Given Surface Area $=1000$

Given NO TOP Triangular prism with side length x

Perimeter of Base $=2\cdot x+x\cdot\sqrt{2}=\left(\sqrt{2}+2\right)\cdot x$

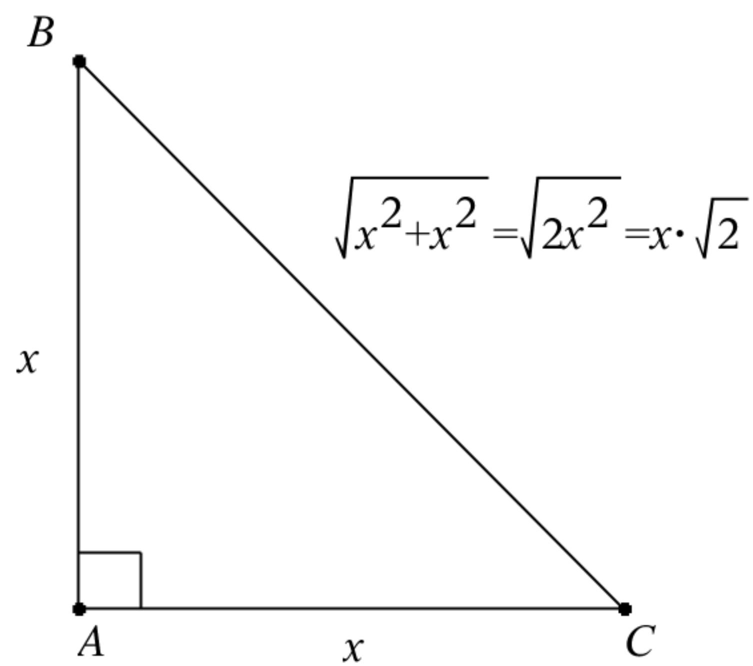
Area of Base $=\dfrac{x^2}{2}$

Solve $1000=\left(\left(\sqrt{2}+2\right)\cdot x\right)\mathbf{h}+\dfrac{x^2}{2}$ **for h**

$$\mathbf{h}=\frac{1000-\frac{1}{2}\cdot x^2}{\left(2+\sqrt{2}\right)x}=\frac{1000}{\left(2+\sqrt{2}\right)\cdot x}-\frac{1}{2\left(2+\sqrt{2}\right)}x$$

$$=\frac{\left(\sqrt{2}-2\right)\cdot\left(x^2-2000\right)}{4\cdot x}$$

$$=\frac{\sqrt{2}\cdot x}{4}-\frac{x}{2}-\frac{500\cdot\sqrt{2}}{x}+\frac{1000}{x}$$



Steps in the process of maximizing volume when given surface area allowed

1) Write surface area function

2) solve for missing variable in volume formula

3) replace constraint from surface area formula in volume formula

4) write volume formula in terms of single variable.

5) find $\frac{dV}{dx}$

6) solve $\frac{dV}{dx} = 0$

7) find feasible domain for x

8) Maximum or minimum will occur at

x = minimum of feasible domain, or

x = maximum of feasible domain, or

x = solutions to $\frac{dV}{dx} = 0$

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing volume when given surface area allowed

1) Write surface area function (no TOP)

SA = PH+B

$$1000 = \frac{x^2}{2} + h \cdot \left(\sqrt{2}+2\right) \cdot x$$

2) solve for missing variable in volume formula

$$1000 - \frac{x^2}{2} = h \cdot \left(\sqrt{2}+2\right) \cdot x$$

$$\left(1000 - \frac{x^2}{2}\right) / \left(\left(\sqrt{2}+2\right) \cdot x\right) = \mathbf{h}$$

$$\mathbf{h} = \frac{1000 - \frac{1}{2} \cdot x^2}{\left(2+\sqrt{2}\right)x} = \frac{1000}{\left(2+\sqrt{2}\right) \cdot x} - \frac{1}{2\left(2+\sqrt{2}\right)} x = \frac{\left(\sqrt{2}-2\right) \cdot \left(x^2 - 2000\right)}{4 \cdot x}$$

Steps in the process of maximizing volume when given surface area allowed

3) replace constraint from surface area formula in volume formula $V=\frac{1}{2}x^2\cdot h$

$$\mathbf{h}=\frac{2000-\frac{1}{2}\cdot x^2}{(2+\sqrt{2})x}=\frac{2000}{(2+\sqrt{2})\cdot x}-\frac{1}{2(2+\sqrt{2})}x$$

$$V=\frac{1}{2}x^2\cdot\left(\frac{2000-\frac{1}{2}\cdot x^2}{(2+\sqrt{2})x}\right)$$

4) write volume formula in terms of single variable.

$$V(x)=\frac{1000}{(2+\sqrt{2})\cdot}x-\frac{1}{4(2+\sqrt{2})}x^3$$

$$V(x)=\frac{(\sqrt{2}-2)\cdot x\cdot(x^2-2000)}{8}$$

Steps in the process of maximizing volume when given surface area allowed

5) find $\frac{dV}{dx}$

$$V(x) = \frac{500}{(2+\sqrt{2})}x - \frac{1}{4(2+\sqrt{2})}x^3 \quad \text{yields} \quad \frac{dV}{dx} = \frac{500}{(2+\sqrt{2})} - \frac{3}{4(2+\sqrt{2})}x^2$$

6) solve $\frac{dV}{dx} = 0$

$$\frac{500}{(2+\sqrt{2})} - \frac{3}{4(2+\sqrt{2})}x^2 = 0$$

$$\frac{500}{(2+\sqrt{2})\cdot} = \frac{3}{4(2+\sqrt{2})}x^2$$

$$\frac{500}{(2+\sqrt{2})\cdot} \cdot \frac{4(2+\sqrt{2})}{3} = x^2$$

$$x = \sqrt{\frac{2000}{3.}} \approx 25.82$$

Steps in the process of maximizing volume when given surface area allowed

7) find feasible domain for x

This is probably the hardest conceptual thing to do

For this volume model, the feasible domain is related to the zero of the height function

$$\textbf{height} = \frac{(\sqrt{2}-2)\cdot(x^2-2000)}{4\cdot x}$$ This zero is $\sqrt{2000.} \approx 44.72$

