

LT 3.4 Book 377

$$\textcircled{1} \sqrt{8} \sqrt{32} = \sqrt{2^3} \sqrt{2^5} = \sqrt{2^3 \cdot 2^5} = \sqrt{2^8} \\ = 2^{\frac{8}{2}} = \textcircled{2^4} = \textcircled{16}$$

$$\sqrt{8} \sqrt{8} \sqrt{4} = \sqrt{64} \sqrt{4} = 8 \cdot 2 = \textcircled{16}$$

$$\sqrt{4} \sqrt{2} \sqrt{16} \sqrt{2} = (2\sqrt{2})(4\sqrt{2}) = (2)(4)(\sqrt{2})(\sqrt{2}) \\ = 8 \sqrt{4} = 8 \cdot 2 = \textcircled{16}$$

$$\sqrt{8 \cdot 32} = \sqrt{256} = \textcircled{16}$$

$$\textcircled{5} \sqrt{-5} \sqrt{5} = \sqrt{-25} = \sqrt{-1} \sqrt{25} = i \cdot 5 = \textcircled{5i}$$

↓
not real

↓
imaginary

$$\sqrt{-5} \sqrt{5} = \sqrt{-1} \sqrt{5} \sqrt{5} = i \sqrt{5^2} = i \cdot 5 = \textcircled{5i}$$

↑
not real

$$\textcircled{17} \sqrt[3]{6} \sqrt[3]{16} = \sqrt[3]{96} \sqrt[3]{8} \sqrt[3]{12} = \sqrt[3]{2^3 \cdot 3} \sqrt[3]{12} \\ = \textcircled{2 \sqrt[3]{12}}$$

↓
perfect cube

①7 method ②

$$\begin{aligned}\sqrt[3]{6} \sqrt[3]{16} &= \sqrt[3]{2^1 \cdot 3^1} \cdot \sqrt[3]{2^4} \\ &= \sqrt[3]{2^5 \cdot 3^1} \\ &= \sqrt[3]{2^3} \sqrt[3]{2^2 \cdot 3^1} \\ &= 2 \cdot \sqrt[3]{2^2 \cdot 3^1} \\ &= \boxed{2 \cdot \sqrt[3]{12}}\end{aligned}$$

①7 Method ③

$$\begin{aligned}\sqrt[3]{6} \sqrt[3]{16} &= 6^{\frac{1}{3}} 16^{\frac{1}{3}} = (2^1 \cdot 3^1)^{\frac{1}{3}} (2^4)^{\frac{1}{3}} \\ &= 2^{\frac{1}{3}} 3^{\frac{1}{3}} 2^{\frac{4}{3}} \\ &= 2^{\frac{1}{3}} 2^{\frac{4}{3}} 3^{\frac{1}{3}} \\ &= 2^{\frac{5}{3}} 3^{\frac{1}{3}} \\ &= 2^{\frac{2}{3}} 3^{\frac{1}{3}} \\ &= 2^{1 + \frac{2}{3}} 3^{\frac{1}{3}} \\ &= 2^1 2^{\frac{2}{3}} 3^{\frac{1}{3}} \\ &= 2 \left(2^{\frac{2}{3}} 3^{\frac{1}{3}} \right)^{\frac{1}{3}} \\ &= 2 \sqrt[3]{2^2 \cdot 3} = \boxed{2 \sqrt[3]{12}}\end{aligned}$$

$$\begin{aligned}
 (19) \quad \sqrt{7x^5} \sqrt{42x^1y^9} &= \sqrt{7 \cdot 42 x^5 x^1 y^9} \\
 &= \sqrt{294} \sqrt{x^6} \sqrt{y^9} \\
 &= \sqrt{49 \cdot 6} \sqrt{x^6} \sqrt{y^8 \cdot y^1} \\
 &= \sqrt{49} \sqrt{6} \sqrt{x^6} \sqrt{y^8} \sqrt{y^1} \\
 &= 7\sqrt{6} x^{6/2} y^{8/2} \sqrt{y} \\
 &= 7\sqrt{6} x^3 y^4 \sqrt{y} \\
 &= \boxed{7x^3y^4\sqrt{6y}}
 \end{aligned}$$

