

8.6 Rational Exponents

$$\textcircled{1} m^{\overset{\text{power}}{3}}_{\underset{\text{index}}{5}} = \boxed{\sqrt[5]{m^3}} \leftarrow \text{power}$$

$$\begin{aligned} \textcircled{2} (7^1 x^1)^{\frac{3}{2}} &= 7^{\frac{3}{2}} x^{\frac{3}{2}} = \sqrt{7^3} \sqrt{x^3} = \sqrt{7^3 x^3} \\ &= (7^3 x^3)^{\frac{1}{2}} = \sqrt{7^3 x^3} \\ &= \sqrt{7^2 \cdot 7^1} \cdot \sqrt{x^2 x^1} \\ &= \sqrt{7^2} \sqrt{x^2} \sqrt{7x} = \boxed{7x\sqrt{7x}} \end{aligned}$$

$$\text{OR } 7^{\frac{3}{2}} = 7^{1\frac{1}{2}} = 7^{1+\frac{1}{2}} = 7^1 \cdot 7^{\frac{1}{2}} = 7\sqrt{7}$$

$$x^{\frac{3}{2}} = x^{1\frac{1}{2}} = x^{1+\frac{1}{2}} = x^1 x^{\frac{1}{2}} = x\sqrt{x}$$

$$\boxed{7^{\frac{3}{2}} x^{\frac{3}{2}} = 7\sqrt{7} x\sqrt{x} = 7x\sqrt{7x}}$$

$$\begin{aligned} \textcircled{5} \frac{1}{(\sqrt{6x})^3} &= \frac{1}{(\sqrt{6^1 x^1})^3} = \frac{1}{(6^{\frac{1}{2}} x^{\frac{1}{2}})^3} = \boxed{\frac{1}{6^{\frac{3}{2}} x^{\frac{3}{2}}}} \\ &= \boxed{6^{-\frac{3}{2}} x^{-\frac{3}{2}}} \end{aligned}$$

$$\textcircled{7} \quad \frac{1}{(\sqrt[4]{n^{-1}})^7} = \frac{1}{(n^{-1/4})^7} = \frac{1}{n^{-7/4}} = n^{7/4}$$

$$\textcircled{9} \quad 8^{2/3} = (2^3)^{2/3} = 2^{2} = 4$$

$$\textcircled{11} \quad 4^{3/2} = (2^2)^{3/2} = 2^3 = 8$$

$$\begin{aligned} \textcircled{13} \quad y^1 x^{1/2} x^1 y^{2/3} &= y^1 y^{2/3} \cdot x^1 x^{1/2} = y^{1+2/3} x^{1+1/2} \\ &= y^{5/3} x^{3/2} = \boxed{y^{5/3} x^{3/2}} \end{aligned}$$

$$\textcircled{15} \quad (a^{1/2} b^{1/2})^{-1} = a^{-1/2} b^{-1/2} = \frac{1}{a^{1/2}} \frac{1}{b^{1/2}} = \frac{1}{a^{1/2} b^{1/2}}$$

$$\textcircled{17} \quad \frac{a^2 b^0}{3^1 a^4} = \frac{a^2 \cdot 1}{a^4 \cdot 3^1} = \frac{1}{3} \frac{1}{a^{4-2}} = \frac{1}{3a^2}$$

$$\text{or} \quad \frac{a^2 \cdot 1}{3^1 a^4} = \frac{1 a^2}{3 a^4} = \frac{1}{3} a^{-2} = \frac{1}{3a^2}$$

$$(19) u'v' \cdot u' \cdot \left(\sqrt{\frac{3}{2}}\right)^3$$

$$u'u'v' \sqrt{\frac{9}{2}} = u'^{1+1} v'^{1+\frac{9}{2}} = u'^2 v'^{\frac{11}{2}}$$

$$= \boxed{u'^2 v'^{\frac{11}{2}}}$$

$$(21) (x^0 y^{\frac{1}{3}})^{NN} x^0 = (1 \cdot y^{\frac{1}{3}})^{NN} \cdot 1 = (y^{\frac{1}{3}})^{NN}$$

$$= y^{\frac{2}{3}} = \boxed{y^{\frac{2}{3}}}$$

$$(23) \frac{a^{\frac{3}{4}} b^{-1} b^{\frac{7}{4}}}{3b^{-1}} = \frac{a^{\frac{3}{4}}}{3} \cdot \frac{b^{-1} b^{\frac{7}{4}}}{b^{-1}}$$