Feasible Domain $0 < x < \sqrt{2000.} \approx 44.72$

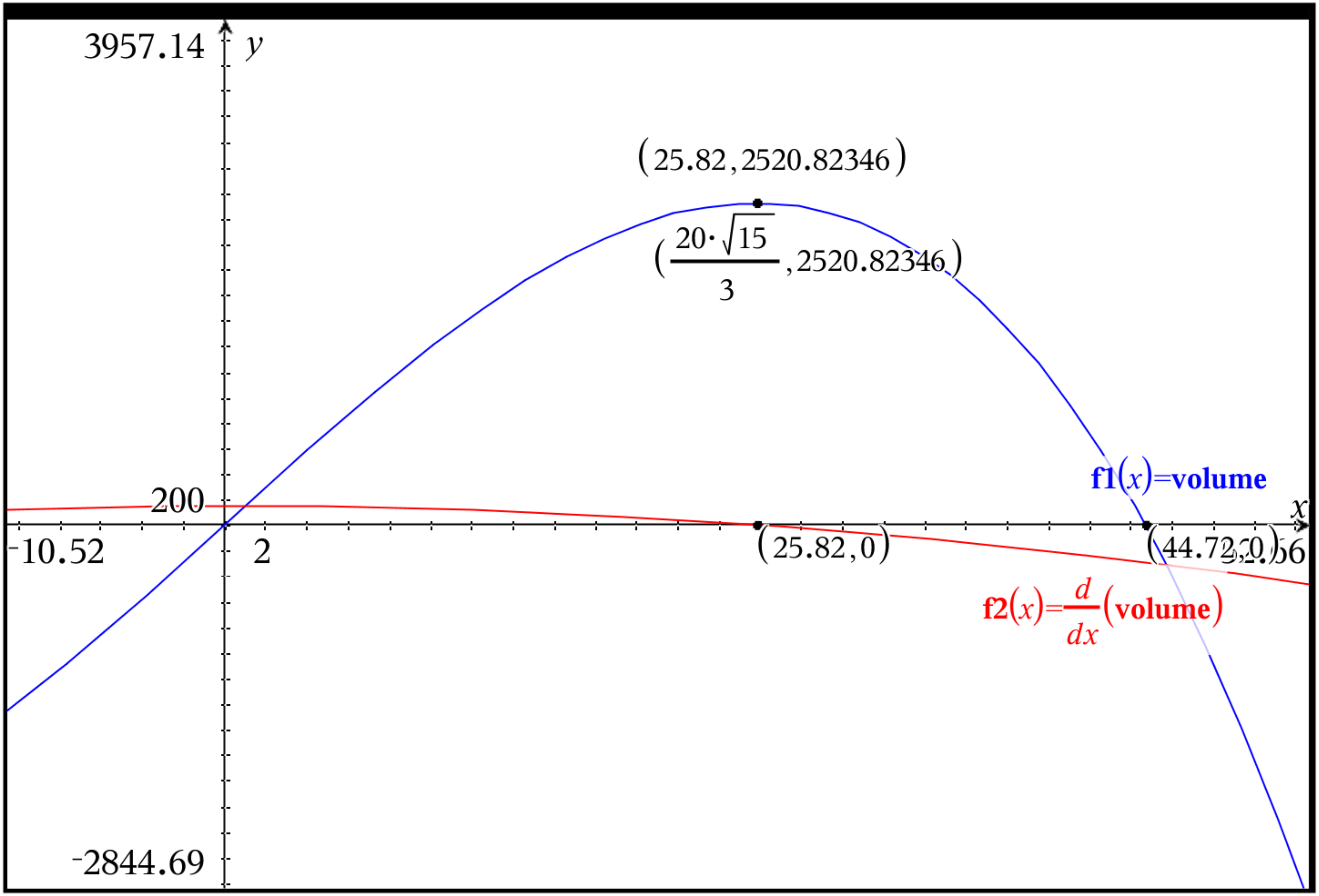
8) Maximum or minimum will occur at

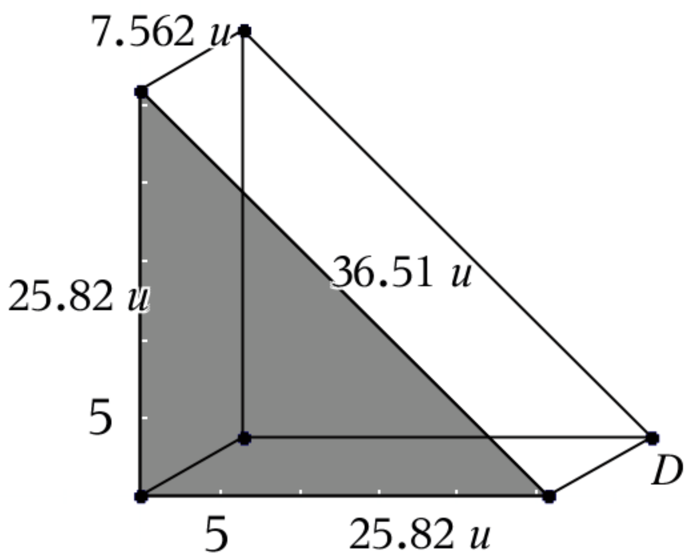
x = minimum of feasible domain, or V(0) = 0

x = maximum of feasible domain, or $V(\sqrt{2000.}) = 0$

x = solutions to $\frac{dV}{dx} = 0$ $V\left(\sqrt{\frac{2000}{3.}}\right) = 2520.82$

(this is a consequence of EVT Extreme Value Theorem)