$$\sqrt{7x^4x^1} \sqrt{7 \cdot 6 x^1 y^8 y^1}$$

$$\sqrt{x^4} \sqrt{7x} \sqrt{y^8} \sqrt{7 \cdot 6xy}$$

$$x^2 \sqrt{7x} y^4 \sqrt{7x} \sqrt{6y}$$

$$x^2 y^4 \sqrt{7x} \sqrt{7x} \sqrt{6y}$$

$$x^2 y^4 \sqrt{49x^2} \sqrt{6y}$$

$$x^2 y^4 \cdot 7x \sqrt{6y}$$

$$\boxed{7x^3y^4\sqrt{6y}}$$

$$\sqrt{7x^5} \sqrt{42x^1y^9}$$

$$\sqrt{7^1 x^5} \sqrt{7^1 \cdot 6^1 x^1 y^9}$$

$$7^{1/2} x^{5/2} 7^{1/2} 6^{1/2} x^{1/2} y^{9/2}$$

$$7^{1/2} 7^{1/2} 6^{1/2} x^{5/2} x^{1/2} y^{9/2}$$

$$7^{1/2+1/2} 6^{1/2} x^{5/2+1/2} y^{9/2}$$

$$7^1 6^{1/2} x^3 y^{9/2}$$

$$7^1 6^{1/2} x^3 y^{4\frac{1}{2}}$$

$$7^1 x^3 y^{4+\frac{1}{2}} 6^{1/2}$$

$$7^1 x^3 y^{4\frac{1}{2}} 6^{1/2} = \boxed{7x^3y^4\sqrt{6y}}$$

$$(21) \quad 3 \sqrt[3]{5y^3} \cdot 2 \sqrt[3]{50y^4}$$

$$3 \cdot 2 \sqrt[3]{5 \cdot 50 \cdot y^3 y^4}$$

$$6 \sqrt[3]{250 y^7}$$

$$6 \cdot \sqrt[3]{125} \sqrt[3]{2} \sqrt[3]{y^6} \sqrt[3]{y}$$

$$6 \cdot \sqrt[3]{5^3} \sqrt[3]{2} \sqrt[3]{y^6} \sqrt[3]{y}$$

$$6 \cdot 5^{3/3} \sqrt[3]{2} y^{6/3} \sqrt[3]{y}$$

$$6 \cdot 5^1 \cdot \sqrt[3]{2} y^2 \sqrt[3]{y}$$

$$\boxed{30 y^2 \sqrt[3]{2y}}$$

$$3 \cdot \sqrt[3]{5y^3} \cdot 2 \sqrt[3]{50y^4}$$

$$3 \cdot 2 \sqrt[3]{5} \sqrt[3]{y^3} \sqrt[3]{50} \sqrt[3]{y^4}$$

$$6 \sqrt[3]{5} y^{3/3} \sqrt[3]{50} \sqrt[3]{y^3} \sqrt[3]{y}$$

$$6 \cdot \sqrt[3]{5} y^1 \sqrt[3]{50} y^{3/3} \sqrt[3]{y}$$

$$6 \cdot y \sqrt[3]{5} \sqrt[3]{50} y^1 \sqrt[3]{y}$$

$$6 \cdot y \cdot y^1 \sqrt[3]{250} \sqrt[3]{y} = 6 \cdot y^2 \sqrt[3]{125} \sqrt[3]{2} \sqrt[3]{y}$$

$$\begin{array}{l} 6y^2 \cdot 5 \sqrt[3]{2y} \\ \hline 30y^2 \sqrt[3]{2y} \end{array}$$

$$(21) 3 \cdot \sqrt[3]{5y^3}, 2 \sqrt[3]{50y^4}$$

$$3 \cdot \sqrt[3]{5^1 y^3} \cdot 2 \cdot \sqrt[3]{2 \cdot 5^2 y^4}$$

$$3 \cdot 2 \cdot \sqrt[3]{5^1 5^2} \sqrt[3]{y^3 y^4} \sqrt[3]{2}$$

$$6 \cdot \sqrt[3]{5^3} \sqrt[3]{y^7} \sqrt[3]{2}$$

$$6 \cdot 5^{3/3} \sqrt[3]{y^6} \sqrt[3]{y^1} \sqrt[3]{2}$$

$$6 \cdot 5^1 y^{6/3} \sqrt[3]{2y}$$

$$\boxed{30 y^2 \sqrt[3]{2y}}$$

$$(22) \frac{\sqrt{500}}{\sqrt{5}} = \sqrt{\frac{500}{5}} = \sqrt{100} = \sqrt{10^2} = (10)$$