$$= \frac{a^{\frac{3}{4}} b^{\frac{3}{4}}}{3}$$

OR

$$\frac{1}{3} \cdot \frac{a^{\frac{3}{4}}}{1} \cdot \frac{b^{-1} b^{\frac{7}{4}}}{b^{-1}} = \frac{1}{3} \cdot \frac{a^{\frac{3}{4}}}{1} \cdot \frac{b^{\frac{3}{4}} b^{\frac{7}{4}}}{b^{\frac{1}{4}} b^{\frac{1}{4}}} = \frac{1}{3} \cdot \frac{a^{\frac{3}{4}}}{1} \cdot \frac{b^{\frac{11}{4}}}{b^{\frac{2}{4}}}$$

$$= \frac{1}{3} \cdot \frac{a^{\frac{3}{4}}}{1} \cdot \frac{b^{\frac{9}{4}}}{1} = \frac{a^{\frac{3}{4}} b^{\frac{9}{4}}}{3}$$

$$(25) \frac{3' y^{-\frac{5}{4}}}{y^{-1} \cdot 2' y^{-\frac{1}{3}}}$$

$$= \frac{3}{2} \cdot \frac{y^{-1} y^{-\frac{5}{4}}}{y^{-\frac{1}{4}} y^{-\frac{1}{3}}} = \frac{3}{2} \cdot \frac{y^{-1-\frac{5}{4}}}{y^{-\frac{1}{4}-\frac{1}{3}}}$$

$$= \frac{3}{2} \cdot \frac{y^{-\frac{9}{4}-\frac{1}{4}}}{y^{-\frac{3}{12}-\frac{4}{12}}} = \frac{3}{2} \cdot \frac{y^{-\frac{10}{4}}}{y^{-\frac{7}{12}}}$$

$$= \frac{3}{2} \cdot \frac{y^{-\frac{15}{12}}}{y^{-\frac{7}{12}}} = \frac{3}{2} y^{-\frac{15}{12} + \frac{7}{12}} = \boxed{\frac{3}{2} y^{-\frac{4}{12}}}$$

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$$\left[\frac{m^{\frac{3}{2}} h^{-2}}{(m^1 h^{\frac{4}{3}})^{-1}} \right]^{\frac{7}{4}}$$

Deal w/ inside()

1st !

$$\left[\frac{m^{\frac{3}{2}} h^{-2}}{m^{-1} h^{-\frac{4}{3}}} \right]^{\frac{7}{4}}$$

outside/h

$$\frac{m^{\frac{21}{8}} h^{-\frac{14}{4}}}{m^{-\frac{7}{4}} h^{-\frac{28}{12}}}$$

$$\frac{m^{\frac{21}{8}} m^{\frac{7}{4}}}{1} \cdot \frac{h^{\frac{28}{12}}}{h^{\frac{14}{4}}}$$

$$m^{\frac{21}{8}} m^{\frac{14}{8}} \cdot \frac{h^{\frac{28}{12}}}{h^{\frac{42}{12}}}$$

$$\frac{m^{\frac{35}{8}}}{1} \cdot \frac{1}{h^{\frac{42}{12} - \frac{28}{12}}}$$

$$\frac{m^{\frac{35}{8}}}{1} \cdot \frac{1}{h^{\frac{14}{12}}} = \boxed{\frac{m^{\frac{35}{8}}}{h^{\frac{7}{6}}}}$$

(27)

$$\left[\frac{m^{\frac{3}{2}} n^{-2}}{(m^{-1} n^{\frac{4}{3}})^{-1}} \right]^{\frac{7}{4}}$$

Deal w/ () inside
1st !

$$\left[\frac{m^{\frac{3}{2}} n^{-2}}{m^{-1} n^{-\frac{4}{3}}} \right]^{\frac{7}{4}}$$

inside/out

$$\left[\frac{m^{\frac{3}{2}} m^1}{1} \frac{n^{\frac{15}{3}}}{n^2} \right]^{\frac{7}{4}}$$

$$\left[\frac{m^{\frac{3}{2}+1}}{1} \frac{1}{n^{2-\frac{4}{3}}} \right]^{\frac{7}{4}}$$

$$\left[\frac{m^{\frac{3}{2}+\frac{2}{2}}}{1} \frac{1}{n^{\frac{6}{3}-\frac{4}{3}}} \right]^{\frac{7}{4}}$$

$$\left[\frac{m^{\frac{5}{2}}}{1} \frac{1}{n^{\frac{2}{3}}} \right]^{\frac{7}{4}}$$

$$\frac{m^{\frac{35}{8}}}{n^{\frac{14}{12}}}$$

$$= \frac{m^{\frac{35}{8}}}{n^{\frac{7}{6}}}$$

(29)

$$\frac{(m^2 n^{\frac{1}{2}})^0}{n^{\frac{3}{4}}}$$

outside/in

$$\frac{m^0 n^0}{n^{\frac{3}{4}}} = \frac{1 \cdot 1}{n^{\frac{3}{4}}} = \boxed{\frac{1}{n^{\frac{3}{4}}}}$$