3957.14
y
(25.82,2520.82346)
(20·√15/3,2520.82346)
f1(x)=volume
200
x
-10.52
2
(25.82,0)
(44.72,0)
f2(x)=d/dx(volume)
-2844.69



Dimensions of Isosceles Triangular Prism with open top that has a maximum volume when it uses a total surface area of 1000

x = 25.82

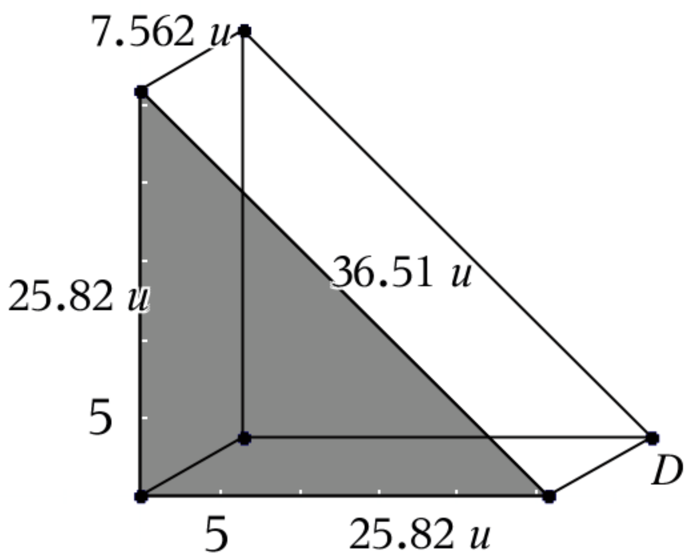
x = 25.82

$x\sqrt{2}$ = 36.51

$$\text{height} = \frac{(\sqrt{2}-2)\cdot(x^2-2000)}{4\cdot x} = 7.562$$

Maximum Volume

2520.82



Finding the rate of change in the Volume when x = 2

$$\frac{dV}{dx}=\frac{500}{\left(2+\sqrt{2}\right)}-\frac{3}{4\left(2+\sqrt{2}\right)}x^2$$

V'(2)=145.568

A	B	C x_list	D y_list	E
=				
1 sa_1	1000	0	0	
2 x_1	x	25.82	0	
3 missing_side	√(2)*x	0	25.82	
4 height	(√(2)−2)*(x^2−2000)/(4*x)	6.549	3.781	
5 volume	(√(2)−2)*x*(x^2−2000)/8	6.549	29.6	
6 p_1	(√(2)+2)*x	32.37	3.781	
7 b_1	x^2/2			
8 x_solve	20*√(15)/3			
9 h_solve	⁻10*(√(2)−2)*√(15)/3			
10 given_x	2			
11				

A1 sa_1

Given Surface Area = 2000

Given NO TOP Triangular prism with side length x

Perimeter of Base = $2\cdot x+x\cdot\sqrt{2} = (\sqrt{2}+2)\cdot x$

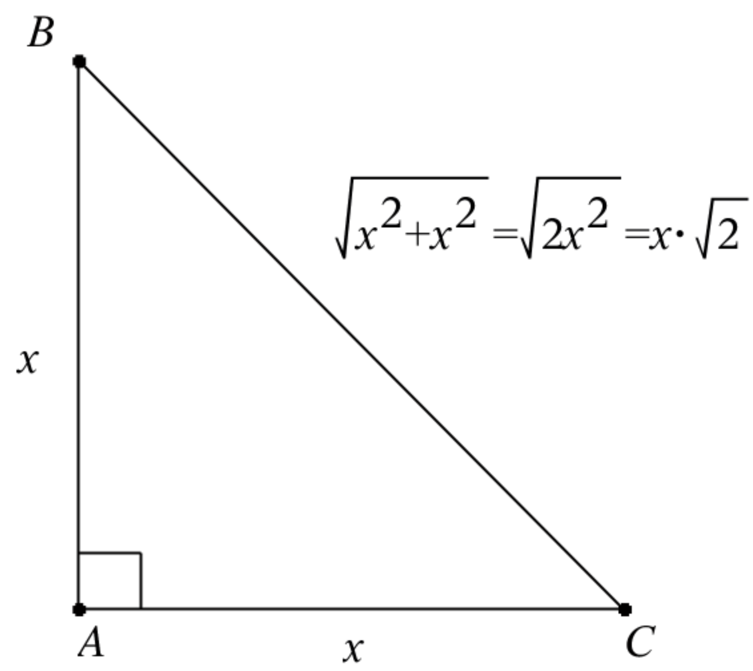
Area of Base = $\frac{x^2}{2}$

Solve $2000 = ((\sqrt{2}+2)\cdot x)h + \frac{x^2}{2}$ **for h**

$$h = \frac{2000-\frac{1}{2}\cdot x^2}{(2+\sqrt{2})x} = \frac{2000}{(2+\sqrt{2})\cdot x} - \frac{1}{2(2+\sqrt{2})}x$$

$$= \frac{(\sqrt{2}-2)\cdot(x^2-4000)}{4\cdot x}$$

$$= \frac{\sqrt{2}\cdot x}{4} - \frac{x}{2} - \frac{1000\cdot\sqrt{2}}{x} + \frac{2000}{x}$$



Steps in the process of maximizing volume when given surface area allowed

1) Write surface area function

2) solve for missing variable in volume formula

3) replace constraint from surface area formula in volume formula

4) write volume formula in terms of single variable.