$$\text{OR } \frac{\cancel{\sqrt{5}} \sqrt{100}}{\cancel{\sqrt{5}}} = \sqrt{100} = (10)$$

$$\text{OR } \frac{\sqrt{5^3 \cdot 2^2}}{\sqrt{5^1}} = \frac{\sqrt{5^3}}{\sqrt{5^1}} \frac{\sqrt{2^2}}{1} = \sqrt{5^2} \sqrt{2^2} = 5 \cdot 2 = (10)$$

$$\textcircled{37} \sqrt{5} \sqrt{40} = \sqrt{200} = \sqrt{100} \sqrt{2} = \textcircled{10\sqrt{2}}$$

$$\begin{aligned} \sqrt{5} \sqrt{40} &= \sqrt{5} \sqrt{2^3 \cdot 5} = \sqrt{5^2 \cdot 2^3} = \sqrt{5^2 \cdot 2^2 \cdot 2} \\ &= 5^{2/2} 2^{2/2} \sqrt{2} \\ &= 5 \cdot 2 \sqrt{2} \\ &= \textcircled{10\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \textcircled{40} 5^{1/2} 40^{1/2} &= 5^{1/2} \cdot (2^3 \cdot 5)^{1/2} = 5^{1/2} 2^{3/2} 5^{1/2} \\ &= 5^{1/2+1/2} 2^{3/2} \\ &= 5 \cdot 2^{1\frac{1}{2}} \\ &= 5 \cdot 2 \cdot 2^{1/2} \\ &= 5 \cdot 2 \sqrt{2} \\ &= \textcircled{10\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \textcircled{40} 5 \sqrt{2x^4y^6} \cdot 2 \sqrt{2x^3y} &= 5 \cdot 2 \sqrt{2 \cdot 2 \cdot x^4 \cdot x^3 \cdot y^6 \cdot y^1} \\ &= 10 \sqrt{2^2 x^4 y^6 y^1} = 10 \cdot 2^{2/2} x^{4/2} y^{6/2} y^{1/2} \\ &= 10 \cdot 2 x^2 y^3 y^{1/2} \\ &= \boxed{20 x^2 y^3 \sqrt{y}} \end{aligned}$$

$$(40) 5\sqrt{2xy^6} \cdot 2\sqrt{2x^3y}$$

$$5\sqrt{2x}\sqrt{y^6} \cdot 2\sqrt{2y}\sqrt{x^2x^1}$$

$$2 \cdot 5\sqrt{2x}y^{\frac{6}{2}}\sqrt{2y} \cdot x^{\frac{2}{2}}\sqrt{x^1}$$

$$10\sqrt{2x}y^3\sqrt{2y}x^1\sqrt{x}$$

$$10xy^3\sqrt{2x}\sqrt{2y}\sqrt{x}$$

$$10xy^3\sqrt{2 \cdot 2}\sqrt{xx}\sqrt{y}$$

$$10xy^3\sqrt{4}\sqrt{x^2}\sqrt{y}$$

$$10xy^3 \cdot 2 \cdot x \cdot \sqrt{y}$$

$$\boxed{20x^2y^3\sqrt{y}}$$

$$(43) \sqrt{5}(\sqrt{5} + \sqrt{75})$$

$$\sqrt{25} + \sqrt{75}$$

$$5 + \sqrt{25}\sqrt{3}$$

$$\boxed{5 + 5\sqrt{3}}$$

$$\sqrt{5}(\sqrt{5} + \sqrt{5}\sqrt{3})$$

$$\sqrt{25} + \sqrt{25}\sqrt{3}$$

$$\boxed{5 + 5\sqrt{3}}$$

$$5^{\frac{1}{2}}(5^{\frac{1}{2}} + 5^{\frac{1}{2}}3^{\frac{1}{2}})$$

$$5^{\frac{1}{2}+\frac{1}{2}} + 5^{\frac{1}{2}+\frac{1}{2}}3^{\frac{1}{2}}$$

$$5^1 + 5^1 \cdot 2^{\frac{1}{2}} = 5 + 5\sqrt{2}$$

(46)

