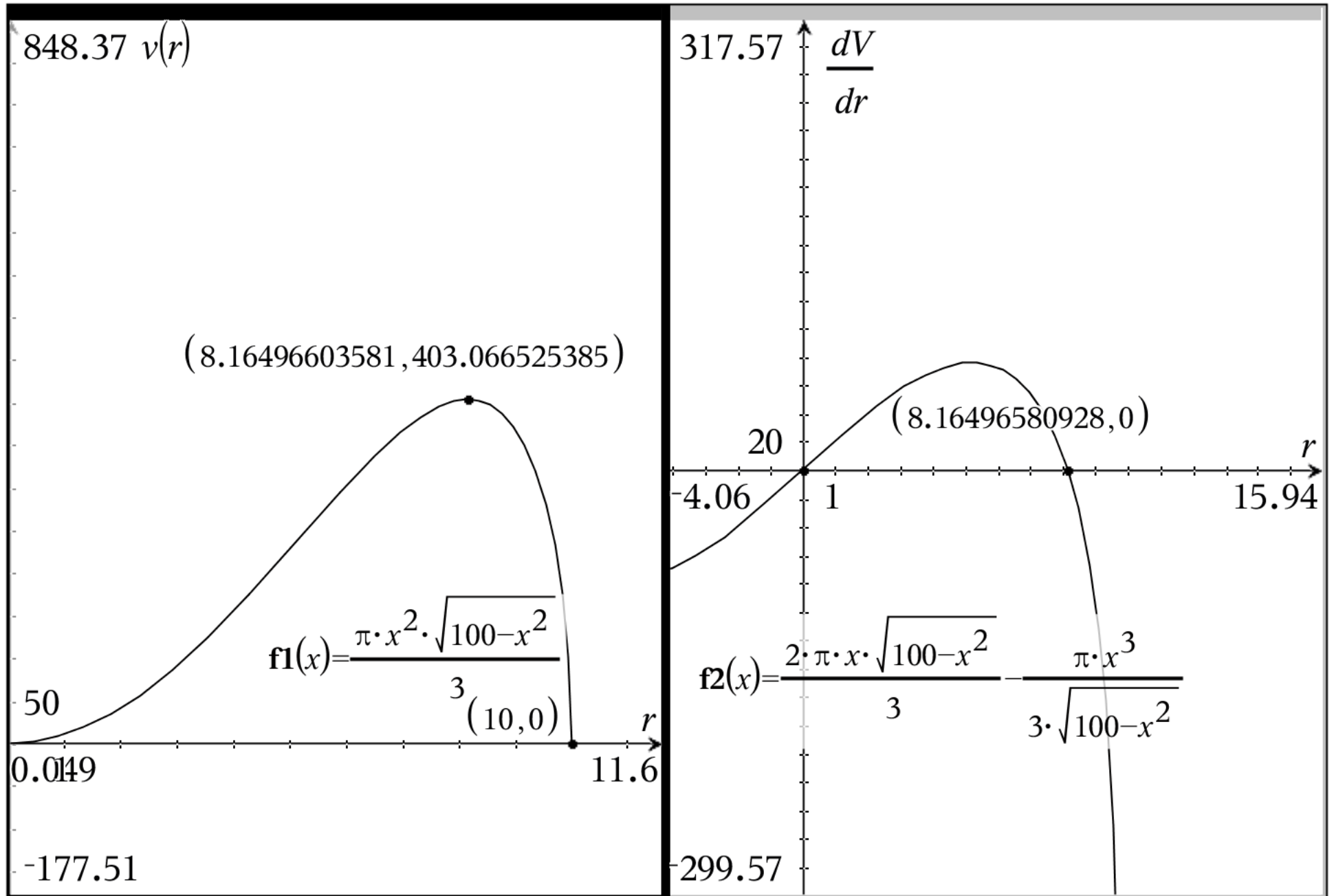


cone slant height 10



$$f1(x) := \frac{\pi \cdot x^2 \cdot \sqrt{100 - x^2}}{3}$$

Done

$$\frac{d}{dx}(f1(x))$$

$$\frac{2 \cdot \pi \cdot x \cdot \sqrt{100 - x^2}}{3} - \frac{\pi \cdot x^3}{3 \cdot \sqrt{100 - x^2}}$$

$$f2(x) := \frac{2 \cdot \pi \cdot x \cdot \sqrt{100 - x^2}}{3} - \frac{\pi \cdot x^3}{3 \cdot \sqrt{100 - x^2}}$$

