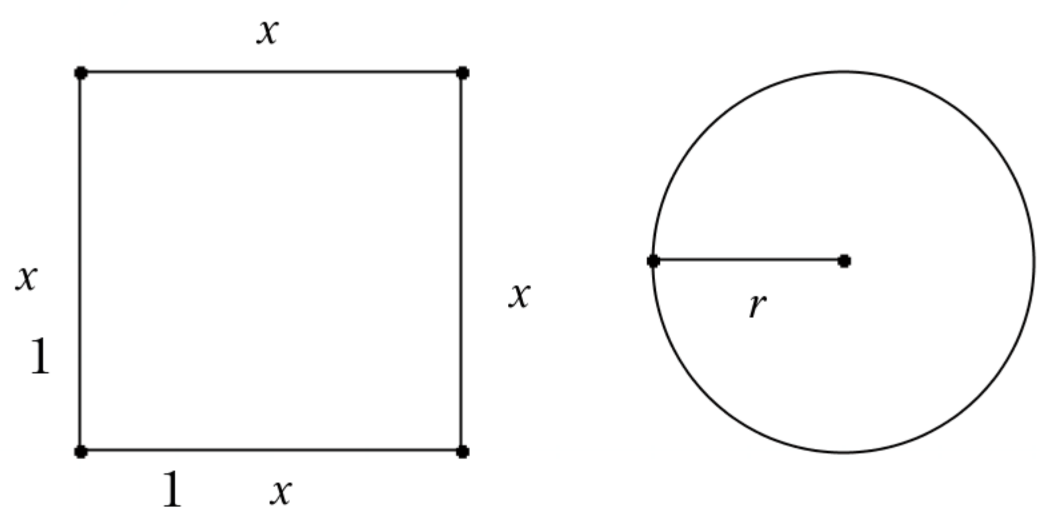


Perimeter of Square $=4\cdot x$ Perimeter of Circle $=2\cdot\pi\cdot r$ Total Perimeter $=2\cdot\pi\cdot r+4\cdot x$

Given wire $=100$

Area of Square $=x^2$ Area of Circle $=\pi\cdot r^2$ Total Area $=\pi\cdot r^2+x^2$

Steps in the process of maximizing area when given perimeter allowed

1) Write perimeter function

2) solve for missing variable in area formula

3) replace constraint from perimeter formula in area formula

4) write area formula in terms of single variable. either A(r) or A(x)

5) find $\frac{dA}{dx}$ or $\frac{dA}{dr}$

6) solve $\frac{dA}{dx} = 0$ or $\frac{dA}{dr} = 0$

7) find feasible domain for x or r

8) Maximum or minimum will occur at

x or r = minimum of feasible domain, or

x or r= maximum of feasible domain, or

x = solutions to $\frac{dA}{dx} = 0$ or $\frac{dA}{dr} = 0$

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing area when given perimeter allowed

1) Write perimeter function

Total Perimeter $=2\cdot\pi\cdot r+4\cdot x$ Given wire $=100$

$100 = 2\cdot\pi\cdot r+4\cdot x$

2) solve for missing variable in area formula

This leads to $r=\frac{50}{\pi}-\frac{2\cdot x}{\pi}$ **or** $x=25-\frac{\pi\cdot r}{2}$

3) replace constraint from perimeter formula in area formula

Total Area $=\pi\cdot r^2+x^2$

Area formula in terms of radius $= \pi\cdot r^2+\left(25-\frac{\pi\cdot r}{2}\right)^2$

Area formula in terms of square side $= x^2+\pi\left(\frac{50}{\pi}-\frac{2\cdot x}{\pi}\right)^2$

Steps in the process of maximizing area when given perimeter allowed

4) write area formula in terms of single variable. either A(r) or A(x)

Area formula in terms of radius $= \pi \cdot r^2 + \left(25 - \frac{\pi \cdot r}{2}\right)^2$

$$A(r) = \left(\frac{\pi^2}{4} + \pi\right) \cdot r^2 - 25 \cdot \pi \cdot r + 625$$

Area formula in terms of square side $= a\_s + \pi (r\_1)^2$

$$A(x) = \frac{(\pi+4) \cdot x^2 - 200 \cdot x + 2500}{\pi}$$

5) find $\frac{dA}{dx}$ or $\frac{dA}{dr}$

$$A'(r) = \frac{dA}{dr} = \frac{\pi^2 \cdot r}{2} + 2 \cdot \pi \cdot r - 25 \cdot \pi$$

$$A'(x) = \frac{dA}{dx} = \frac{8 \cdot x}{\pi} + 2 \cdot x - \frac{200}{\pi}$$

Steps in the process of maximizing area when given perimeter allowed

6) solve $\frac{dA}{dr} = 0$ $\quad$ $A'(r) = \frac{dA}{dr} = \frac{\pi^2 \cdot r}{2} + 2\cdot\pi\cdot r - 25\cdot\pi$

$\frac{dA}{dr} = \frac{\pi^2 \cdot r}{2} + 2\cdot\pi\cdot r - 25\cdot\pi = 0$ at $\frac{50}{\pi\cdot(\pi+4)}$ $\quad$ $r = \frac{50}{\pi\cdot(\pi+4)} \approx 2.229$

6) solve $\frac{dA}{dx} = 0$ $\quad$ $A'(x) = \frac{dA}{dx} = \frac{8\cdot x}{\pi} + 2\cdot x - \frac{200}{\pi}$

$\frac{dA}{dx} = \frac{8\cdot x}{\pi} + 2\cdot x - \frac{200}{\pi} = 0$ at $\frac{100}{\pi+4}$ $\quad$ $x = \frac{100}{\pi+4} \approx 14.0025$

