

Integrated III Even Solutions

② $\sqrt[3]{p^2} = (p^2)^{1/3} = \boxed{p^{2/3}}$ ④ $(\sqrt[3]{x^1})^2 = (x^{1/3})^2 = \boxed{x^{2/3}}$

⑥ $\frac{1}{\sqrt[3]{7}} = \frac{1}{7^{1/3}} = \boxed{7^{-1/3}}$ ⑧ $(\sqrt[5]{21})^7 = (21^{1/5})^7 = \boxed{21^{7/5}}$

⑩ $6^{1/3} = \sqrt[3]{6^1}$ ⑫ $10^{5/3} = \sqrt[3]{10^5}$ not simplest form

⑫ method ①

$$\begin{aligned} 10^{5/3} &= 10^{1\frac{2}{3}} \\ &= 10^{1 + \frac{2}{3}} \\ &= 10^1 \cdot 10^{2/3} \\ &= 10 \sqrt[3]{10^2} \\ &= \boxed{10 \sqrt[3]{100}} \end{aligned}$$

simplest form

method ①

$$\begin{aligned} \sqrt[3]{10^5} &= \sqrt[3]{10^{3+2}} = \sqrt[3]{10^3} \sqrt[3]{10^2} \\ &= 10^{3/3} \sqrt[3]{10^2} = \boxed{10 \sqrt[3]{10^2}} \\ &= \boxed{10 \sqrt[3]{100}} \end{aligned}$$

simplest form

method ②

$$\begin{aligned} \sqrt[3]{10^5} &= \sqrt[3]{100000} = \sqrt[3]{1000} \sqrt[3]{100} \\ &= \sqrt[3]{10^3} \sqrt[3]{100} \\ &= 10^{3/3} \sqrt[3]{100} \\ &= \boxed{10 \sqrt[3]{100}} \end{aligned}$$

simplest form

⑭ $(2^1 p^1)^{2/3} = 2^{2/3} p^{2/3} = (2^2 p^2)^{1/3}$

$$\begin{aligned} \boxed{(2^1 p^1)^2}^{1/3} &= \boxed{\sqrt[3]{4 p^2}} \\ \rightarrow (2^2 p^2)^{1/3} &= \boxed{\sqrt[3]{4 p^2}} \end{aligned}$$

$$\textcircled{16} (S'X')^{\frac{S}{4}} = S^{\frac{S}{4}} X^{\frac{S}{4}} = (S^S X^S)^{\frac{1}{4}} = \sqrt[4]{S^S X^S}$$

$$= \sqrt[4]{3125 X^S}$$

not simplest form

method ①

$$(S'X')^{\frac{S}{4}} = (S'X')^{1\frac{1}{4}} = (S'X')^{1+\frac{1}{4}}$$

$$= (S'X')^1 \cdot (S'X')^{\frac{1}{4}}$$

$$= \boxed{S'X' \sqrt[4]{S'X'}} \text{ simplest form}$$

method ②

$$(S'X')^{\frac{S}{4}} = S^{\frac{S}{4}} X^{\frac{S}{4}} = S^{\frac{4}{4}+\frac{1}{4}} X^{\frac{4}{4}+\frac{1}{4}}$$

$$= S^{1+\frac{1}{4}} X^{1+\frac{1}{4}}$$

$$= S^1 \cdot S^{\frac{1}{4}} X^1 \cdot X^{\frac{1}{4}}$$

$$= S'X' S^{\frac{1}{4}} X^{\frac{1}{4}}$$

$$= S'X' (SX)^{\frac{1}{4}}$$

$$= \boxed{S'X' \sqrt[4]{SX}} \text{ simplest form}$$

method ③

$$(S'X')^{\frac{S}{4}} = [(S'X')^S]^{\frac{1}{4}} = (S^S X^S)^{\frac{1}{4}}$$

$$= \sqrt[4]{3125 X^S} = \sqrt[4]{625} \sqrt[4]{X^4} \sqrt[4]{S'X'}$$

$$= \sqrt[4]{5^4} \sqrt[4]{X^4} \sqrt[4]{S'X'}$$

$$= S^{\frac{4}{4}} X^{\frac{4}{4}} \sqrt[4]{S'X'}$$

$$= \boxed{S'X' \sqrt[4]{SX}} \text{ simplest form}$$

$$625 \cdot 5 = 3125$$

$$X^S = X^4 X^1$$

$$(18) (P^9)^{1/3} = P^{9/3} = (P^3)$$

$$(20) (32x^{10})^{2/5} = (2^5 x^{10})^{2/5} = 2^{10/5} x^{20/5}$$

$$= (2^2 x^4)$$

Method (2)