5) find $\frac{dV}{dx}$

6) solve $\frac{dV}{dx} = 0$

7) find feasible domain for x

8) Maximum or minimum will occur at

x = minimum of feasible domain, or

x = maximum of feasible domain, or

x = solutions to $\frac{dV}{dx} = 0$

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing volume when given surface area allowed

1) Write surface area function (no TOP)

SA = PH+B

$$2000 = \frac{x^2}{2} + h\cdot\left(\sqrt{2}+2\right)\cdot x$$

2) solve for missing variable in volume formula

$$2000 - \frac{x^2}{2} = h\cdot\left(\sqrt{2}+2\right)\cdot x$$

$$(2000 - \frac{x^2}{2})/(\left(\sqrt{2}+2\right)\cdot x = \mathbf{h}$$

$$\mathbf{h} = \frac{2000-\frac{1}{2}\cdot x^2}{\left(2+\sqrt{2}\right)x} = \frac{2000}{\left(2+\sqrt{2}\right)\cdot x} - \frac{1}{2\left(2+\sqrt{2}\right)}x = \frac{\left(\sqrt{2}-2\right)\cdot\left(x^2-4000\right)}{4\cdot x}$$

Steps in the process of maximizing volume when given surface area allowed

3) replace constraint from surface area formula in volume formula $V=\frac{1}{2}x^2 \cdot h$

$$\mathbf{h} = \frac{2000-\frac{1}{2}\cdot x^2}{\left(2+\sqrt{2}\right)x} = \frac{2000}{\left(2+\sqrt{2}\right)\cdot x} - \frac{1}{2\left(2+\sqrt{2}\right)}x$$

$$V=\frac{1}{2}x^2 \cdot \left(\frac{2000-\frac{1}{2}\cdot x^2}{\left(2+\sqrt{2}\right)x}\right)$$

4) write volume formula in terms of single variable.

$$V(x) = \frac{1000}{\left(2+\sqrt{2}\right)\cdot}x - \frac{1}{4\left(2+\sqrt{2}\right)}x^3$$

$$V(x) = \frac{\left(\sqrt{2}-2\right)\cdot x\cdot \left(x^2-4000\right)}{8}$$

Steps in the process of maximizing volume when given surface area allowed

5) find $\frac{dV}{dx}$

V(x) $=\frac{1000}{(2+\sqrt{2})}x-\frac{1}{4(2+\sqrt{2})}x^3$ yields $\frac{dV}{dx}=\frac{1000}{(2+\sqrt{2})}-\frac{3}{4(2+\sqrt{2})}x^2$

6) solve $\frac{dV}{dx}=0$

$$\frac{1000}{(2+\sqrt{2})}-\frac{3}{4(2+\sqrt{2})}x^2=0$$

$$\frac{1000}{(2+\sqrt{2})\cdot}=\frac{3}{4(2+\sqrt{2})}x^2$$

$$\frac{1000}{(2+\sqrt{2})\cdot}\cdot\frac{4(2+\sqrt{2})}{3}=x^2$$

$$x=\sqrt{\frac{4000}{3.}}\approx 36.51$$

Steps in the process of maximizing volume when given surface area allowed

7) find feasible domain for x

This is probably the hardest conceptual thing to do

For this volume model, the feasible domain is related to the zero of the height function

$\textbf{height} = \frac{(\sqrt{2}-2)\cdot(x^2-4000)}{4\cdot x}$ This zero is $\sqrt{4000.} \approx 63.25$