Done

$$\text{solve}(f2(x)=0, x)$$

$$x = \frac{-10 \cdot \sqrt{6}}{3} \text{ or } x=0 \text{ or } x = \frac{10 \cdot \sqrt{6}}{3}$$

$$\text{solve}(f2(x)=0., x)$$

$$x = -8.16497 \text{ or } x=0. \text{ or } x=8.16497$$

$$f1(8.165)$$

403.067

$$f5(x) := \sqrt{100 - x^2}$$

Done



$$V(r) = \frac{\pi \cdot r^2 \cdot \sqrt{100-r^2}}{3}$$

Feasible Domain of r: $0 \leq r \leq 10$

$$\frac{dV}{dr} = \frac{2 \cdot \pi \cdot r \cdot \sqrt{100-r^2}}{3} - \frac{\pi \cdot r^3}{3 \cdot \sqrt{100-r^2}}$$

If $V(r)$ has a maximum over its feasible domain then it occurs at one of the following:

$V(0)$, $V(10)$ these are values of volume at the extremes

Or at the solutions of $\frac{dV}{dr} = 0$

$$V(0) = 0$$

$$V(10) = 0$$

$$\text{or at } r = \sqrt{\frac{200}{3}} = \frac{10 \cdot \sqrt{6}}{3} = 8.16497$$

$$V\left(\sqrt{\frac{200}{3}}\right) = \frac{2000 \cdot \pi \cdot \sqrt{3}}{27} = 403.067$$

Volume formula in terms of radius $V(r) = \frac{\pi \cdot r^2 \cdot \sqrt{100-r^2}}{3}$

Feasible Domain of r: $0 \leq r \leq 10$

$$\frac{dV}{dr} = \frac{2 \cdot \pi \cdot r \cdot \sqrt{100-r^2}}{3} - \frac{\pi \cdot r^3}{3 \cdot \sqrt{100-r^2}}$$

The maximum volume for this scenario occurs at $r = \sqrt{\frac{200}{3}} \approx 8.16497$

The height that maximizes the volume is $h = \sqrt{\frac{100}{3}} = \frac{10 \cdot \sqrt{3}}{3} \approx 5.7735$

The maximum volume is $V\left(\sqrt{\frac{200}{3}}\right) = \frac{2000 \cdot \pi \cdot \sqrt{3}}{27} = 403.067$

$$f3(x) := \frac{\pi}{3} \cdot (100 - x^2) \cdot x$$

Done

$$\frac{d}{dx}(f3(x))$$

$$\frac{-\pi \cdot (3 \cdot x^2 - 100)}{3}$$

$$\text{expand}(f3(x))$$

$$\frac{100 \cdot \pi \cdot x}{3} - \frac{\pi \cdot x^3}{3}$$

$$f4(x) := \frac{d}{dx}(f3(x))$$

Done

$$\text{expand}(f4(x))$$

$$\frac{100 \cdot \pi}{3} - \pi \cdot x^2$$

$$\text{solve}(f4(x)=0, x)$$

$$x = \frac{-10 \cdot \sqrt{3}}{3} \text{ or } x = \frac{10 \cdot \sqrt{3}}{3}$$

$$\sqrt{\frac{100}{3}}$$

$$\frac{10 \cdot \sqrt{3}}{3}$$



Volume formula in terms of radius $V(h) = \frac{100 \cdot h \cdot \pi}{3} - \frac{h^3 \cdot \pi}{3}$