$$(32x^{10})^{2/5} = \left[(32x^{10})^2 \right]^{1/5}$$

$$= \left[(2^5 x^{10})^2 \right]^{1/5}$$

$$= \left[2^{10} x^{20} \right]^{1/5}$$

$$= 2^{10/5} x^{20/5} = \boxed{2^2 x^4} = \boxed{4x^4}$$



Method (3)

$$(32x^{10})^{2/5} = \left[(32^1 x^{10})^2 \right]^{1/5} = (32^2 x^{20})^{1/5}$$

$$= [1024 x^{20}]^{1/5} = [1024^{1/5} x^{20/5}]$$

$$= \sqrt[5]{1024} x^4 = \sqrt[5]{32} \sqrt[5]{32} x^4$$

$$= \sqrt[5]{2^5} \sqrt[5]{2^5} x^4$$

$$= 2^1 2^1 x^4 = \boxed{2^2 x^4 = 4x^4}$$

Note

$$4^5 = 1024$$

$$\textcircled{22} (81a^4)^{\frac{3}{2}} = (81^1 a^4)^{\frac{3}{2}} = (3^4 a^4)^{\frac{3}{2}} = 3^{\frac{12}{2}} a^{\frac{12}{2}}$$

$$= \boxed{3^6 a^6} = \boxed{729 a^6}$$

$$\begin{array}{c} 81 \\ \wedge \\ 9 \quad 9 \\ \wedge \quad \wedge \\ 3 \quad 3 \quad 3 \quad 3 \end{array}$$

Method ② $(81a^4)^{\frac{3}{2}} = (3^4 a^4)^{1\frac{1}{2}}$

$$= (3^4 a^4)^{1+\frac{1}{2}}$$

$$= (3^4 a^4)^1 (3^4 a^4)^{\frac{1}{2}}$$

$$= 3^4 a^4 \cdot 3^{\frac{4}{2}} a^{\frac{4}{2}}$$

$$= 3^4 a^4 \cdot 3^2 a^2$$

$$= 3^4 \cdot 3^2 a^4 a^2$$

$$= \boxed{3^6 a^6} = \boxed{729 a^6}$$

Method ③ $(81^1 a^4)^{\frac{3}{2}} = \left[(81^3 a^{12})^{\frac{1}{2}} \right]^{\frac{1}{2}}$

$$= (81^3 a^{12})^{\frac{1}{2}} = \sqrt{81^3 a^{12}}$$

$$= \sqrt{81^3} \sqrt{a^{12}}$$

$$= \sqrt{81^{2+1}} a^{\frac{12}{2}}$$

$$= \sqrt{81^2} \sqrt{81} a^6$$

$$= 81 \cdot 9 \cdot 9 a^6$$

$$= \boxed{729 a^6}$$

$$\textcircled{24} \quad 49^{\frac{3}{2}} = (7^2)^{\frac{3}{2}} = 7^{\frac{6}{2}} = \boxed{7^3 = 343}$$

Method (2)

$$\begin{aligned} 49^{\frac{3}{2}} &= (7^2)^{1\frac{1}{2}} = [7^2]^{1+\frac{1}{2}} \\ &= [7^2]^1 \cdot [7^2]^{\frac{1}{2}} \\ &= 7^2 \cdot 7^{\frac{2}{2}} \\ &= 7^2 \cdot 7^1 \\ &= \boxed{7^3 = 343} \end{aligned}$$

$$\textcircled{26} \quad 4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{(2^2)^{\frac{3}{2}}} = \frac{1}{2^{\frac{6}{2}}} = \boxed{\frac{1}{2^3}} = \boxed{\frac{1}{8}}$$
$$= \boxed{2^{-3}}$$