Steps in the process of maximizing area when given perimeter allowed

7) find feasible domain for x or r

A(r) has a feasible domain of $0 \le r \le \frac{50}{\pi}$ **Why? The maximum radius means no square**

A(x) has a feasible domain of $0 \le x \le 25$ **Why? The maximum side means no circle**

8) Maximum or minimum will occur at

A(r) will maximize or minimize at $r = 0$, $r = \frac{50}{\pi}$, **or** $r = \frac{50}{\pi \cdot (\pi+4)}$

$A(0) = 625$ $A\left(\frac{50}{\pi}\right) = \frac{1250 \cdot (\pi+1)}{\pi}$ $A\left(\frac{50}{\pi \cdot (\pi+4)}\right) = 625 - \frac{625}{\pi \cdot (\pi+4)}$

A(x) will maximize or minimize at $x = 0$, $x = 25$, **or** $x = \frac{100}{\pi+4}$

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing area when given perimeter allowed

8) Maximum or minimum will occur at

A(r) will maximize or minimize at r = 0, r = $\frac{50}{\pi}$, or r=$\frac{50}{\pi\cdot(\pi+4)}$

$A(0)=625$

$A(\frac{50}{\pi})=\frac{1250\cdot(\pi+1)}{\pi}$ also $A(15.92)=1647.89$

$A(\frac{50}{\pi\cdot(\pi+4)})=625-\frac{625}{\pi\cdot(\pi+4)}$ also $A(2.229)=597.143$

A(x) will maximize or minimize at x = 0, x = x_smax , or x=x_solve

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing area when given perimeter allowed

8) Maximum or minimum will occur at

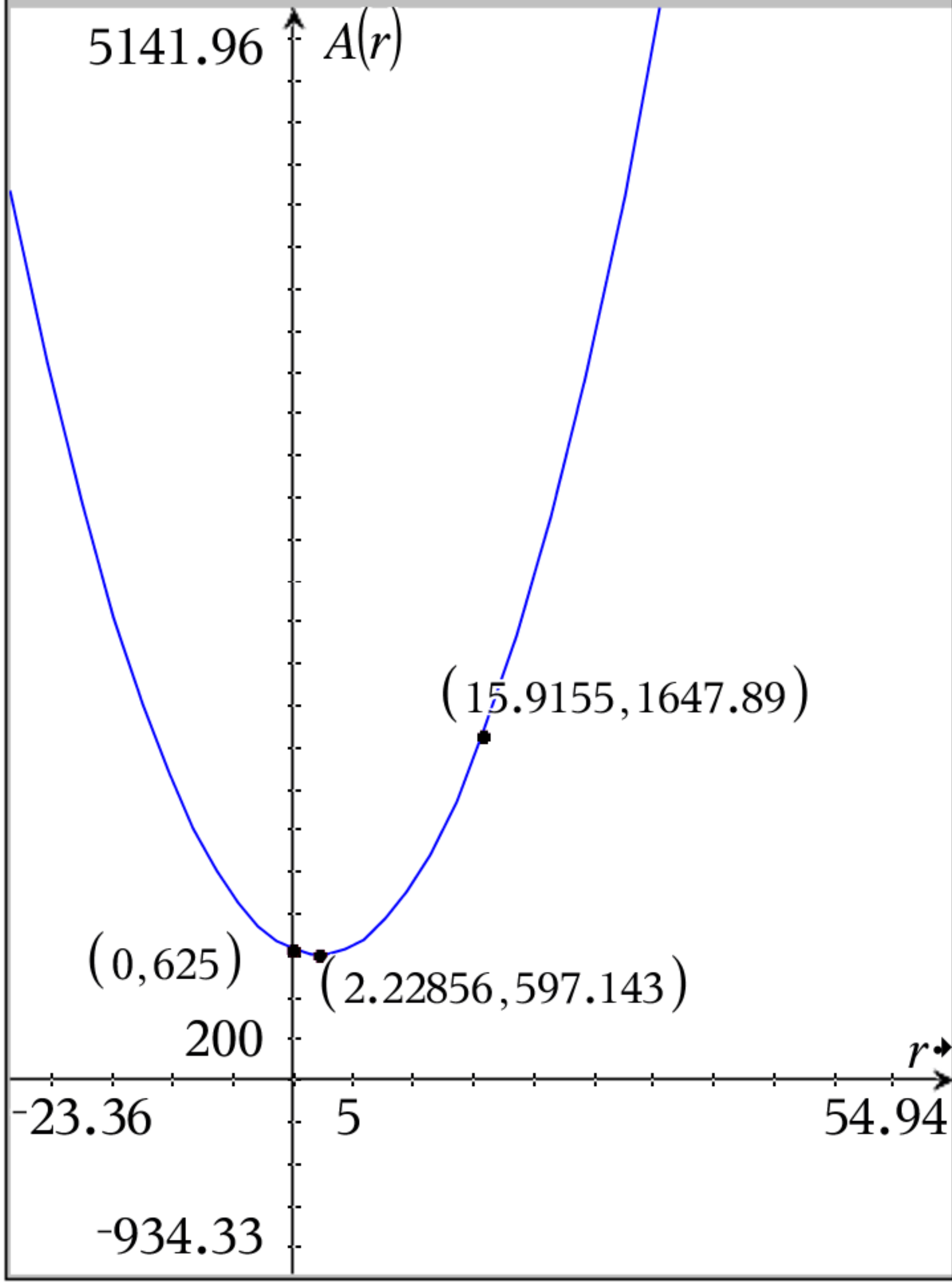
A(x) will maximize or minimize at x = 0, x = 25, or $x=\frac{100}{\pi+4}$

