

Integrated III

Exponential Form = no radical

$$\textcircled{1} (\sqrt{\sqrt{r}})^3 = (r^{1/2})^3 = \boxed{r^{3/2}}$$

$$\textcircled{3} \sqrt{6r} = \sqrt{6} \sqrt{r} = \boxed{6^{1/2} r^{1/2}}$$

$$\textcircled{5} \sqrt[3]{5r} = \boxed{(5r)^{1/3}}$$

$$\textcircled{7} (\sqrt{3r})^5 = (3^{1/2} r^{1/2})^5 = \boxed{3^{5/2} r^{5/2}}$$

Radical Form = no exponent bigger than index under radical
= no radical in denominator

$$\textcircled{9} 5^{-1/2} = \frac{1}{5^{1/2}} = \boxed{\frac{1}{\sqrt{5}}} \text{ not simplified}$$

Rationalize Denominator

$$\frac{1}{\sqrt{5}} \cdot 1 = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{1\sqrt{5}}{\sqrt{5}\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$$

completely simplified ↗

(11) $10^{-7/5} = \frac{1}{10^{7/5}} = \frac{1}{\sqrt[5]{10^7}}$

Annotations: "index" points to the 5 in the root; "exponent" points to the 7 in the power; "not simplified" points to the entire radical expression.

$$\frac{1}{\sqrt[5]{10^7}} \cdot 1 = \frac{1}{\sqrt[5]{10^7}} \frac{\sqrt[5]{10^3}}{\sqrt[5]{10^3}} = \frac{\sqrt[5]{10^3}}{\sqrt[5]{10^{10}}}$$

Note: $\sqrt[5]{10^{10}} = 10^{10/5} = 10^2$

$$\frac{\sqrt[5]{10^3}}{10^2} = \frac{\sqrt[5]{1000}}{100}$$

Annotation: "completely simplified" points to the boxed fraction.

Alternate method $\frac{1}{\sqrt[5]{10^7}} = \frac{1}{\sqrt[5]{10^5}} \cdot \frac{1}{\sqrt[5]{10^2}} = \frac{1}{10 \sqrt[5]{10^2}}$

Note: "missing" 10^3 "under radical"

$$\begin{aligned} \frac{1}{10} \frac{1}{\sqrt[5]{10^2}} \cdot 1 &= \frac{1}{10} \cdot \frac{1}{\sqrt[5]{10^2}} \frac{\sqrt[5]{10^3}}{\sqrt[5]{10^3}} = \frac{1}{10} \frac{\sqrt[5]{10^3}}{\sqrt[5]{10^5}} \\ &= \frac{1}{10} \cdot \frac{\sqrt[5]{10^3}}{10} = \frac{\sqrt[5]{10^3}}{10} \end{aligned}$$

$$\textcircled{13} (3^1 p^1)^{1/2} = 3^{1/2} p^{1/2} = \sqrt{3} \sqrt{p} = \boxed{\sqrt{3p}}$$

$$\begin{aligned} \textcircled{15} (7^1 n^1)^{5/2} &= 7^{5/2} n^{5/2} = \sqrt[2]{7^5} \sqrt[2]{n^5} \\ &= \sqrt[2]{7^4} \sqrt[2]{7^1} \sqrt[2]{n^4} \sqrt[2]{n^1} \\ &= 7^{4/2} \sqrt{7^1} n^{4/2} \sqrt{n^1} \\ &= 7^2 \sqrt{7} n^2 \sqrt{n} \\ &= 7^2 n^2 \sqrt{7} \sqrt{n} \\ &= \boxed{49n^2 \sqrt{7n}} = \boxed{7^2 n^2 \sqrt{7n}} \end{aligned}$$