$$\frac{\sqrt{5x^4}}{\sqrt{2x^2y^3}} = \sqrt{\frac{5}{2}} \sqrt{\frac{x^4}{x^2}} \sqrt{\frac{1}{y^3}}$$

$$= \frac{\sqrt{5}}{\sqrt{2}} \frac{\sqrt{x^2}}{1} \frac{1}{\sqrt{y^2}\sqrt{y}}$$

$$= \frac{\sqrt{5}}{\sqrt{2}} \frac{x}{1} \frac{1}{y} \frac{1}{\sqrt{y}}$$

$$= \frac{x\sqrt{5}}{y} \cdot \frac{1}{\sqrt{2y}} \frac{\sqrt{2y}}{\sqrt{2y}}$$

$$= \frac{x\sqrt{10y}}{y\sqrt{4y^2}} = \frac{x\sqrt{10y}}{y \cdot 2y}$$

$$= \frac{x\sqrt{10y}}{2y^2}$$

(46)

$$\frac{\sqrt{5x^4}}{\sqrt{2x^2y^3}} \cdot \frac{\sqrt{2y}}{\sqrt{2y}} = \frac{\sqrt{10x^4y}}{\sqrt{4x^2y^4}}$$

$$= \frac{\sqrt{x^4} \sqrt{10y}}{\sqrt{4} \sqrt{x^2} \sqrt{y^4}}$$

$$= \frac{x^2 \sqrt{10y}}{2xy^2}$$

$$= \frac{1}{2} \frac{x^2}{x} \frac{\sqrt{10y}}{y^2}$$

$$= \frac{1}{2} \frac{x}{1} \frac{\sqrt{10y}}{y^2}$$

$$= \frac{x\sqrt{10y}}{2y^2}$$

$$\begin{aligned}
 (49) \quad \frac{10}{\sqrt[3]{5x^2}} \frac{\sqrt[3]{5^2x}}{\sqrt[3]{5^2x}} &= \frac{10 \sqrt[3]{5^2x}}{\sqrt[3]{5^3x^3}} \\
 &= \frac{10 \sqrt[3]{25x}}{5^{3/3} x^{3/3}} = \frac{10}{5} \frac{1}{x} \frac{\sqrt[3]{25x}}{1} \\
 &= \frac{2}{1} \frac{1}{x} \sqrt[3]{25x} = \frac{2 \sqrt[3]{25x}}{x}
 \end{aligned}$$

$$\begin{aligned}
 \frac{10}{5^{1/3} x^{2/3}} \frac{5^{2/3} x^{1/3}}{5^{2/3} x^{1/3}} &= \frac{10 \cdot 5^{2/3} x^{1/3}}{5^{3/3} x^{3/3}} \\
 &= \frac{10 \sqrt[3]{5^2x^1}}{5x} \\
 &= \frac{10}{5} \frac{1}{x} \frac{\sqrt[3]{25x}}{1} \\
 &= \frac{2}{1} \frac{1}{x} \frac{\sqrt[3]{25x}}{1} \\
 &= \frac{2 \sqrt[3]{25x}}{x}
 \end{aligned}$$