$A(0)=\frac{2500}{\pi}$ also $A(0)=795.775$

$A(25)=625$ also $A(25.)=625.$

$A\left(\frac{100}{\pi+4}\right)=\frac{2500}{\pi+4}$ also $A(14.0025)=350.062$

(this is a consequence of EVT Extreme Value Theorem)



Given length of wire 100

$$A(r) = \left(\frac{\pi^2}{4}+\pi\right)\cdot r^2-25\cdot\pi\cdot r+625$$

$$\frac{dA}{dr} =$$

$$\text{expand}\left(\frac{d}{dr}(\mathbf{a\_1r})\right) \blacktriangleright \frac{\pi^2\cdot r}{2}+2\cdot\pi\cdot r-25\cdot\pi$$

$$\frac{dA}{dr} = \frac{\pi^2\cdot r}{2}+2\cdot\pi\cdot r-25\cdot\pi = 0 \text{ at } \frac{50}{\pi\cdot(\pi+4)}$$

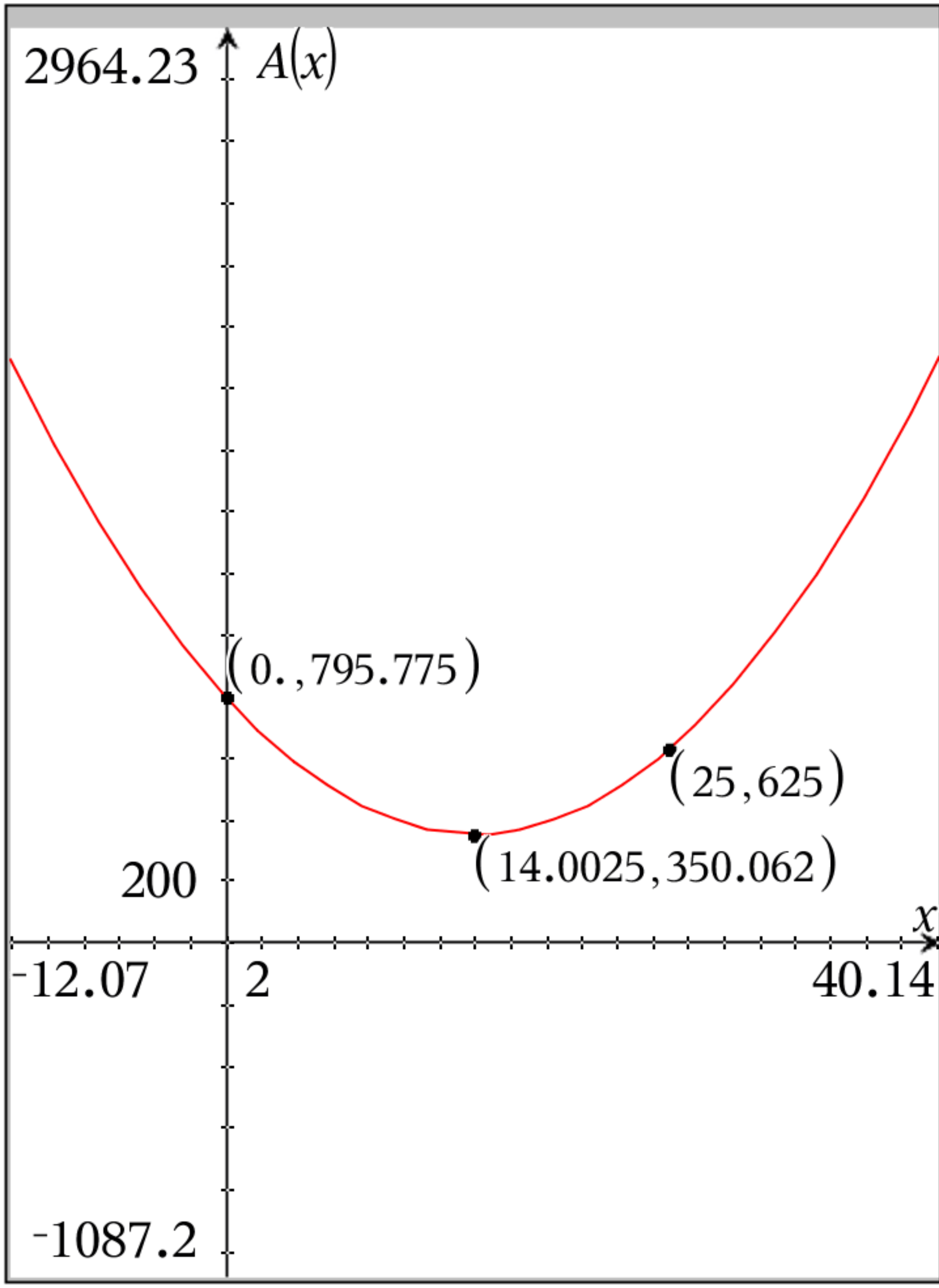
$$r = \frac{50}{\pi\cdot(\pi+4)} \approx 2.229$$

A(r) has a feasible domain of

$$0\le r\le\frac{50}{\pi}$$

A(r) will maximize or minimize at

r = 0, r = $\frac{50}{\pi}$**, or r=** $\frac{50}{\ldots}$



Given length of wire 100

$$A(x) = \frac{(\pi+4)\cdot x^2 - 200\cdot x + 2500}{\pi}$$

$$\frac{dA}{dx} = \frac{8\cdot x}{\pi} + 2\cdot x - \frac{200}{\pi}$$

$$\frac{dA}{dx} = \frac{8\cdot x}{\pi} + 2\cdot x - \frac{200}{\pi} = 0 \text{ at } \frac{100}{\pi+4}$$