Feasible Domain $0 < x < \sqrt{4000.} \approx 63.25$

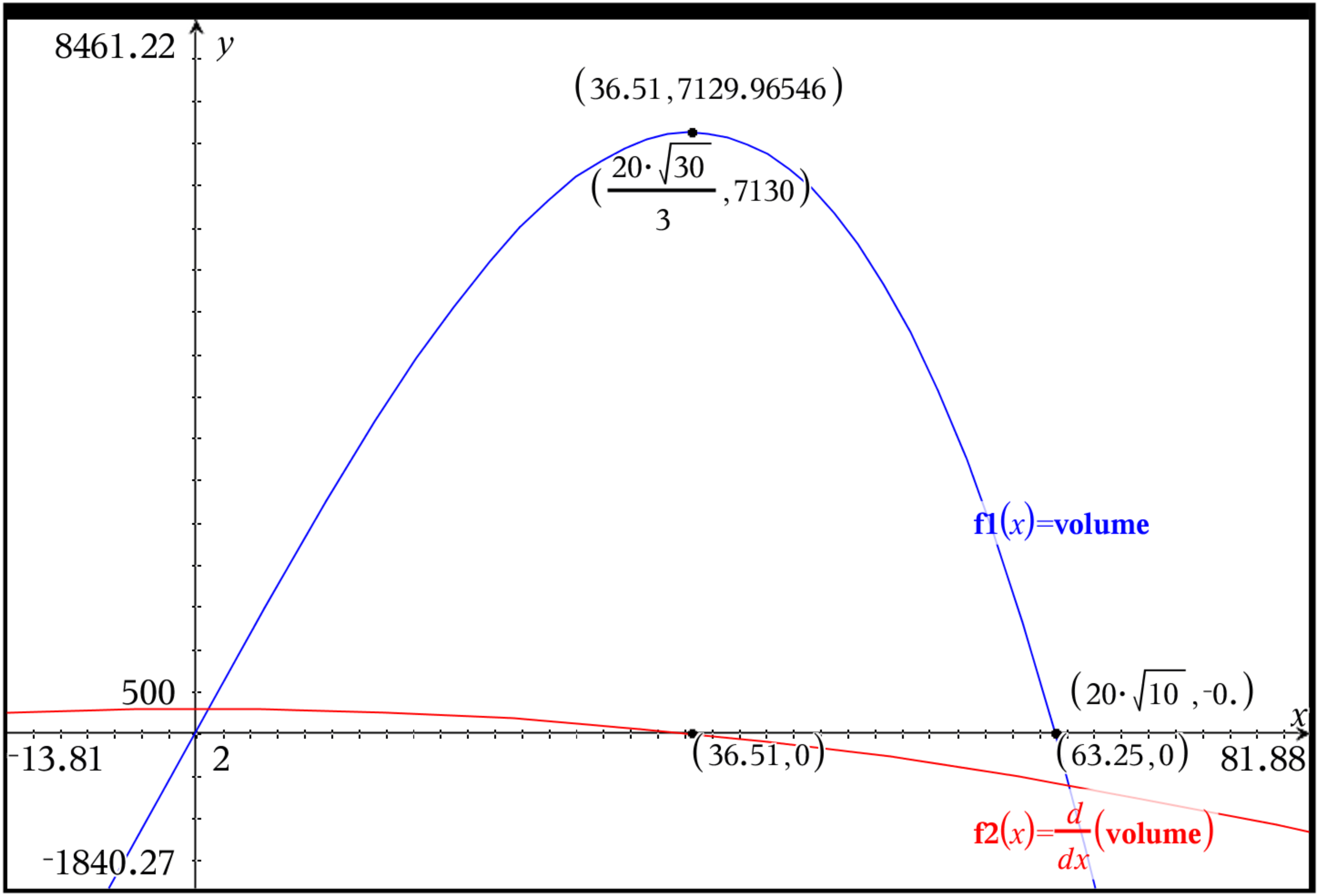
8) Maximum or minimum will occur at

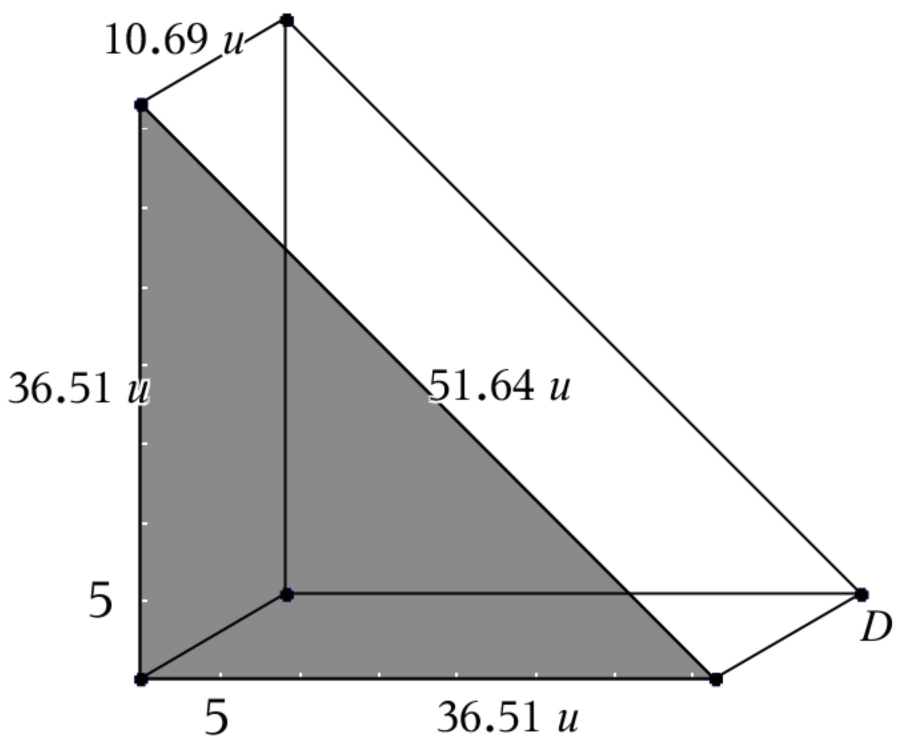
x = minimum of feasible domain, or V(0) = 0

x = maximum of feasible domain, or $V(\sqrt{4000.}) = 0.$

x = solutions to $\frac{dV}{dx} = 0$ $V\left(\sqrt{\frac{4000}{3.}}\right) = 7129.97$

(this is a consequence of EVT Extreme Value Theorem)


8461.22
y
(36.51,7129.96546)
$\left(\frac{20\cdot\sqrt{30}}{3},7130\right)$
f1(x)=volume
500
(20·√10 ,⁻0.)
x
⁻13.81
2
(36.51,0)
(63.25,0)
81.88
f2(x)=$\frac{d}{dx}$(volume)
⁻1840.27



Dimensions of Isosceles Triangular Prism with open top that has a maximum volume when it uses a total surface area of 2000