Simplified

$$\downarrow$$

$$(7^5 n^5)^{1/2}$$

$$\boxed{\sqrt{7^5 n^5}}$$

$$\downarrow$$

not simplified

$$\textcircled{17} (243 \times 15)^{7/5} = [3^5 \times 15]^{7/5}$$

Note

$$243 = 3^5$$

$$\begin{array}{r} / \quad \backslash \\ 3 \quad 81 \\ / \quad \backslash \\ 3 \quad 9 \quad 9 \\ / \quad \backslash \quad / \quad \backslash \\ 3 \quad 3 \quad 3 \quad 3 \end{array}$$

$$\text{or } \begin{array}{r} 3 \overline{) 243} \\ \underline{3} \\ 3 \\ \underline{3} \\ 3 \\ \underline{3} \\ 0 \end{array}$$

$$3^5 = 243$$

$$= (3^5)^{7/5} (15)^{7/5}$$

$$= 3^{35/5} \times 15^{7/5}$$

$$= 3^7 \times 21$$

$$= \boxed{2187 \times 21 = 3^7 \times 21}$$

(17) Alternate method

$$(243 \times^{15})^{7/5} = [243 \times^{15}]^{1 \frac{2}{5}}$$

$$= [243 \times^{15}]^1 \cdot [243 \times^{15}]^{2/5}$$

$$= 243 \times^{15} [243^{2/5} \times^{30/5}]$$

$$= 243 \times^{15} [(3^5)^{2/5} \times^6]$$

$$= 243 \times^{15} 3^{10/5} \times^6$$

$$= 243 \times^{15} 3^2 \times^6$$

$$= 243 \cdot 9 \times^{15} \times^6$$

$$= 2187 \times^{15+6}$$

$$= \boxed{2187 \times^{21}} = \boxed{3^7 \times^{21}}$$

$$\begin{array}{r} 243 \\ / \quad \backslash \\ 3 \quad 81 \end{array}$$

$$\begin{array}{r} / \quad / \quad \backslash \\ 3 \quad 9 \quad 9 \\ / \quad / \quad / \quad \backslash \\ 3 \quad 3 \quad 3 \quad 3 \end{array}$$

$$3^5$$

$$\begin{array}{r} 3 \overline{) 243} \\ \underline{81} \\ 3 \overline{) 27} \\ \underline{9} \\ 3 \overline{) 9} \\ \underline{3} \end{array}$$

$$3^5$$

$$2187$$

$$\begin{array}{r} / \quad \backslash \\ 3 \quad 729 \end{array}$$

$$\begin{array}{r} / \quad / \quad \backslash \\ 3 \quad 9 \quad 9 \quad 9 \\ / \quad / \quad / \quad \backslash \\ 3 \quad 3 \quad 3 \quad 3 \end{array}$$

$$3^7$$

$$\begin{array}{r} 3 \overline{) 2187} \\ \underline{729} \\ 3 \overline{) 243} \\ \underline{81} \\ 3 \overline{) 27} \\ \underline{9} \\ 3 \overline{) 9} \\ \underline{3} \end{array}$$

$$3^7$$

(15) Alternate method

$$(7'n')^{5/2} = (7'n')^{2 \frac{1}{2}} = (7'n')^{2 + \frac{1}{2}} = (7'n')^2 (7'n')^{1/2}$$

$$= 7^2 n'^2 \cdot 7^{1/2} n'^{1/2}$$

$$= \boxed{49 n'^2 \sqrt{7'n'}}$$

$$= \boxed{7^2 n'^2 \sqrt{7'n'}}$$

$$\begin{aligned}
 (21) \quad (81b^4)^{5/4} &= (81^1 b^4)^{5/4} \\
 &= 81^{5/4} b^{20/4} \\
 &= 81^{5/4} b^5 \\
 &= (3^4)^{5/4} b^5 \\
 &= 3^{20/4} b^5 \\
 &= \boxed{3^5 b^5} = \boxed{243 b^5}
 \end{aligned}$$

$$\begin{array}{r}
 81 \\
 \sqrt{\quad} \\
 9 \quad 9 \\
 \sqrt{\quad} \quad \sqrt{\quad} \\
 3 \quad 3 \quad 3 \quad 3 \\
 3^4
 \end{array}
 \qquad
 \begin{array}{r}
 3 \overline{) 81} \\
 \underline{27} \\
 9 \\
 \underline{3} \\
 3^4
 \end{array}$$