$$x = \frac{100}{\pi+4} \approx 14.0025$$

A(x) has a feasible domain of

$0 \le x \le 25$

A(x) will maximize or minimize at

$x = 0$, $x = 25$, or $x = \frac{100}{\pi+4}$

When r = 5 find rate of change in area

This really depends on which function you use to answer this question

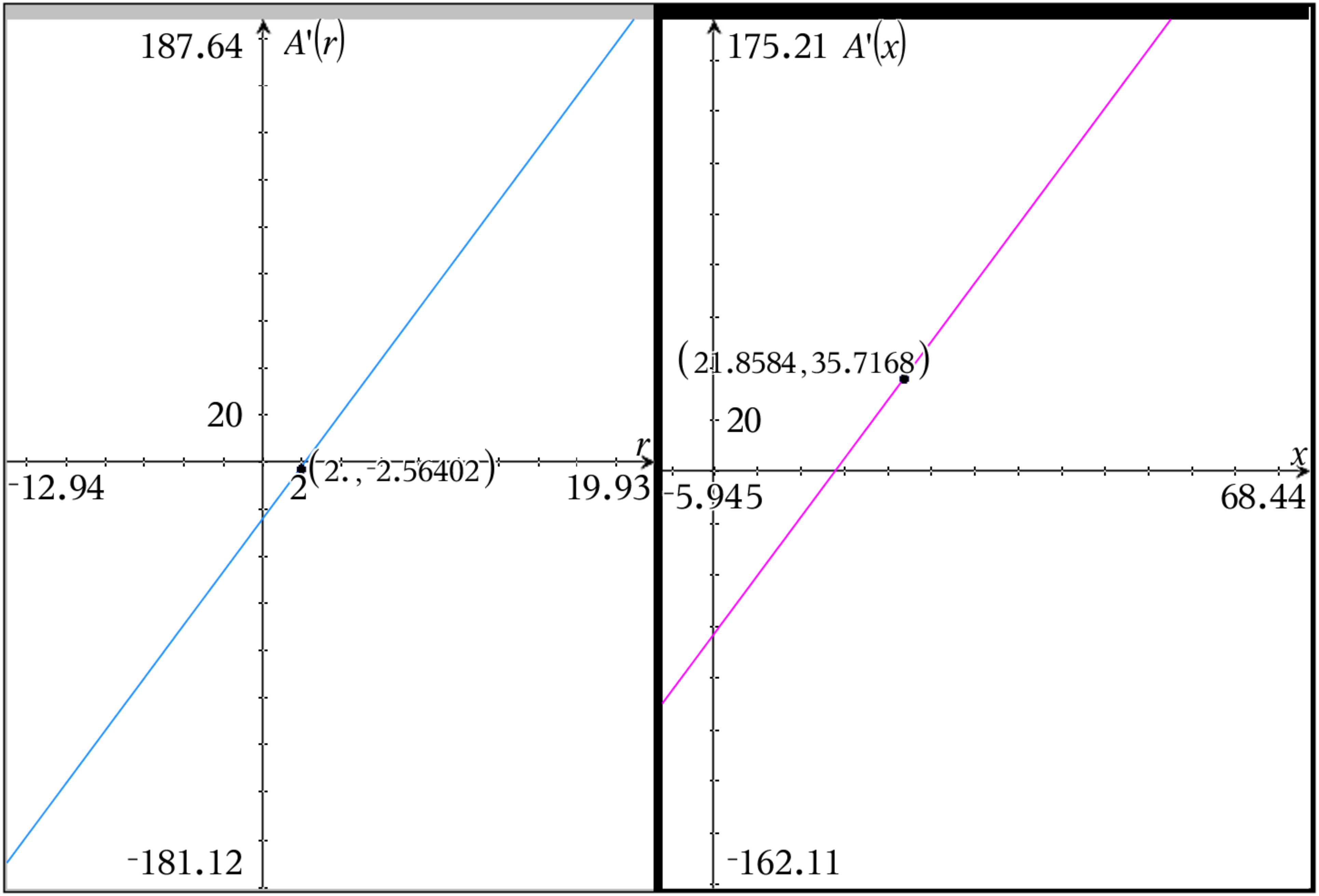
The intent is to use $\frac{dA}{dr} = \frac{\pi^2 \cdot r}{2} + 2\cdot\pi\cdot r - 25\cdot\pi$

Find $\frac{dA}{dr}$ at r = 2 $\mathbf{A'(}2\mathbf{)=}\pi^2 + 4\cdot\pi - 25 \approx {}^{-}2.564$

You can also use $\mathbf{A'(x)=}\frac{dA}{dx} = \frac{8\cdot x}{\pi} + 2\cdot x - \frac{200}{\pi}$ but indirectly

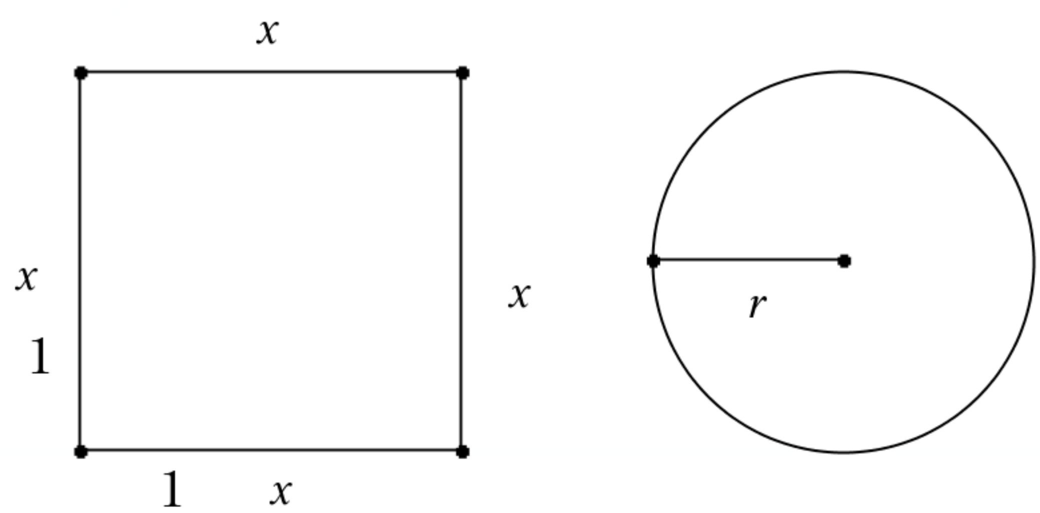
Find $\frac{dA}{dx}$ at r = 2 **which implies x** $= 25 - \pi$

