$$\sqrt{24\,x^3\,y^{-8}} = \sqrt{24}\,\sqrt{x^3}\,\sqrt{y^{-8}}$$

$$= \sqrt{4}\,\sqrt{6}\,\sqrt{x^2}\,\sqrt{x^1}\,\sqrt{y^{-8}}$$

$$= 2\sqrt{6}\,\,x^{2/2}\sqrt{x}\,\,\,y^{-8/2}$$

$$= 2\sqrt{6}\,\,x^1\,\sqrt{x}\,\,y^{-4}$$

$$= 2x^1\,y^{-4}\,\sqrt{6x}$$

$$= \frac{2x}{y^4}\,\frac{\sqrt{6x}}{1} = \boxed{\frac{2x\sqrt{6x}}{y^4}}$$

$$\boxed{\text{Version } 24} \quad \underline{\text{Method} \textcircled{2}} \quad 24 = 2^3 3^1$$

$$\sqrt{24 x^3 y^{-8}} = \sqrt{\frac{24 x^3}{y^8}} = \frac{\sqrt{24 x^3}}{\sqrt{y^8}}$$

$$= \frac{\sqrt{24}\sqrt{x^3}}{\sqrt{y^8}} = \frac{\sqrt{2^3 3^1}\sqrt{x^3}}{\sqrt{y^8}} = \frac{2^{\frac{3}{2}} 3^{\frac{1}{2}} x^{\frac{3}{2}}}{y^{\frac{8}{2}}}$$

$$= \frac{2^{\frac{3}{2}} 3^{\frac{1}{2}} x^{\frac{3}{2}}}{y^4} = \frac{2^{1\frac{1}{2}} 3^{\frac{1}{2}} x^{1\frac{1}{2}}}{y^4}$$

$$= \frac{2^{1+\frac{1}{2}} 3^{\frac{1}{2}} x^{1+\frac{1}{2}}}{y^4} = \frac{2^1 2^{\frac{1}{2}} 3^{\frac{1}{2}} x^1 x^{\frac{1}{2}}}{y^4}$$

$$= \frac{2^1 x^1 2^{\frac{1}{2}} 3^{\frac{1}{2}} x^{\frac{1}{2}}}{y^4} = \frac{2x \sqrt{2^1 3^1 x^1}}{y^4} = \boxed{\frac{2x\sqrt{6x}}{y^4}}$$

$$\sqrt[3]{24x^3y^{-8}} = \sqrt[3]{24}\,\sqrt[3]{x^3}\,\sqrt[3]{y^{-8}}$$

$$= \sqrt[3]{8}\,\sqrt[3]{3}\,\sqrt[3]{x^3}\,\sqrt[3]{y^{-6}}\,\sqrt[3]{y^{-2}}$$

$$= 2\,\sqrt[3]{3}\,x^{3/3}\,y^{-6/3}\,\sqrt[3]{y^{-2}}$$

$$= 2\,\sqrt[3]{3}\,x^1\,y^{-2}\,\sqrt[3]{y^{-2}}$$

$$= 2x^1y^{-2}\,\sqrt[3]{3y^{-2}}$$

$$= \frac{2x}{y^2}\cdot\frac{\sqrt[3]{3}}{\sqrt[3]{y^2}} \rightarrow \frac{2x}{y^2}\,\frac{\sqrt[3]{3}}{\sqrt[3]{y^2}}\,\frac{\sqrt[3]{y^1}}{\sqrt[3]{y^1}}$$

$$= \frac{2x}{y^2}\,\frac{\sqrt[3]{3y}}{\sqrt[3]{y^3}} = \frac{2x\,\sqrt[3]{3y}}{y^2\,y^{3/3}} = \frac{2x\,\sqrt[3]{3y}}{y^2\,y^1}$$

$$= \boxed{\frac{2x\,\sqrt[3]{3y}}{y^3}}$$

VERSION 24 Method 2

$$24 = 2^3 \cdot 3^1 \qquad \sqrt[3]{24x^3y^{-8}} = \sqrt[3]{\frac{24x^3}{y^8}}$$

$$= \frac{\sqrt[3]{2^3 \cdot 3^1}\ \sqrt[3]{x^3}}{\sqrt[3]{y^8}} = \frac{2^{\frac{3}{3}} 3^{\frac{1}{3}} x^{\frac{3}{3}}}{y^{8/3}}$$

$$= \frac{2^1 3^{\frac{1}{3}} x^1}{y^{2\frac{2}{3}}} = \frac{2^1 3^{\frac{1}{3}} x^1}{y^{2+\frac{2}{3}}}$$