Feasible Domain of h : $0 \leq h \leq 10$

$$\frac{dV}{dh} = \frac{100 \cdot \pi}{3} - h^2 \cdot \pi$$

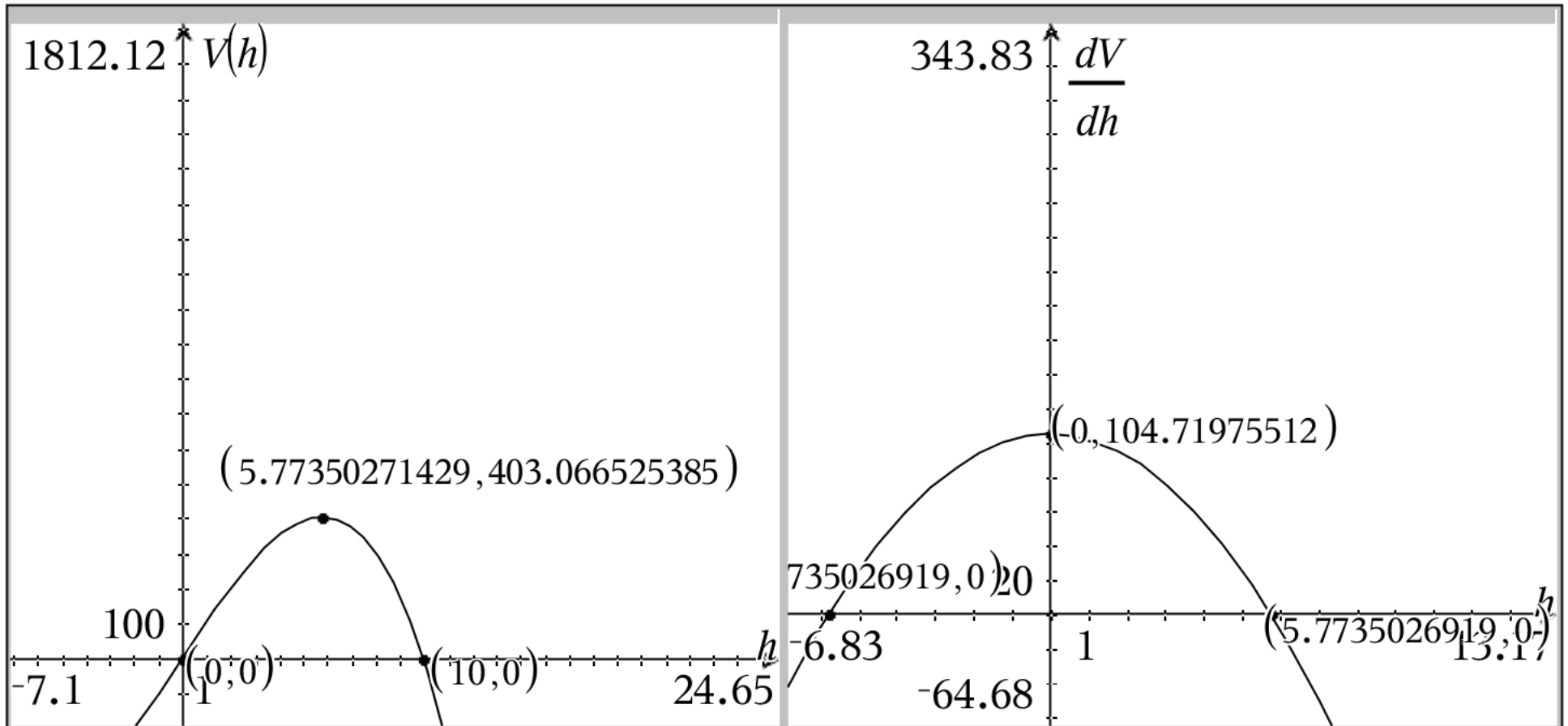
The maximum volume for this scenario occurs at $h = \sqrt{\frac{100}{3}} \approx 5.7735$

The radius that maximizes the volume is $r = \frac{10 \cdot \sqrt{6}}{3} \approx 8.16497$