$\mathbf{A'(}25-\pi\mathbf{)=}42 - 2\cdot\pi \approx 35.72$


187.64
A'(r)
20
r
-12.94
2
(2., -2.56402)
19.93
-181.12
175.21
A'(x)
(21.8584, 35.7168)
20
x
-5.945
68.44
-162.11

	A	B	C	D
=				
1	p_s	4*x	a_1r	(π^2/4+π)*r^2−25*π*r+625
2	p_c	2*π*r	a_1s	((π+4)*x^2−200*x+2500)/(π)
3	a_s	x^2	x_smax	25
4	a_c	π*r^2	r_cmax	50/(π)
5	p_1	2*π*r+4*x	a_r	π^2/4+π
6	a_1	π*r^2+x^2	b_r	-25
7	x_1	25−π*r/2	c_r	625
8	p_given	100	d_r	-625*π^2−2500*π+625
9	r_1	50/(π)−2*x/(π)	r_solve	50/(π*(π+4))
10	r_given	2	a_sq	(π+4)/(π)
11	x_implied	25−π	b_sq	-200/(π)

A1 p_s



Perimeter of Square $=4\cdot x$ Perimeter of Circle $=2\cdot \pi\cdot r$ Total Perimeter $=2\cdot \pi\cdot r+4\cdot x$

Given wire $=200$

Area of Square $=x^2$ Area of Circle $=\pi\cdot r^2$ Total Area $=\pi\cdot r^2+x^2$

Steps in the process of maximizing area when given perimeter allowed

1) Write perimeter function

2) solve for missing variable in area formula

3) replace constraint from perimeter formula in area formula

4) write area formula in terms of single variable. either A(r) or A(x)

5) find $\frac{dA}{dx}$ or $\frac{dA}{dr}$

6) solve $\frac{dA}{dx} = 0$ or $\frac{dA}{dr} = 0$

7) find feasible domain for x or r

8) Maximum or minimum will occur at

x or r = minimum of feasible domain, or

x or r= maximum of feasible domain, or

x = solutions to $\frac{dA}{dx} = 0$ or $\frac{dA}{dr} = 0$

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing area when given perimeter allowed

1) Write perimeter function

Total Perimeter $= 2 \cdot \pi \cdot r + 4 \cdot x$ Given wire $= 200$

$200 = 2 \cdot \pi \cdot r + 4 \cdot x$

2) solve for missing variable in area formula

This leads to $r = \frac{100}{\pi} - \frac{2 \cdot x}{\pi}$ **or** $x = 50 - \frac{\pi \cdot r}{2}$

3) replace constraint from perimeter formula in area formula

Total Area $= \pi \cdot r^2 + x^2$

Area formula in terms of radius $= \pi \cdot r^2 + \left(50 - \frac{\pi \cdot r}{2}\right)^2$

Area formula in terms of square side $= x^2 + \pi \left(\frac{100}{\pi} - \frac{2 \cdot x}{\pi}\right)^2$

Steps in the process of maximizing area when given perimeter allowed

4) write area formula in terms of single variable. either A(r) or A(x)

Area formula in terms of radius $= \pi \cdot r^2 + \left(50 - \frac{\pi \cdot r}{2}\right)^2$

$$A(r) = \left(\frac{\pi^2}{4} + \pi\right) \cdot r^2 - 50 \cdot \pi \cdot r + 2500$$

Area formula in terms of square side = a_s + π(r_1)^2

$$A(x) = \frac{(\pi+4) \cdot x^2 - 400 \cdot x + 10000}{\pi}$$

5) find $\frac{dA}{dx}$ or $\frac{dA}{dr}$

$$A'(r) = \frac{dA}{dr} = \frac{\pi^2 \cdot r}{2} + 2 \cdot \pi \cdot r - 50 \cdot \pi$$

$$A'(x) = \frac{dA}{dx} = \frac{8 \cdot x}{\pi} + 2 \cdot x - \frac{400}{\pi}$$

Steps in the process of maximizing area when given perimeter allowed

6) solve $\frac{dA}{dr} = 0$ $\quad$ **A'(r)=** $\frac{dA}{dr} = \frac{\pi^2 \cdot r}{2} + 2 \cdot \pi \cdot r - 50 \cdot \pi$

$\frac{dA}{dr} = \frac{\pi^2 \cdot r}{2} + 2 \cdot \pi \cdot r - 50 \cdot \pi = 0$ at $\frac{100}{\pi \cdot (\pi+4)}$ $\quad$ $r = \frac{100}{\pi \cdot (\pi+4)} \approx 4.457$

6) solve $\frac{dA}{dx} = 0$ $\quad$ **A'(x)=** $\frac{dA}{dx} = \frac{8 \cdot x}{\pi} + 2 \cdot x - \frac{400}{\pi}$

$\frac{dA}{dx} = \frac{8 \cdot x}{\pi} + 2 \cdot x - \frac{400}{\pi} = 0$ at $\frac{200}{\pi+4}$ $\quad$ $x = \frac{200}{\pi+4} \approx 28.005$