Method (2)

$$4^{-\frac{3}{2}} = \frac{1}{4^{\frac{3}{2}}} = \frac{1}{\sqrt{4^3}} = \frac{1}{\sqrt{64}} = \textcircled{\frac{1}{8}}$$

Method (3)

$$\begin{aligned} 4^{-\frac{3}{2}} &= \frac{1}{4^{\frac{3}{2}}} = \frac{1}{4^{1+\frac{1}{2}}} = \frac{1}{4^1} \cdot \frac{1}{4^{\frac{1}{2}}} = \frac{1}{4} \cdot \frac{1}{\sqrt{4}} \\ &= \frac{1}{4} \cdot \frac{1}{2} = \textcircled{\frac{1}{8}} \end{aligned}$$

$$(28) (343)^{\frac{5}{3}} = (7^3)^{\frac{5}{3}} = 7^{\frac{15}{3}} = 7^5 = \boxed{16807}$$

Note

$$343 = 7^3$$

$$\begin{array}{c} 343 \\ \swarrow \quad \searrow \\ 49 \quad 7 \\ \swarrow \quad \searrow \\ 7 \quad 7 \quad 7 \end{array}$$

Method (2)

$$(343)^{\frac{5}{3}} = (7^3)^{\frac{5}{3}}$$

$$= (7^3)^{1\frac{2}{3}} = (7^3)^{1+\frac{2}{3}}$$

$$= (7^3)^1 \cdot (7^3)^{\frac{2}{3}}$$

$$= 7^3 \cdot 7^{\frac{6}{3}} = 7^3 \cdot 7^2$$

$$= \boxed{7^5 = 16807}$$

Method (3)

$$(343)^{\frac{5}{3}} = \left((343)^{\frac{1}{3}} \right)^5 = \left(7^3 \right)^{\frac{1}{3}}^5$$

$$= \left[7^{\frac{3}{3}} \right]^5 = (7^1)^5 = \boxed{7^5 = 16807}$$

$$(30) 2^1 x^2 y^2 \cdot 2^1 x^1 y^1 = 2^1 \cdot 2^1 \cdot x^2 \cdot x^1 \cdot y^2 \cdot y^1 = \boxed{2^2 x^3 y^3}$$

$$= \boxed{4x^3y^3}$$

$$(32) x^{\frac{3}{2}} y^{\frac{1}{2}} \cdot 2^1 \cdot x^{\frac{5}{4}} = 2^1 x^{\frac{3}{2}} x^{\frac{5}{4}} y^{\frac{1}{2}} = 2^1 \cdot x^{\frac{3}{2} + \frac{5}{4}} y^{\frac{1}{2}}$$

$$= 2^1 x^{\frac{6}{4} + \frac{5}{4}} y^{\frac{1}{2}} = \boxed{2 x^{\frac{11}{4}} y^{\frac{1}{2}}}$$

$$= \boxed{2^1 x^{\frac{11}{4}} y^{\frac{2}{4}}} = \boxed{2 \sqrt[4]{x^{11} y^2}}$$

not simplest form

simplest form \downarrow

$$= 2 \sqrt[4]{x^{11}} \sqrt[4]{y^2} = 2 \sqrt[4]{y^2} \sqrt[4]{x^{11} x^3} = 2 \sqrt[4]{y^2} \cdot \sqrt[4]{x^{14}} = \boxed{2y^{\frac{1}{2}} \sqrt[4]{x^{14}}}$$

$$(34) \left(a^{\frac{3}{2}}\right)^{\frac{1}{2}} = \boxed{a^{\frac{3}{4}} = \sqrt[4]{a^3}}$$

$$(36) \left(y^{\frac{5}{3}}\right)^{\frac{1}{4}} = \boxed{y^{\frac{5}{12}} = \sqrt[12]{y^5}}$$

$$(38) \left(\frac{a^2 a^1 b^{\frac{1}{4}}}{a^{\frac{7}{4}}}\right)^{\frac{1}{3}} = \frac{a^{\frac{2}{3}} a^{\frac{1}{3}} b^{\frac{1}{12}}}{a^{\frac{7}{12}}} = \frac{a^{\frac{2}{3} + \frac{1}{3}} b^{\frac{1}{12}}}{a^{\frac{7}{12}}}$$