x = 36.51

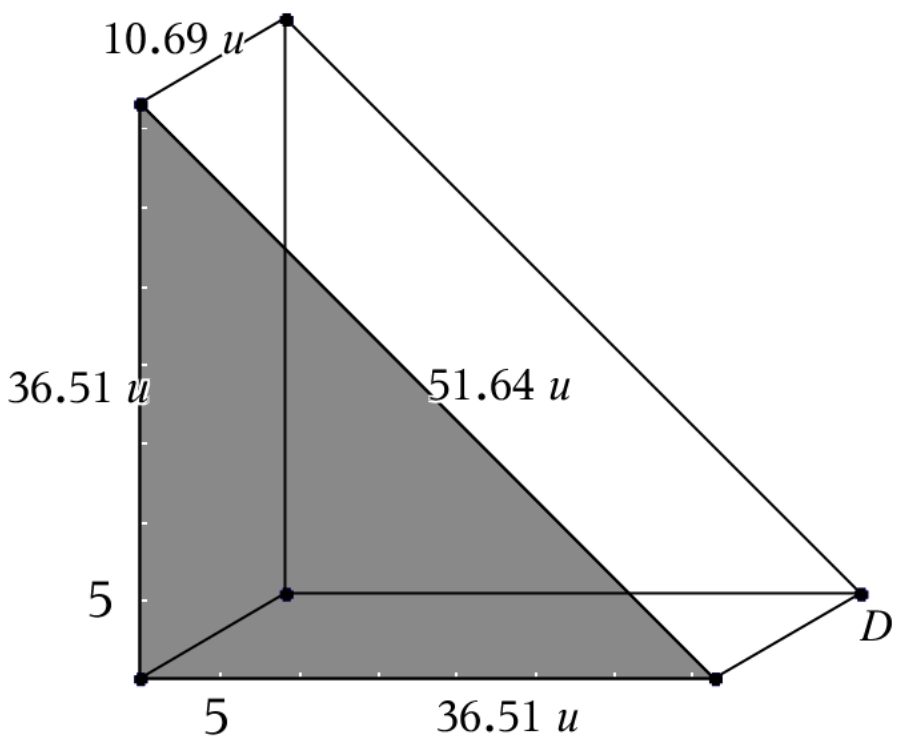
x = 36.51

$x\sqrt{2} = 51.64$

$$\text{height} = \frac{(\sqrt{2}-2)\cdot(x^2-4000)}{4\cdot x} = 10.69$$

Maximum Volume

7129.97



Finding the rate of change in the Volume when x = 5

$$\frac{dV}{dx}=\frac{1000}{(2+\sqrt{2})}-\frac{3}{4(2+\sqrt{2})}x^2$$

V'(5)=287.401

	A	B	C x_list	D y_list	E
=					
1	sa_1	2000	0	0	
2	x_1	x	36.51	0	
3	missing_side	√(2)*x	0	36.51	
4	height	(√(2)−2)*(x^2−4000)/(4*x)	9.262	5.347	
5	volume	(√(2)−2)*x*(x^2−4000)/8	9.262	41.86	
6	p_1	(√(2)+2)*x	45.78	5.347	
7	b_1	x^2/2			
8	x_solve	20*√(30)/3			
9	h_solve	20*(√(2)−1)*√(15)/3			
10	given_x	5			
11					

A1 sa_1

Given Surface Area = 4000

Given NO TOP Triangular prism with side length x

Perimeter of Base $= 2\cdot x + x\cdot\sqrt{2} = (\sqrt{2}+2)\cdot x$

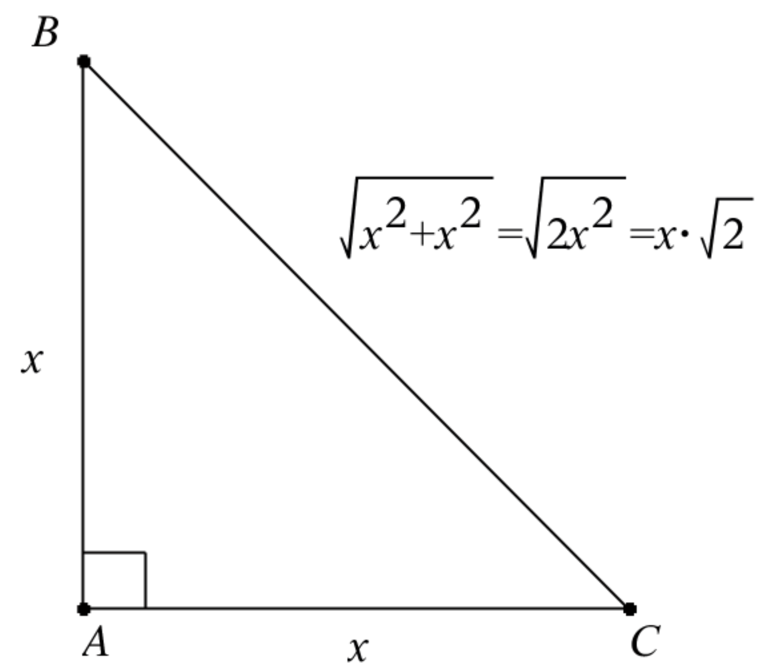
Area of Base $= \dfrac{x^2}{2}$

Solve $4000 = ((\sqrt{2}+2)\cdot x)h + \dfrac{x^2}{2}$ **for h**

$$h = \frac{4000-\frac{1}{2}\cdot x^2}{(2+\sqrt{2})x} = \frac{4000}{(2+\sqrt{2})\cdot x} - \frac{1}{2(2+\sqrt{2})}x$$

$$= \frac{(\sqrt{2}-2)\cdot(x^2-8000)}{4\cdot x}$$

$$= \frac{\sqrt{2}\cdot x}{4} - \frac{x}{2} - \frac{2000\cdot\sqrt{2}}{x} + \frac{4000}{x}$$



Steps in the process of maximizing volume when given surface area allowed

1) Write surface area function

2) solve for missing variable in volume formula

3) replace constraint from surface area formula in volume formula

4) write volume formula in terms of single variable.

5) find $\frac{dV}{dx}$

6) solve $\frac{dV}{dx} = 0$

7) find feasible domain for x

8) Maximum or minimum will occur at

x = minimum of feasible domain, or

x = maximum of feasible domain, or

x = solutions to $\frac{dV}{dx} = 0$

(this is a consequence of EVT Extreme Value Theorem)

Steps in the process of maximizing volume when given surface area allowed

1) Write surface area function (no TOP)

SA = PH+B

$$4000 = \frac{x^2}{2} + h\cdot\left(\sqrt{2}+2\right)\cdot x$$

2) solve for missing variable in volume formula

$$4000 - \frac{x^2}{2} = h\cdot\left(\sqrt{2}+2\right)\cdot x$$

$$\left(4000 - \frac{x^2}{2}\right)/\left(\left(\sqrt{2}+2\right)\cdot x\right) = \mathbf{h}$$

$$\mathbf{h} = \frac{4000 - \frac{1}{2}\cdot x^2}{\left(2+\sqrt{2}\right)x} = \frac{4000}{\left(2+\sqrt{2}\right)\cdot x} - \frac{1}{2\left(2+\sqrt{2}\right)}x = \frac{\left(\sqrt{2}-2\right)\cdot\left(x^2-8000\right)}{4\cdot x}$$

Steps in the process of maximizing volume when given surface area allowed

3) replace constraint from surface area formula in volume formula $V=\frac{1}{2}x^2 \cdot h$

$$\mathbf{h} = \frac{4000-\frac{1}{2}\cdot x^2}{(2+\sqrt{2})x} = \frac{4000}{(2+\sqrt{2})\cdot x} - \frac{1}{2(2+\sqrt{2})}x$$

$$V=\frac{1}{2}x^2\cdot\left(\frac{4000-\frac{1}{2}\cdot x^2}{(2+\sqrt{2})x}\right)$$