Steps in the process of maximizing area when given perimeter allowed

7) find feasible domain for x or r

A(r) has a feasible domain of $0 \le r \le \frac{100}{\pi}$ **Why? The maximum radius means no square**

A(x) has a feasible domain of $0 \le x \le 50$ **Why? The maximum side means no circle**

8) Maximum or minimum will occur at

A(r) will maximize or minimize at $r = 0$, $r = \frac{100}{\pi}$, **or** $r = \frac{100}{\pi \cdot (\pi+4)}$

$A(0) = 2500 \quad A\left(\frac{100}{\pi}\right) = \frac{5000 \cdot (\pi+1)}{\pi} \quad A\left(\frac{100}{\pi \cdot (\pi+4)}\right) = 2500 - \frac{2500}{\pi \cdot (\pi+4)}$

A(x) will maximize or minimize at $x = 0$, $x = 50$, **or** $x = \frac{200}{\pi+4}$

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing area when given perimeter allowed

8) Maximum or minimum will occur at

A(r) will maximize or minimize at r = 0, r = $\frac{100}{\pi}$, or r=$\frac{100}{\pi\cdot(\pi+4)}$

A(0)=2500

$A\left(\frac{100}{\pi}\right)=\frac{5000\cdot(\pi+1)}{\pi}$ also A(31.83)=6591.55

$A\left(\frac{100}{\pi\cdot(\pi+4)}\right)=2500-\frac{2500}{\pi\cdot(\pi+4)}$ also A(4.457)=2388.57

A(x) will maximize or minimize at x = 0, x = x_smax , or x=x_solve

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing area when given perimeter allowed

8) Maximum or minimum will occur at

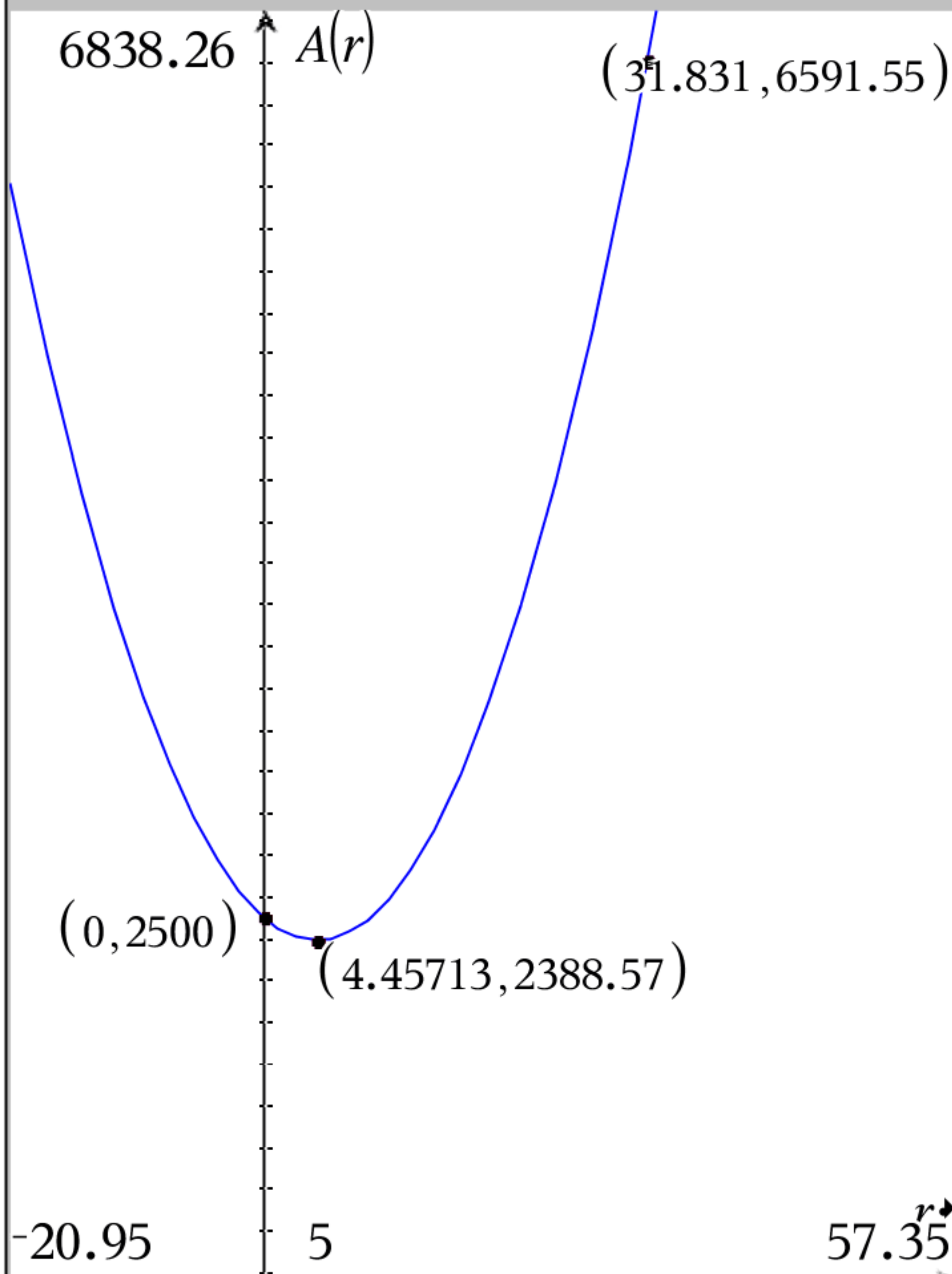
A(x) will maximize or minimize at x = 0, x = 50, or $x=\frac{200}{\pi+4}$