The maximum volume is $V\left(\sqrt{\frac{100}{3}}\right) = \frac{2000 \cdot \pi \cdot \sqrt{3}}{27} \approx 403.067$

$$\frac{dV}{dh} \text{ at } x = 3$$

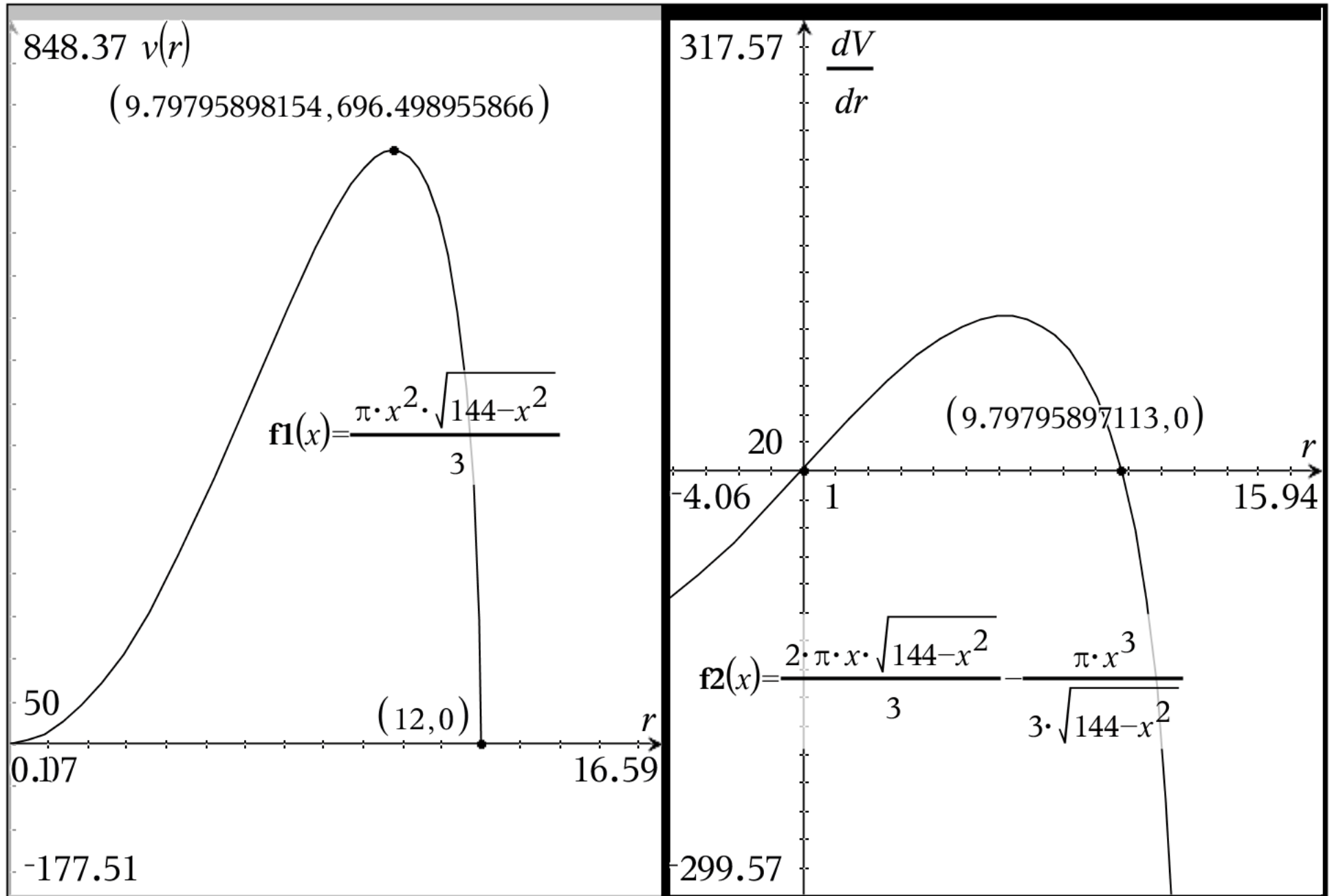
$$\frac{dV}{dh} = \frac{73 \cdot \pi}{3} \approx 76.4454$$



Volume formula in terms of radius $V(h) = \frac{100 \cdot h \cdot \pi}{3} - \frac{h^3 \cdot \pi}{3}$