4) write volume formula in terms of single variable.

$$V(x) = \frac{2000}{(2+\sqrt{2})\cdot}x - \frac{1}{4(2+\sqrt{2})}x^3$$

$$V(x) = \frac{(\sqrt{2}-2)\cdot x\cdot(x^2-8000)}{8}$$

Steps in the process of maximizing volume when given surface area allowed

5) find $\frac{dV}{dx}$

V(x) $= \frac{2000}{(2+\sqrt{2})}x - \frac{1}{4(2+\sqrt{2})}x^3$ yields $\frac{dV}{dx} = \frac{2000}{(2+\sqrt{2})} - \frac{3}{4(2+\sqrt{2})}x^2$

6) solve $\frac{dV}{dx} = 0$

$$\frac{2000}{(2+\sqrt{2})} - \frac{3}{4(2+\sqrt{2})}x^2 = 0$$

$$\frac{2000}{(2+\sqrt{2})} = \frac{3}{4(2+\sqrt{2})}x^2$$

$$\frac{2000}{(2+\sqrt{2})} \cdot \frac{4(2+\sqrt{2})}{3} = x^2$$

$$x = \sqrt{\frac{8000}{3.}} \approx 51.64$$

Steps in the process of maximizing volume when given surface area allowed

7) find feasible domain for x

This is probably the hardest conceptual thing to do

For this volume model, the feasible domain is related to the zero of the height function

$$\textbf{height} = \frac{(\sqrt{2}-2)\cdot(x^2-8000)}{4\cdot x}$$ This zero is $\sqrt{8000.} \approx 89.44$