$A(0)=\frac{10000}{\pi}$ also $A(0)=3183.1$

$A(50)=2500$ also $A(50.)=2500.$

$A\left(\frac{200}{\pi+4}\right)=\frac{10000}{\pi+4}$ also $A(28.005)=1400.25$

(this is a consequence of EVT Extreme Value Theorem)



Given length of wire 200

$$A(r) = \left(\frac{\pi^2}{4}+\pi\right)\cdot r^2-50\cdot\pi\cdot r+2500$$

$$\frac{dA}{dr} =$$

$$\text{expand}\left(\frac{d}{dr}(\mathbf{a\_1r})\right) ▸ \frac{\pi^2\cdot r}{2}+2\cdot\pi\cdot r-50\cdot\pi$$

$$\frac{dA}{dr} = \frac{\pi^2\cdot r}{2}+2\cdot\pi\cdot r-50\cdot\pi = 0 \text{ at } \frac{100}{\pi\cdot(\pi+4)}$$

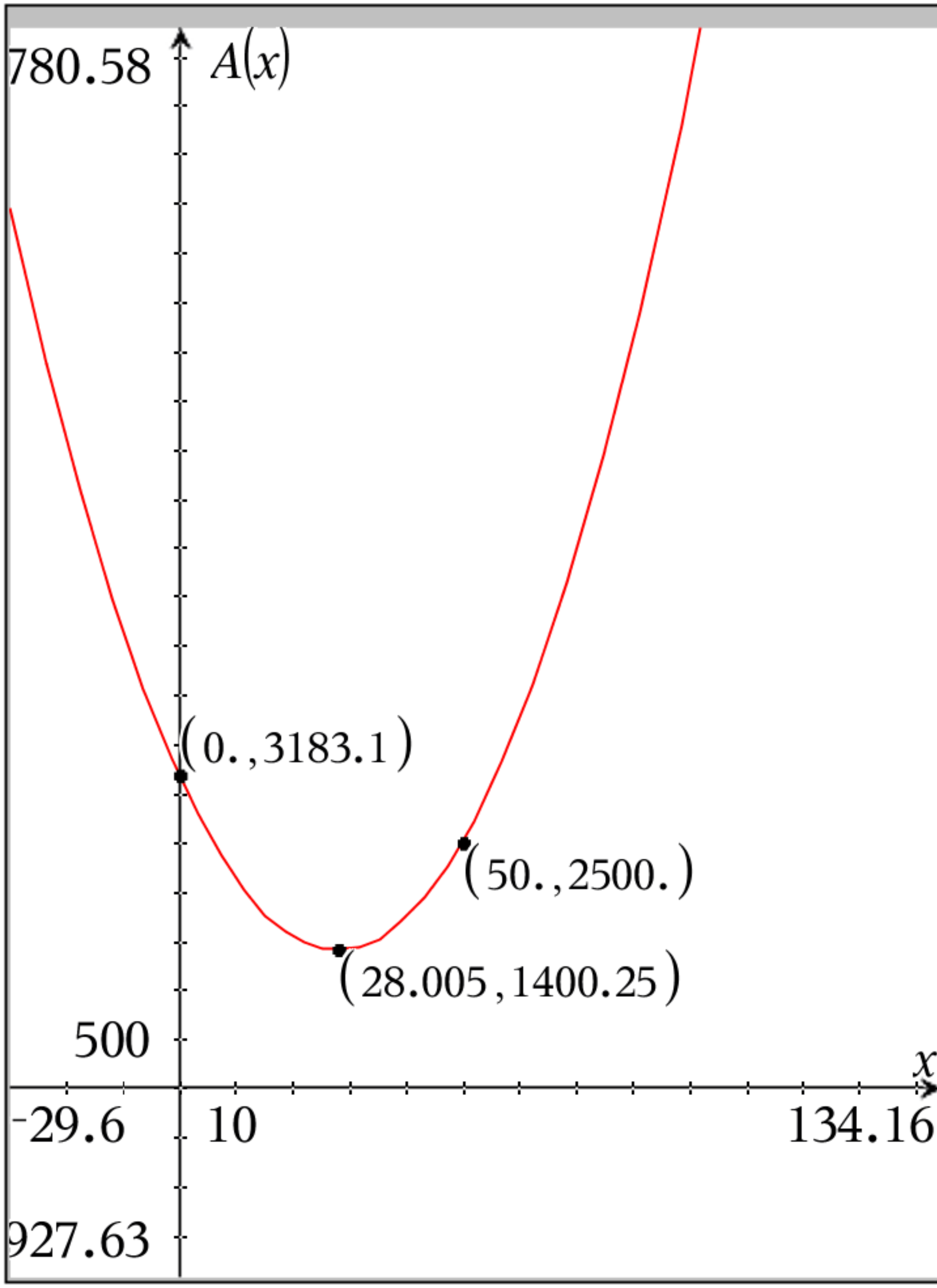
$$r = \frac{100}{\pi\cdot(\pi+4)} \approx 4.457$$

A(r) has a feasible domain of

$$0 \le r \le \frac{100}{\pi}$$

A(r) will maximize or minimize at

r = 0, r = $\frac{100}{\ }$**, or r=** $\frac{100}{(\ \ )}$



Given length of wire 200

$$A(x) = \frac{(\pi+4)\cdot x^2 - 400\cdot x + 10000}{\pi}$$

$$\frac{dA}{dx} = \frac{8\cdot x}{\pi} + 2\cdot x - \frac{400}{\pi}$$

$$\frac{dA}{dx} = \frac{8\cdot x}{\pi} + 2\cdot x - \frac{400}{\pi} = 0 \text{ at } \frac{200}{\pi+4}$$