51

$$\frac{3\sqrt{11x^3y}}{-2\sqrt{12x^4y}}$$

$$= \frac{3}{-2} \sqrt{\frac{11}{12}} \cdot \sqrt{\frac{x^3}{x^4}} \sqrt{\frac{y}{y}}$$

$$= -\frac{3}{2} \frac{\sqrt{11}}{\sqrt{4}\sqrt{3}} \sqrt{\frac{1}{x}} \cdot 1$$

$$= \frac{-3\sqrt{11}}{2 \cdot 2\sqrt{3}x} = \frac{-3\sqrt{11}}{4\sqrt{3}x} \frac{\sqrt{3}x}{\sqrt{3}x}$$

$$= \frac{-3\sqrt{11}}{4\sqrt{3}x} \frac{\sqrt{3}x}{\sqrt{3}x} = \frac{-3\sqrt{33}x}{4\sqrt{9x^2}}$$

$$= \frac{-3\sqrt{33}x}{4 \cdot 3x} = \frac{-3\sqrt{33}x}{12x}$$

$$= \frac{-3}{12} \cdot \frac{\sqrt{33}x}{x} = \frac{-1}{4} \frac{\sqrt{33}x}{x}$$

$$= \frac{-\sqrt{33}x}{4x}$$

51

$$\frac{3\sqrt{11x^3y}}{-2\sqrt{12x^4y}} = \frac{3\sqrt{x^2}\sqrt{11xy}}{-2\sqrt{4x^4}\sqrt{3y}}$$

$$= \frac{3x\sqrt{11xy}}{-2(2x^2)\sqrt{3y}} = \frac{3}{-4} \frac{x}{x^2} \frac{\sqrt{y}}{\sqrt{y}} \frac{\sqrt{11x}}{\sqrt{3}}$$

$$= \frac{-3}{4} \frac{1}{x} \cdot 1 \frac{\sqrt{11x}}{\sqrt{3}} = \frac{-3\sqrt{11x}}{4x\sqrt{3}}$$

$$= \frac{-3\sqrt{11x}}{4x\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{33x}}{4x\sqrt{9}}$$

$$= \frac{-3\sqrt{33x}}{4x \cdot 3} = \frac{-3}{3} \cdot \frac{\sqrt{33x}}{4x}$$

$$= \frac{-1\sqrt{33x}}{4x}$$

$$\begin{aligned}
 & \textcircled{52} \quad -2(\sqrt[3]{32} + \sqrt[3]{54}) \\
 & \quad -2(\sqrt[3]{8 \cdot 4} + \sqrt[3]{27 \cdot 2}) \\
 & \quad -2(\sqrt[3]{8} \sqrt[3]{4} + \sqrt[3]{27} \sqrt[3]{2}) \\
 & \quad -2(2 \sqrt[3]{4} + 3 \sqrt[3]{2})
 \end{aligned}$$

$$\boxed{-4 \sqrt[3]{4} - 6 \sqrt[3]{2}}$$

$$-2(\sqrt[3]{32} + \sqrt[3]{54})$$

$$-2(\sqrt[3]{2^5} + \sqrt[3]{3^3 \cdot 2})$$

$$-2(2^{\frac{5}{3}} + 3^{\frac{3}{3}} \cdot 2^{\frac{1}{3}})$$

$$-2(2^{1\frac{2}{3}} + 3^1 \cdot 2^{\frac{1}{3}})$$

$$-2(2^{1+\frac{2}{3}} + 3^1 \cdot 2^{\frac{1}{3}})$$

$$-2(2^1 2^{\frac{2}{3}} + 3^1 \cdot 2^{\frac{1}{3}})$$

$$-2(2 \sqrt[3]{2^2} + 3 \sqrt[3]{2})$$

$$\boxed{-4 \sqrt[3]{4} - 6 \sqrt[3]{2}}$$

$$(84) \left(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{8}} \right) \frac{\sqrt{8}}{\sqrt{8}}$$

$$\frac{\sqrt{24} - \sqrt{16}}{\sqrt{64}} = \frac{\sqrt{24} - 4}{8}$$

$$= \frac{\sqrt{4}\sqrt{6} - 4}{8} = \frac{2\sqrt{6} - 4}{8}$$

$$= \frac{2}{2} \left(\frac{\sqrt{6} - 2}{4} \right) = \frac{\sqrt{6} - 2}{4}$$

$$\left(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{8}} \right) \frac{\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{24} - \sqrt{16}}{\sqrt{64}} = \frac{\sqrt{24} - 4}{8}$$

$$= \frac{\sqrt{4}\sqrt{6} - 4}{8} = \frac{2\sqrt{6}}{8} - \frac{4}{8} = \frac{\sqrt{6}}{4} - \frac{1}{2}$$