Feasible Domain $0 < x < \sqrt{8000.} \approx 89.44$

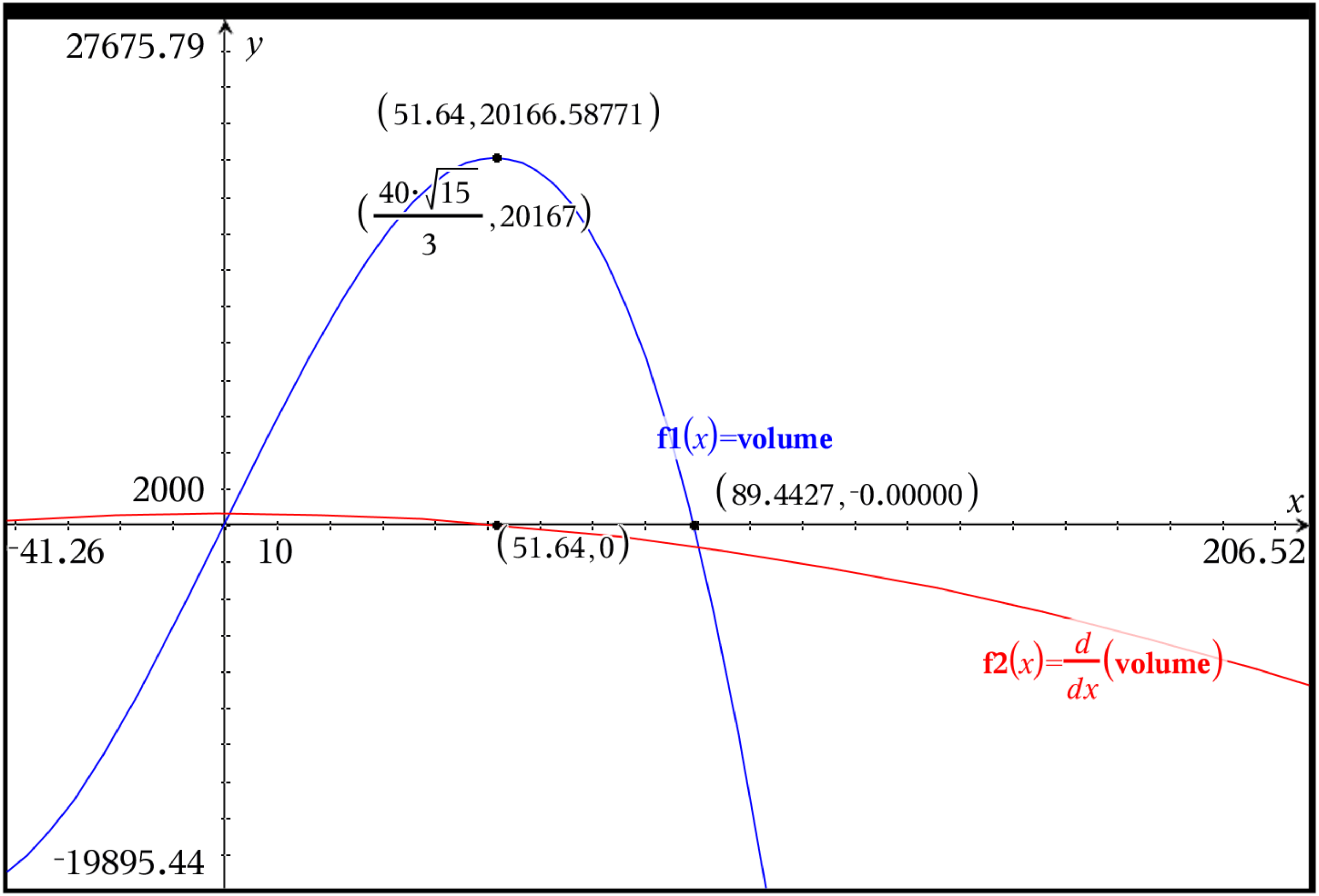
8) Maximum or minimum will occur at

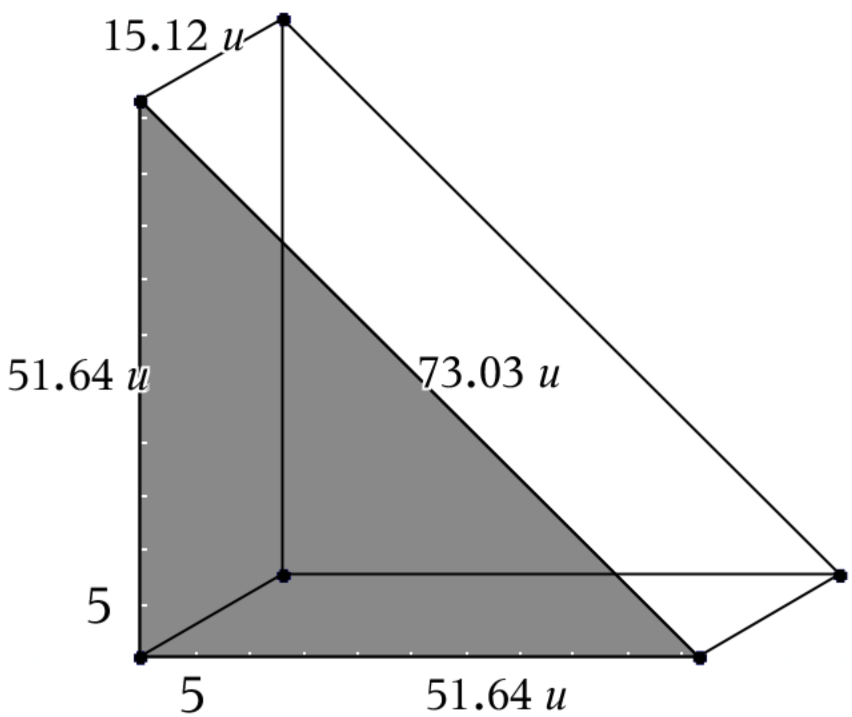
x = minimum of feasible domain, or V(0) = 0

x = maximum of feasible domain, or $V(\sqrt{8000.}) = 0$

x = solutions to $\frac{dV}{dx} = 0$ $\quad V\left(\sqrt{\frac{8000}{3.}}\right) = 20166.6$

(this is a consequence of EVT Extreme Value Theorem)


27675.79
y
(51.64,20166.58771)
$\left(\frac{40\cdot\sqrt{15}}{3},20167\right)$
f1(x)=volume
2000
(89.4427,⁻0.00000)
x
⁻41.26
10
(51.64,0)
206.52
$f2(x)=\frac{d}{dx}(\text{volume})$
⁻19895.44



Dimensions of Isosceles Triangular Prism with open top that has a maximum volume when it uses a total surface area of 4000

x = 51.64

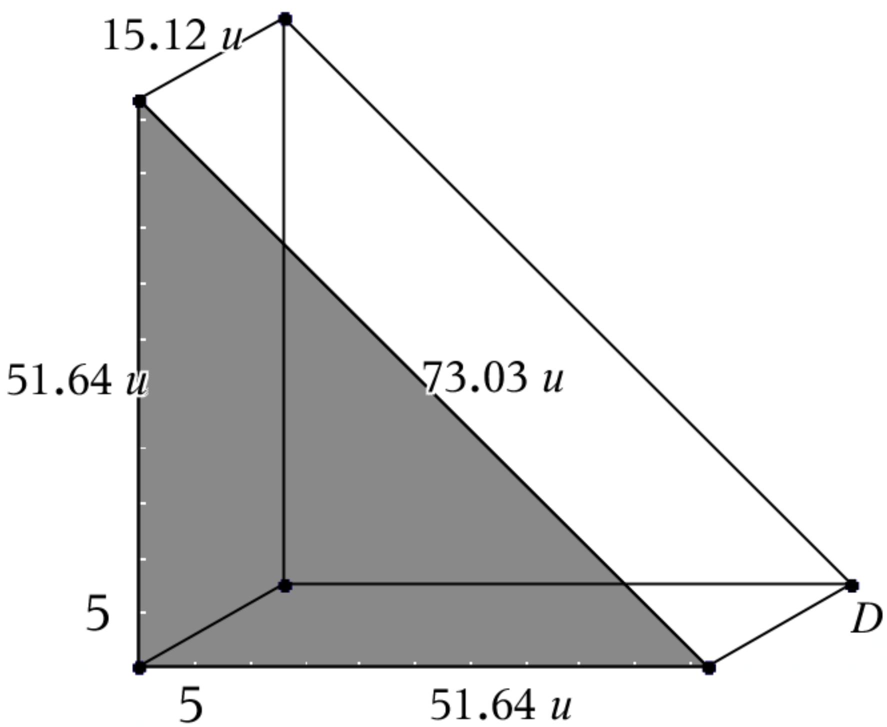
x = 51.64

$x\sqrt{2}$ = 73.03

$$\text{height} = \frac{(\sqrt{2}-2)\cdot(x^2-8000)}{4\cdot x} = 15.12$$

Maximum Volume

20166.6



Finding the rate of change in the Volume when x = 8

$$\frac{dV}{dx}=\frac{2000}{(2+\sqrt{2})}-\frac{3}{4(2+\sqrt{2})}x^2$$

V'(8)=571.728

	A	B	C x_list	D y_list	E
=					
1	sa_1	**4000**	0	0	
2	x_1	**x**	51.64	0	
3	missing_side	**√(2)*x**	0	51.64	
4	height	**(√(2)−2)*(x^2−8000)/(4*x)**	13.1	7.562	
5	volume	**(√(2)−2)*x*(x^2−8000)/8**	13.1	59.2	
6	p_1	**(√(2)+2)*x**	64.74	7.562	
7	b_1	**x^2/2**			
8	x_solve	**40*√(15)/3**			
9	h_solve	**⁻20*(√(2)−2)*√(15)/3**			
10	given_x	**8**			
11					

B11