Worksheet 7.1 Radicals & Rational Exponents

(1a) $\sqrt{10^1} = 10^{1/2}$

(1b) $\sqrt[3]{x^1} = x^{1/3}$

(1c) $\sqrt{x^3} = x^{\frac{3}{2}}$

(2a) $5^{\frac{1}{2}} = \sqrt{5^1}$

(2b) $100^{\frac{1}{2}} = \sqrt{100} = 10$

$= \sqrt{10^2} = 10^1$

(2c) $y^{\frac{1}{5}} = \sqrt[5]{y^1}$

(3a) $\sqrt{64} = \sqrt{2^6} = 2^{6/2} = 2^3 = 8$

$= \sqrt{8^2} = 8^{2/2} = 8^1 = 8$

$= \sqrt{4^3} = 4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^3 = 8$

(3b) $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$

$= \sqrt[3]{2^6} = 2^{6/3} = 2^2 = 4$

(3c) $\sqrt{-64}$ not real

$\sqrt{-1}\sqrt{64} = 8i$ ← imaginary

(3g) $\sqrt[4]{-1} \rightarrow$ not real

$(-1)^{\frac{1}{4}} = \left[(-1)^{\left(\frac{1}{2}\right)}\right]^{\frac{1}{2}}$

$= i^{\frac{1}{2}}$

$= \boxed{\sqrt{i}}$ imaginary

(3h) $16^{\frac{3}{4}} = \sqrt[4]{16^3}$ or $\left(2^4\right)^{\frac{3}{4}} = 2^{\frac{12}{4}} = 2^3$

$= \left(\sqrt[4]{16}\right)^3$ $= (8)$

$= \left(\sqrt[4]{2^4}\right)^3$

$= \left(2^{4/4}\right)^3$

$= \left(2^1\right)^3$

$= 2^3$

$= (8)$

cops skipped (3i)

(3j) $-16^{\frac{3}{4}} = -1 \cdot 16^{\frac{3}{4}}$ or $-1 \cdot \left(\sqrt[4]{16}\right)^3$

$= -1\left(16^{\frac{3}{4}}\right)$ $-1\left(\sqrt[4]{2^4}\right)^3$

$= -1\left(2^4\right)^{\frac{3}{4}}$ $-1\left(2^{4/4}\right)^3$

$= -1\,2^3$ $-1\left(2^1\right)^3$

$= \boxed{-8}$ $-1\,2^3$

$-1 \cdot 8$

(-8)

(3i) $(16)^{-\frac{3}{4}} = \frac{1}{(16^{\frac{3}{4}})} = \frac{1}{(2^4)^{\frac{3}{4}}} = \frac{1}{2^{\frac{12}{4}}} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$

(3k) $(-16)^{3/4} = (-1 \cdot 2^4)^{3/4} = (-1)^{\frac{3}{4}}(2^4)^{3/4}$

$= (-1)^{3/4} \cdot 2^{\frac{12}{4}} = (-1)^{\frac{3}{4}}(2^3)$

↑ not real ↑ still not real

$= \left((-1)^{\frac{1}{2}}\right)^{\frac{3}{2}} \cdot 2^3$

$= (i)^{\frac{3}{2}} \cdot 8$

$= i^{1\frac{1}{2}} \cdot 8$

$= i^1 \cdot i^{\frac{1}{2}} \cdot 8$

$= i\sqrt{i}\,8$

$= \boxed{8i\sqrt{i}}$ ← complex not real answer

(4a) $x^{\frac{1}{4}} x^{-\frac{5}{4}} = x^{\frac{1}{4} - \frac{5}{4}} = x^{-\frac{4}{4}} = x^{-1} = \boxed{\frac{1}{x^1}}$

(4b) $2^{\frac{2}{3}} 2^{-\frac{5}{3}} = 2^{\frac{2}{3} - \frac{5}{3}} = 2^{-\frac{3}{3}} = 2^{-1} = \boxed{\frac{1}{2^1}}$

(4c) $(x^{\frac{1}{2}})^8 = x^{\frac{8}{2}} = \boxed{x^4}$