(21) Alternate Method $(81b^4)^{5/4} = (81b^4)^{1\frac{1}{4}}$

$$\begin{aligned}
 &= [81b^4]^{1+\frac{1}{4}} \\
 &= [81b^4]^1 [81b^4]^{1/4} \\
 &= 81^1 b^4 \cdot 81^{1/4} b^{4/4} \\
 &= 81^1 b^4 [3^4]^{1/4} b^1 \\
 &= 81 b^4 \cdot 3^{4/4} b^1 \\
 &= 81 b^4 \cdot 3^1 b^1 \\
 &= 81 \cdot 3 b^4 b^1 \\
 &= \boxed{243 b^5} = \boxed{3^5 b^5}
 \end{aligned}$$

$$(23) \quad 64^{7/6} = (2^6)^{7/6} = 2^{42/6} = 2^7 = \boxed{128}$$

64	64
/ /	32
8 8	16
/ / / /	8
222 222	4
26	2
	26

(23) Alternate Method

$$\begin{aligned}
 (64)^{7/6} &= (64)^{1\frac{1}{6}} = 64^{1+\frac{1}{6}} \\
 &= 64^1 \cdot 64^{1/6} \\
 &= 64 \cdot (2^6)^{1/6} \\
 &= 64 \cdot 2^{6/6} \\
 &= 64 \cdot 2^1 \\
 &= \boxed{128}
 \end{aligned}$$

$$(25) \quad 64^{-1/2} = \frac{1}{64^{1/2}} = \frac{1}{\sqrt{64}} = \boxed{\frac{1}{8}}$$

2 64	64
2 32	/ /
2 16	8 8
2 8	/ / / /
2 4	222 222
2 2	26
26	26

$$\begin{aligned}
 64^{-1/2} &= (2^6)^{-1/2} = 2^{-6/2} = 2^{-3} \\
 &= \boxed{\frac{1}{2^3}} = \boxed{\frac{1}{8}}
 \end{aligned}$$

(27) $100000^{-2/5} = \frac{1}{100000^{2/5}}$

$10 \overbrace{10000}^{100000}$
 $10 \overbrace{1000}^{100000}$
 $10 \overbrace{100}^{100000}$
 $10 \overbrace{10}^{100000}$

$10^5 = (2 \cdot 5)^5 = 2^5 5^5$

$= \frac{1}{(10^5)^{2/5}} = \frac{1}{10^{10/5}} = \frac{1}{10^2}$

$= \boxed{\frac{1}{100}}$

(27) $(100000)^{-2/5} = (10^5)^{-2/5} = 10^{-10/5} = 10^{-2}$

$= \boxed{10^{-2} = \frac{1}{10^2} = \frac{1}{100}}$

(29) $3^1 m^1 n^{1/2} 4^1 m^{4/3} n^{1/2}$

$3^1 4^1 m^1 m^{4/3} n^{1/2} n^{1/2}$

$12 m^{1+4/3} n^{1/2+1/2}$

$12 m^{7/3} n^{1/2+1/2}$

$\boxed{12 m^{7/3} n^1}$

$12 m^{2\frac{1}{3}} n^1$

$12 m^2 m^{1/3} n^1$

$\boxed{12 m^2 n^1 m^{1/3} = 12 m^2 n^1 \sqrt[3]{m}}$

$3^1 m^1 n^{1/2} 4^1 m^{4/3} n^{1/2}$

$3 \cdot 4 m^1 m^{4/3} n^{1/2} n^{1/2}$

$12 m^{1+1+1/3} n^{1/2+1/2}$

$\boxed{12 m^{2\frac{1}{3}} n^1}$

$12 m^2 m^{1/3} n^1$

$12 m^2 n^1 m^{1/3}$

$\boxed{12 m^2 n^1 \sqrt[3]{m}}$

$$\begin{aligned}
 \textcircled{31} \quad 2^1 m^{\frac{3}{2}} 2^1 n^{\frac{1}{2}} &= 2^1 2^1 m^{\frac{3}{2}} n^{\frac{1}{2}} \\
 &= 2^2 m^{\frac{3}{2}} n^{\frac{1}{2}} \\
 &= \boxed{4 m^{\frac{3}{2}} n^{\frac{1}{2}}} \\
 &= 4 m^{1\frac{1}{2}} n^{\frac{1}{2}} \\
 &= 4 m^1 m^{\frac{1}{2}} n^{\frac{1}{2}} \\
 &= \boxed{4 m \sqrt{mn}}
 \end{aligned}$$