OR any num zero # to zero power = 1

$$\frac{(m^2 n^{\frac{1}{2}})^0}{n^{\frac{3}{4}}} = \boxed{\frac{1}{n^{\frac{3}{4}}}}$$

(30)

$$\frac{(x^{-\frac{4}{3}} y^{-\frac{1}{3}} y^1)^{-1}}{x^{\frac{1}{3}} y^{-2}}$$

outside/in

$$\frac{x^{\frac{4}{3}} y^{\frac{1}{3}} y^{-1}}{x^{\frac{1}{3}} y^{-2}} = \frac{x^{\frac{4}{3}} y^{\frac{1}{3}} y^2}{x^{\frac{1}{3}} y^1}$$

$$= x^{\frac{4}{3} - \frac{1}{3}} \frac{y^{\frac{1}{3} + 2}}{y^1}$$

$$= \frac{x^{\frac{3}{3}} y^{\frac{1}{3} + \frac{6}{3}}}{y^{\frac{3}{3}}} = x^1 \frac{y^{\frac{7}{3}}}{y^1} = \boxed{x^1 y^{\frac{4}{3}}}$$

$$\textcircled{31} \frac{(x^{-\frac{4}{3}} y^{-\frac{1}{3}} y^1)^{-1}}{x^{\frac{1}{3}} y^{-2}}$$

inside/out

$$\frac{(x^{-\frac{4}{3}} y^{-\frac{1}{3}} y^{\frac{3}{3}})^{-1}}{x^{\frac{1}{3}} y^{-2}} = \frac{(x^{-\frac{4}{3}} y^{-\frac{1}{3} + \frac{3}{3}})^{-1}}{x^{\frac{1}{3}} y^{-2}}$$

$$= \frac{[x^{-\frac{4}{3}} y^{\frac{2}{3}}]^{-1}}{x^{\frac{1}{3}} y^{-2}} = \frac{x^{\frac{4}{3}} y^{-\frac{2}{3}}}{x^{\frac{1}{3}} y^{-2}} = \frac{x^{\frac{4}{3} - \frac{1}{3}}}{1} \quad \frac{y^{-\frac{2}{3} - (-2)}}{1}$$

$$= \frac{x^{\frac{3}{3}} y^{\frac{6}{3}}}{y^{\frac{2}{3}}} = x^1 y^{\frac{6}{3} - \frac{2}{3}} = \boxed{x^1 y^{\frac{4}{3}}}$$

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$$\frac{(u^1 v^2)^{1/2}}{v^{-1/4} v^2} = \frac{u^{1/2} v^{2 \cdot \frac{1}{2}}}{v^{-1/4} v^2} = \frac{u^{1/2}}{1} \cdot \frac{v^1 v^{-1/4}}{v^2}$$

$$= \frac{u^{1/2}}{1} \frac{v^{1 + \frac{1}{4}}}{v^2} = \frac{u^{1/2}}{1} \frac{v^{\frac{5}{4} + \frac{1}{4}}}{v^2}$$

$$= \frac{u^{1/2}}{1} \frac{v^{\frac{6}{4}}}{v^2} = \frac{u^{1/2}}{1} \frac{v^{\frac{3}{2}}}{v^{\frac{8}{4}}} = \frac{u^{1/2}}{v^{\frac{8}{4} - \frac{3}{2}}}$$

$$= \boxed{\frac{u^{1/2}}{v^{3/4}}}$$

$$\frac{(u^1 v^2)^{1/2}}{v^{-1/4} v^2} = \frac{u^{1/2} v^{2 \cdot \frac{1}{2}}}{v^{-1/4} v^2} = \frac{u^{1/2} v^1}{v^{-1/4} v^2} = \frac{u^{1/2} v^1}{v^{\frac{7}{4}}}$$

$$= \frac{u^{1/2}}{1} \cdot \frac{1}{v^{\frac{7}{4} - 1}} = \frac{u^{1/2}}{1} \cdot \frac{1}{v^{\frac{7}{4} - \frac{4}{4}}}$$

$$= \boxed{\frac{u^{1/2}}{v^{3/4}}}$$