Feasible Domain of h : $0 \leq h \leq 10$ $\frac{dV}{dh} = \frac{100 \cdot \pi}{3} - h^2 \cdot \pi$

cone slant height 12



$$f1(x) := \frac{\pi \cdot x^2 \cdot \sqrt{144 - x^2}}{3}$$

Done

$$\frac{d}{dx}(f1(x))$$

$$\frac{2 \cdot \pi \cdot x \cdot \sqrt{144 - x^2}}{3} - \frac{\pi \cdot x^3}{3 \cdot \sqrt{144 - x^2}}$$

$$f2(x) := \frac{2 \cdot \pi \cdot x \cdot \sqrt{144 - x^2}}{3} - \frac{\pi \cdot x^3}{3 \cdot \sqrt{144 - x^2}}$$

Done

$$\text{solve}(f2(x)=0, x)$$

$$x = -4 \cdot \sqrt{6} \text{ or } x = 0 \text{ or } x = 4 \cdot \sqrt{6}$$

$$\text{solve}(f2(x)=0., x)$$

$$x = -9.79796 \text{ or } x = 0. \text{ or } x = 9.79796$$

$$f1(9.798)$$

696.499

$$f5(x) := \sqrt{144 - x^2}$$

Done

□

$$V(r) = \frac{\pi \cdot r^2 \cdot \sqrt{144 - r^2}}{3}$$

Feasible Domain of r : $0 \leq r \leq 12$

$$\frac{dV}{dr} = \frac{2 \cdot \pi \cdot r \cdot \sqrt{144 - r^2}}{3} - \frac{\pi \cdot r^3}{3 \cdot \sqrt{144 - r^2}}$$

If $V(r)$ has a maximum over its feasible domain then it occurs at one of the following:

$V(0)$, $V(12)$ these are values of volume at the extremes

Or at the solutions of $\frac{dV}{dr} = 0$

$$V(0) = 0$$

$$V(12) = 0$$

or at $r = \sqrt{48} = 4 \cdot \sqrt{3} = 6.9282$ h at $r = \sqrt{96} = 4 \cdot \sqrt{6} = 9.79796$

$$V(\sqrt{48}) = 64 \cdot \pi \cdot \sqrt{6} = 492.499$$

Volume formula in terms of radius $V(r) = \frac{\pi \cdot r^2 \cdot \sqrt{144 - r^2}}{3}$

Feasible Domain of r : $0 \leq r \leq 10$

$$\frac{dV}{dr} = \frac{2 \cdot \pi \cdot r \cdot \sqrt{144 - r^2}}{3} - \frac{\pi \cdot r^3}{3 \cdot \sqrt{144 - r^2}}$$

The maximum volume for this scenario occurs at $r = \sqrt{48} \approx 6.9282$

The height that maximizes the volume is $h = \sqrt{48} = 4 \cdot \sqrt{6} \approx 9.79796$

The maximum volume is $V(\sqrt{48}) = 64 \cdot \pi \cdot \sqrt{6} = 492.499$

$$f3(x) := \frac{\pi}{3} \cdot (144 - x^2) \cdot x$$

Done

$$\frac{d}{dx}(f3(x))$$

$$-\pi \cdot (x^2 - 48)$$

$$\text{expand}(f3(x))$$

$$48 \cdot \pi \cdot x - \frac{\pi \cdot x^3}{3}$$

$$f4(x) := \frac{d}{dx}(f3(x))$$

Done

$$\text{expand}(f4(x))$$

$$48 \cdot \pi - \pi \cdot x^2$$

$$\text{solve}(f4(x)=0, x)$$

$$x = -4 \cdot \sqrt{3} \text{ or } x = 4 \cdot \sqrt{3}$$

$$\sqrt{48}$$

$$4 \cdot \sqrt{3}$$



Volume formula in terms of radius $V(h) = 48 \cdot h \cdot \pi - \frac{h^3 \cdot \pi}{3}$