$$4 m^{\frac{3}{2}} n^{\frac{1}{2}}$$

$$4 \sqrt{m^3} \sqrt{n^1}$$

$$4 \sqrt{m^2} \sqrt{m} \sqrt{n}$$

$$\boxed{4 m \sqrt{mn}}$$

$$\textcircled{33} \quad \left(y^{\frac{2}{3}}\right)^{\frac{3}{2}} = y^{\frac{2}{3} \cdot \frac{3}{2}} = \boxed{y^{\frac{2}{1}}} = \boxed{\sqrt{y^2}}$$

$$\textcircled{35} \quad (x^3 y^2)^{\frac{2}{3}} = x^{\frac{6}{3}} y^{\frac{4}{3}} = \boxed{x^2 y^{\frac{4}{3}}}$$

$$x^2 \sqrt[3]{y^4}$$

$$x^2 \sqrt[3]{y^3 y^1}$$

$$\boxed{x^2 y^1 \sqrt[3]{y^1}}$$

$$x^2 y^{\frac{1\frac{1}{3}}{1} + \frac{1}{3}}$$

$$x^2 y^{1 + \frac{1}{3}}$$

$$x^2 y^1 \sqrt[3]{y^1}$$

$$\boxed{x^2 y^1 \sqrt[3]{y}}$$

(27)

$$\begin{aligned} \frac{(x^{\frac{3}{2}} y^2)^{\frac{3}{2}}}{(x^{\frac{1}{2}} y^2)^{\frac{5}{3}} y^{\frac{1}{2}}} &= \frac{x^{\frac{9}{4}} y^{\frac{6}{2}}}{x^{\frac{5}{6}} y^{\frac{10}{3}} y^{\frac{1}{2}}} \\ &= \frac{x^{\frac{27}{12}} y^{\frac{18}{6}}}{x^{\frac{10}{12}} y^{\frac{20}{6}} y^{\frac{3}{6}}} \\ &= \frac{x^{\frac{27}{12}-\frac{10}{12}} y^{\frac{18}{6}}}{y^{\frac{23}{6}}} \\ &= \frac{x^{\frac{17}{12}}}{y^{\frac{23}{6}-\frac{18}{6}}} \\ &= \frac{x^{\frac{17}{12}}}{y^{\frac{5}{6}}} \end{aligned}$$

Now this is where some have issues

← $\frac{x^{\frac{17}{12}}}{y^{\frac{5}{6}}}$

$$\frac{x^{\frac{17}{12}}}{y^{\frac{5}{6}}} \cdot \frac{y^{\frac{1}{6}}}{y^{\frac{1}{6}}}$$

— why $\frac{5}{6} + \frac{1}{6} = 1$

$$\frac{x^{\frac{17}{12}} y^{\frac{1}{6}}}{y^{\frac{5}{6} + \frac{1}{6}}} = \frac{x^{\frac{17}{12}} y^{\frac{1}{6}}}{y^1}$$

$$= \frac{x^{\frac{17}{12}} y^{\frac{1}{6}}}{y^1}$$

(37) Alternate method

$$\frac{(x^{\frac{3}{2}} y^2)^{3/2}}{(x^{\frac{1}{2}} y^2)^{\frac{5}{3}} y^{\frac{1}{2}}} = \frac{x^{\frac{9}{4}} y^3}{x^{\frac{5}{6}} y^{\frac{10}{3}} y^{\frac{1}{2}}}$$

$$= \frac{x^{\frac{9}{4} - \frac{5}{6}} y^3}{y^{\frac{10}{3} + \frac{1}{2}}}$$

$$= \frac{x^{\frac{17}{12}} y^3}{y^{\frac{23}{6}}}$$

→ note all fractions with
done on calc.

$$= x^{\frac{17}{12}} y^{3 - \frac{23}{6}}$$

$$= x^{\frac{17}{12}} y^{-\frac{5}{6}}$$

$$= \frac{x^{\frac{17}{12}}}{y^{\frac{5}{6}}}$$

again this next step bothers
some

$$\frac{x^{\frac{17}{12}}}{y^{\frac{5}{6}}} \cdot \frac{y^{\frac{1}{6}}}{y^{\frac{1}{6}}} = \frac{x^{\frac{17}{12}} y^{\frac{1}{6}}}{y^{\frac{5}{6} + \frac{1}{6}}} = \frac{x^{\frac{17}{12}} y^{\frac{1}{6}}}{y^1}$$

$$= \boxed{\frac{x^{\frac{17}{12}} y^{\frac{1}{6}}}{y^1}}$$

(39)

$$\left[\frac{x^{\frac{1}{3}} y^{\frac{5}{3}}}{x^{\frac{3}{2}} y^{\frac{7}{4}} x^1 y^2} \right]^{\frac{9}{4}} = \frac{x^{\frac{5}{12}} y^{\frac{25}{12}}}{x^{\frac{15}{8}} y^{\frac{25}{16}} x^{\frac{9}{4}} y^{\frac{10}{4}}}$$