$$= \frac{2^1 3^{\frac{1}{3}} x^1}{y^2 y^{\frac{2}{3}}} = \frac{2x \cdot 3^{\frac{1}{3}}}{y^2 y^{\frac{2}{3}}}$$

$$= \frac{2x\sqrt[3]{3}}{y^2 \sqrt[3]{y^2}} \rightarrow \frac{2x\sqrt[3]{3}}{y^2 \sqrt[3]{y^2}} \cdot \frac{\sqrt[3]{y^1}}{\sqrt[3]{y^1}}$$

$$= \frac{2x\sqrt[3]{3y}}{y^2 \sqrt[3]{y^3}} = \frac{2x\sqrt[3]{3y}}{y^2 y^1} = \boxed{\frac{2x\sqrt[3]{3y}}{y^3}}$$

$$\frac{1}{\sqrt{15}} = \frac{1}{\sqrt{15}} \cdot 1 = \frac{1}{\sqrt{15}} \cdot \frac{\sqrt{15}}{\sqrt{15}} = \frac{1\sqrt{15}}{\sqrt{15^2}} = \frac{\sqrt{15}}{(15^{2/2})} = \boxed{\frac{\sqrt{15}}{15}}$$

$$\frac{1}{\sqrt{15}} = \frac{1}{\sqrt{15}} \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{15}}{\sqrt{225}} = \boxed{\frac{\sqrt{15}}{15}}$$

$$\frac{4}{\sqrt{12}} = \frac{4}{\sqrt{12}} \cdot 1 = \frac{4}{\sqrt{12}} \cdot \frac{\sqrt{12}}{\sqrt{12}} = \frac{4\sqrt{12}}{\sqrt{12^2}} = \frac{4\sqrt{12}}{(12^{2/2})} = \frac{4\sqrt{12}}{12}$$

$$= \frac{4}{12} \cdot \frac{\sqrt{12}}{1} = \frac{1}{3} \cdot \frac{\sqrt{12}}{1} = \frac{1}{3} \cdot \frac{\sqrt{4}\sqrt{3}}{1} = \frac{1}{3} \cdot \frac{2}{1} \cdot \frac{\sqrt{3}}{1}$$

$$= \frac{2}{3}\sqrt{3} = \boxed{\frac{2\sqrt{3}}{3}}$$

$$\frac{4}{(12)^{1/2}} = \frac{2^2}{(2^2 \cdot 3)^{1/2}} = \frac{2^2}{2^{2/2} 3^{\frac{1}{2}}} = \frac{2^2}{2^1} \cdot \frac{1}{3^{\frac{1}{2}}} = \frac{2^1}{1} \cdot \frac{1}{3^{\frac{1}{2}}}$$

$$= \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \boxed{\frac{2\sqrt{3}}{3}}$$

$$\frac{4}{\sqrt{12}} = \frac{4}{\sqrt{9}\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{4}{2} \cdot \frac{1}{\sqrt{3}} = \frac{2}{1} \cdot \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \boxed{\frac{2\sqrt{3}}{3}}$$

$$\frac{3}{\sqrt[3]{9}} = \frac{3}{\sqrt[3]{9}} \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{3\sqrt[3]{3}}{\sqrt[3]{27}} = \frac{3\sqrt[3]{3}}{\sqrt[3]{3^3}} = \frac{3\sqrt[3]{3}}{3}$$

$$= \frac{3}{3} \cdot \frac{\sqrt[3]{3}}{1} = 1 \cdot \sqrt[3]{3} = \boxed{\sqrt[3]{3}}$$

$$\frac{3}{\sqrt[3]{9}} \frac{\sqrt[3]{9^2}}{\sqrt[3]{9^2}} = \frac{3\sqrt[3]{9^2}}{\sqrt[3]{9^3}} = \frac{3\sqrt[3]{9^2}}{9^{3/3}} = \frac{3 \cdot \sqrt[3]{9^2}}{9^1}$$

$$= \frac{3}{9} \cdot \frac{\sqrt[3]{81}}{1} = \frac{1}{3} \cdot \frac{\sqrt[3]{27}\sqrt[3]{3}}{1} = \frac{1}{3} \frac{3\sqrt[3]{3}}{1}$$

$$= \frac{3\sqrt[3]{3}}{3} = \frac{3}{3} \cdot \sqrt[3]{3} = \boxed{1\sqrt[3]{3}}$$

$$\frac{3^1}{\sqrt[3]{9}} = \frac{3^1}{(9)^{1/3}} = \frac{3^1}{(3^2)^{1/3}} = \frac{3^1}{3^{2/3}} = 3^{1-2/3} = 3^{1/3}$$

$$= \boxed{\sqrt[3]{3}}$$