$$x = \frac{200}{\pi+4} \approx 28.005$$

A(x) has a feasible domain of

$0 \le x \le 50$

A(x) will maximize or minimize at

$x = 0$, $x = 50$, or $x = \frac{200}{\pi+4}$

When r = 5 find rate of change in area

This really depends on which function you use to answer this question

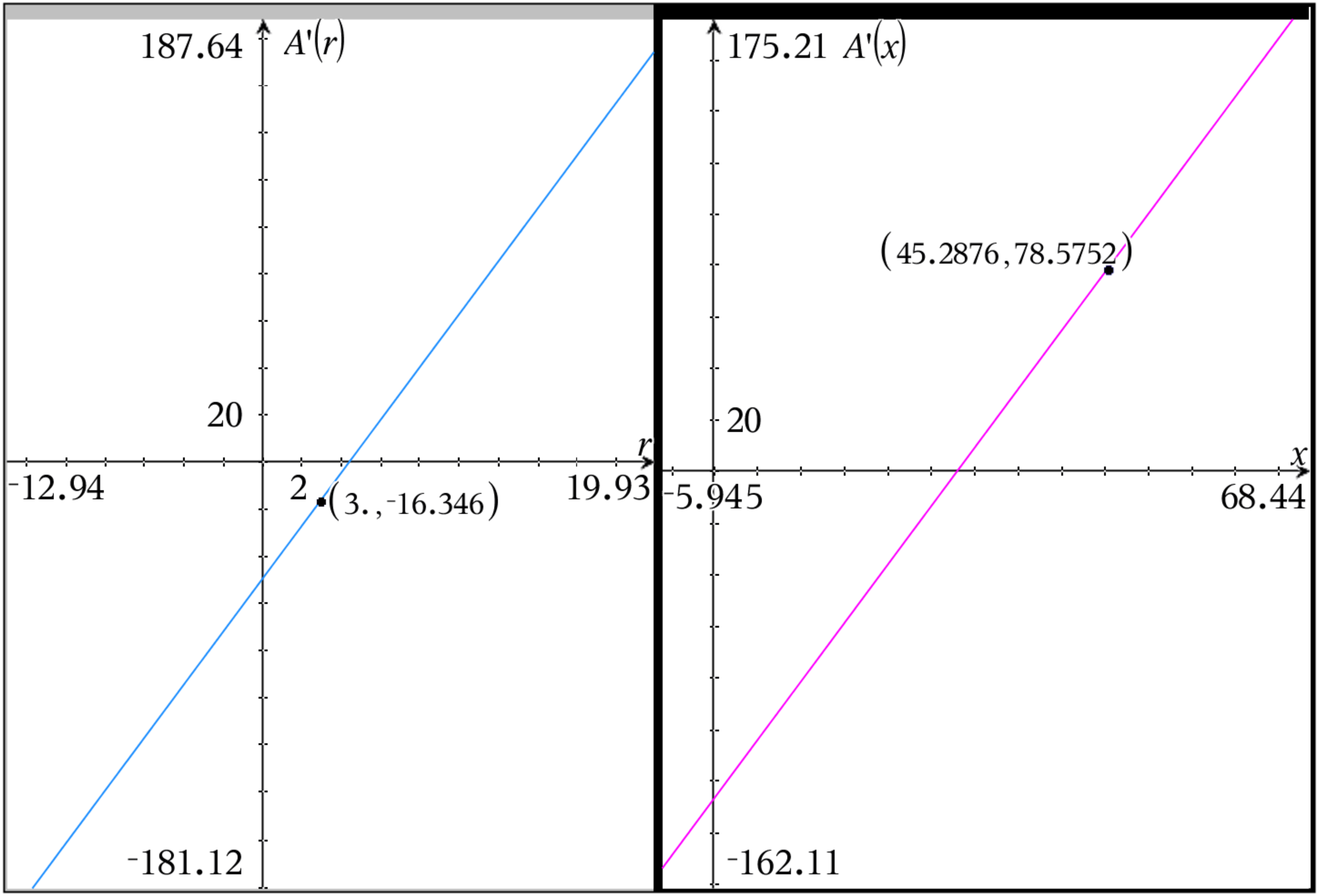
The intent is to use $\frac{dA}{dr} = \frac{\pi^2 \cdot r}{2} + 2\cdot\pi\cdot r - 50\cdot\pi$

Find $\frac{dA}{dr}$ at r = 3 $\quad$ **A'(3)=**$\frac{3\cdot\pi^2}{2} + 6\cdot\pi - 50 \approx -16.35$

You can also use A'(x)=$\frac{dA}{dx} = \frac{8\cdot x}{\pi} + 2\cdot x - \frac{400}{\pi}$ but indirectly

Find $\frac{dA}{dx}$ at r = 3 **which implies x** $= 50 - \frac{3\cdot\pi}{2}$

A'($50 - \frac{3\cdot\pi}{2}$**)=**$88 - 3\cdot\pi \approx 78.58$



187.64
A'(r)
20
r
-12.94
2
19.93
(3., -16.346)
-181.12
175.21
A'(x)
(45.2876, 78.5752)
20
x
-5.945
68.44
-162.11

	A	B	C	D
1	p_s	4*x	a_1r	(π^2/4+π)*r^2−50*π*r+2500
2	p_c	2*π*r	a_1s	((π+4)*x^2−400*x+10000)/(π)
3	a_s	x^2	x_smax	50
4	a_c	π*r^2	r_cmax	100/(π)
5	p_1	2*π*r+4*x	a_r	π^2/4+π
6	a_1	π*r^2+x^2	b_r	-50
7	x_1	50−π*r/2	c_r	2500
8	p_given	200	d_r	-2500*π^2−10000*π+2500
9	r_1	100/(π)−2*x/(π)	r_solve	100/(π*(π+4))
10	r_given	3	a_sq	(π+4)/(π)
11	x_implied	50−3*π/2	b_sq	-400/(π)

A1 p_s