$$= \frac{a^1 b^{\frac{1}{12}}}{a^{\frac{7}{12}}} = a^{1 - \frac{7}{12}} b^{\frac{1}{12}} = a^{\frac{12}{12} - \frac{7}{12}} b^{\frac{1}{12}}$$

$$= \boxed{a^{\frac{5}{12}} b^{\frac{1}{12}} = \sqrt[12]{a^5 b}}$$

$$(40) \frac{m^2 n^{\frac{3}{2}}}{\left(m^{\frac{3}{4}} n^{\frac{1}{2}}\right)^{\frac{1}{2}} m^2 n^{\frac{1}{4}} n^{\frac{1}{4}}} = \frac{m^2 n^{\frac{3}{2}}}{m^{\frac{3}{8}} n^{\frac{1}{4}} m^2 n^{\frac{1}{4}} n^{\frac{1}{4}}} = \frac{m^2}{m^2 m^{\frac{3}{8}} m^{\frac{2}{8}}} \cdot \frac{n^{\frac{18}{12}}}{n^{\frac{3}{12}} n^{\frac{16}{12}} n^{\frac{2}{12}}}$$

$$\text{LCM}(4, 8) = 8$$

$$= \frac{m^2}{m^2} \cdot \frac{1}{m^{\frac{5}{8}}} \cdot \frac{n^{\frac{18}{12}}}{n^{\frac{19}{12}}} \cdot \frac{1}{n^{\frac{1}{2}}}$$

$$\text{LCM}(3, 4) = 12$$

$$= 1 \cdot \frac{1}{m^{\frac{5}{8}}} \cdot \frac{1}{n^{\frac{1}{2}}} \cdot \frac{1}{n^{\frac{1}{2}}} = \frac{1}{m^{\frac{5}{8}} n^2} = \frac{1}{m^{\frac{5}{8}} n^{\frac{25}{12}}}$$

$$= \frac{1}{m^{\frac{5}{8}}} \cdot \frac{m^{\frac{3}{8}}}{m^{\frac{3}{8}}} \cdot \frac{1}{n^{\frac{25}{12}}} \cdot \frac{n^{\frac{23}{12}}}{n^{\frac{23}{12}}} = \frac{m^{\frac{3}{8}}}{m^1} \cdot \frac{n^{\frac{23}{12}}}{n^4} = \boxed{\frac{m^{\frac{3}{8}} n^{\frac{23}{12}}}{m n^4}}$$

40)
$$\frac{m^{\frac{3}{8}} n^{\frac{23}{12}}}{m^1 n^4} = \frac{m^{\frac{3}{8}} n^{\frac{12}{12} + \frac{11}{12}}}{m^1 n^4} = \frac{m^{\frac{3}{8}} n^1 n^{\frac{11}{12}}}{m^1 n^4}$$

$$= \frac{m^{\frac{3}{8}} n^{\frac{11}{12}}}{m^1 n^3} = \frac{m^{\frac{9}{24}} n^{\frac{22}{24}}}{m^1 n^3} = \frac{\sqrt[24]{m^9 n^{22}}}{m^1 n^3}$$

LCM (8, 12) = 24

Simplest radical form.

Note
$$\frac{m^{\frac{3}{8}} n^{\frac{23}{12}}}{m^1 n^4} = \frac{m^{\frac{3}{8}} n^{\frac{12}{12}} n^{\frac{11}{12}}}{m^1 n^4} = \frac{m^{\frac{3}{8}} n^1 n^{\frac{11}{12}}}{m^1 n^4}$$

$$= \frac{m^{\frac{3}{8}} n^{\frac{11}{12}}}{m^1 n^3} \leftarrow \text{given answer}$$

Acceptable Answer via direction

$$\frac{m^{\frac{3}{8}} n^{\frac{23}{12}}}{m^1 n^4} = \frac{m^{\frac{3}{8}} n^{\frac{11}{12}}}{m^1 n^4} \quad \text{or} \quad \frac{\sqrt[24]{m^9 n^{22}}}{m^1 n^3}$$

mathematically speaking

most helpful form

$$\boxed{m^{-\frac{5}{8}} n^{-\frac{1}{12}}}$$

or
$$\frac{1}{m^{\frac{5}{8}} n^{\frac{1}{12}}}$$

