

$$\tan x = 1$$

$$x = 45^\circ, 225^\circ$$



26.) $2 \cos^2 x - 5 \cos x + 2 = 0$

$$(2 \cos x - 1)(\cos x - 2) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = 60^\circ, 300^\circ$$

$$\cos x - 2 = 0$$

$$\cos x = 2$$

NO SOLUTION!