Note LCM(12, 8, 4) = 24

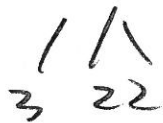
LCM(12, 16, 4) = 48

Why?



$$24 = 3^1 \cdot 2^3$$

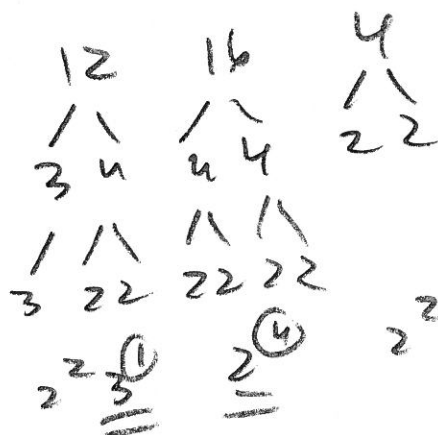
$$= 24$$



<u>2²</u>	<u>3^①</u>	<u>2^③</u>	2 ³
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$$48 = 2^4 \cdot 3^1$$

$$= 48$$



$$\begin{aligned}
& \frac{x^{\frac{5}{12}} y^{\frac{25}{12}}}{x^{\frac{15}{8}} y^{\frac{25}{16}} x^{\frac{5}{4}} y^{\frac{10}{4}}} = \frac{x^{\frac{5}{12}}}{x^{\frac{15}{8}} x^{\frac{5}{4}}} \cdot \frac{y^{\frac{25}{12}}}{y^{\frac{25}{16}} y^{\frac{10}{4}}} \\
& = \frac{x^{\frac{10}{24}}}{x^{\frac{45}{24}} x^{\frac{30}{24}}} \cdot \frac{y^{\frac{100}{48}}}{y^{\frac{75}{48}} y^{\frac{120}{48}}} \\
& = \frac{x^{\frac{10}{24}}}{x^{\frac{75}{24}}} \cdot \frac{y^{\frac{100}{48}}}{y^{\frac{195}{48}}} \\
& = \frac{1}{x^{\frac{65}{24}}} \cdot \frac{1}{y^{\frac{95}{48}}} \\
& = \frac{1}{x^{\frac{48}{24} + \frac{17}{24}}} \cdot \frac{1}{y^{\frac{48}{48} + \frac{47}{48}}} \\
& = \frac{1}{x^{\frac{48}{24}} \cdot x^{\frac{17}{24}}} \cdot \frac{1}{y^{\frac{48}{48}} y^{\frac{47}{48}}} \\
& = \frac{1}{x^2} \frac{1}{x^{\frac{17}{24}}} \cdot \frac{1}{y} \frac{1}{y^{\frac{47}{48}}} \\
& = \frac{1}{x^2 y^1} \frac{1}{x^{\frac{17}{24}}} \frac{1}{y^{\frac{47}{48}}} \\
& = \frac{1}{x^2 y^1} \cdot \frac{1}{x^{\frac{17}{24}}} \frac{x^{\frac{7}{24}}}{x^{\frac{7}{24}}} \cdot \frac{1}{y^{\frac{47}{48}}} \frac{y^{\frac{1}{48}}}{y^{\frac{1}{48}}} \\
& = \frac{1}{x^2 y^1} \frac{x^{\frac{7}{24}}}{x^{\frac{24}{24}}} \cdot \frac{y^{\frac{1}{48}}}{y^{\frac{48}{48}}} = \frac{x^{\frac{7}{24}}}{x^2 x^1} \cdot \frac{y^{\frac{1}{48}}}{y^1 y^1}
\end{aligned}$$

Answer $\frac{x^{\frac{7}{24}} y^{\frac{1}{48}}}{x^3 y^2}$

Notp
 $17+7=24$
 $47+1=48$