Feasible Domain of h: $0 \leq h \leq 10$

$$\frac{dV}{dh} = 48 \cdot \pi - h^2 \cdot \pi$$

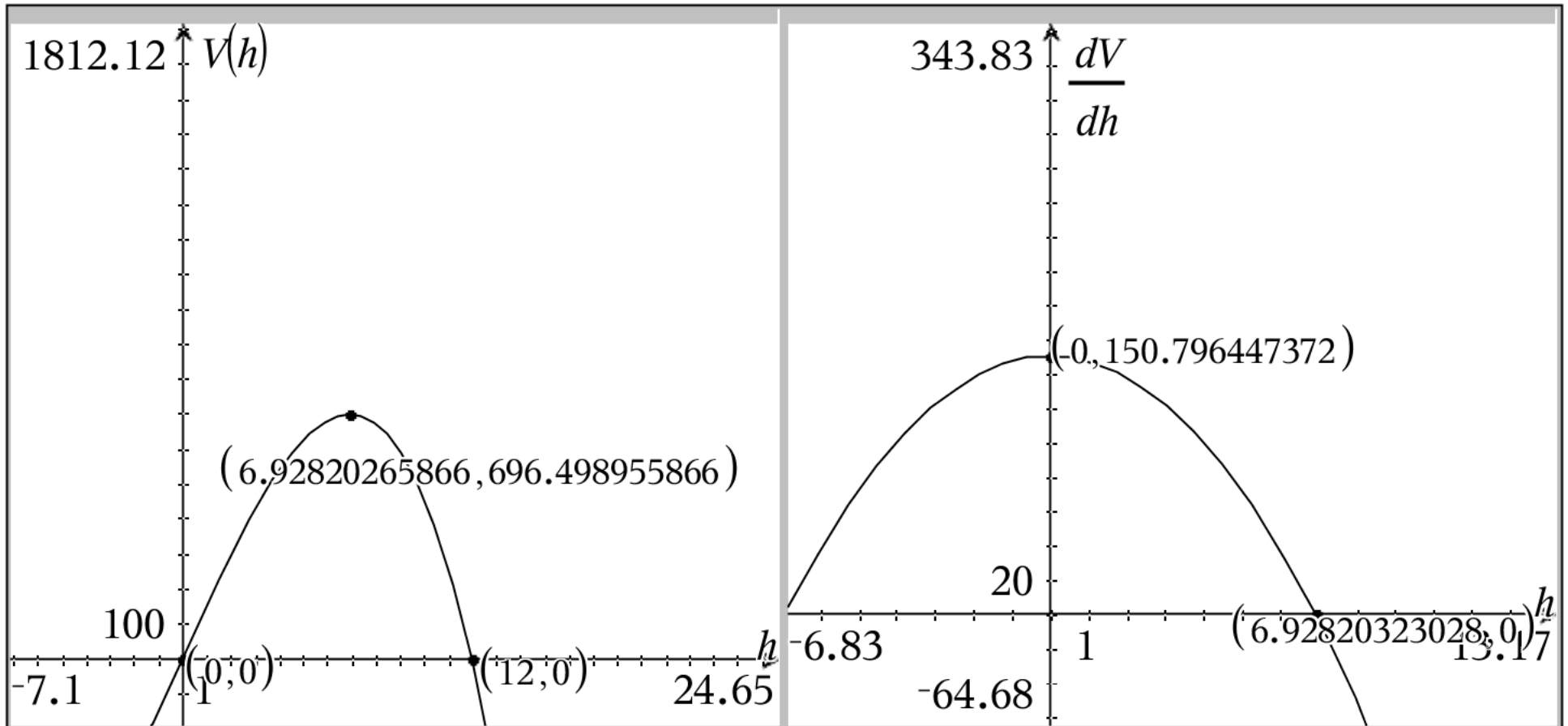
The maximum volume for this scenario occurs at $h = \sqrt{96} \approx 9.79796$

The radius that maximizes the volume is $r = 4 \cdot \sqrt{6} \approx 6.9282$

The maximum volume is $V(\sqrt{96}) = 64 \cdot \pi \cdot \sqrt{6} \approx 492.499$

$$\frac{dV}{dh} \text{ at } h = 4$$

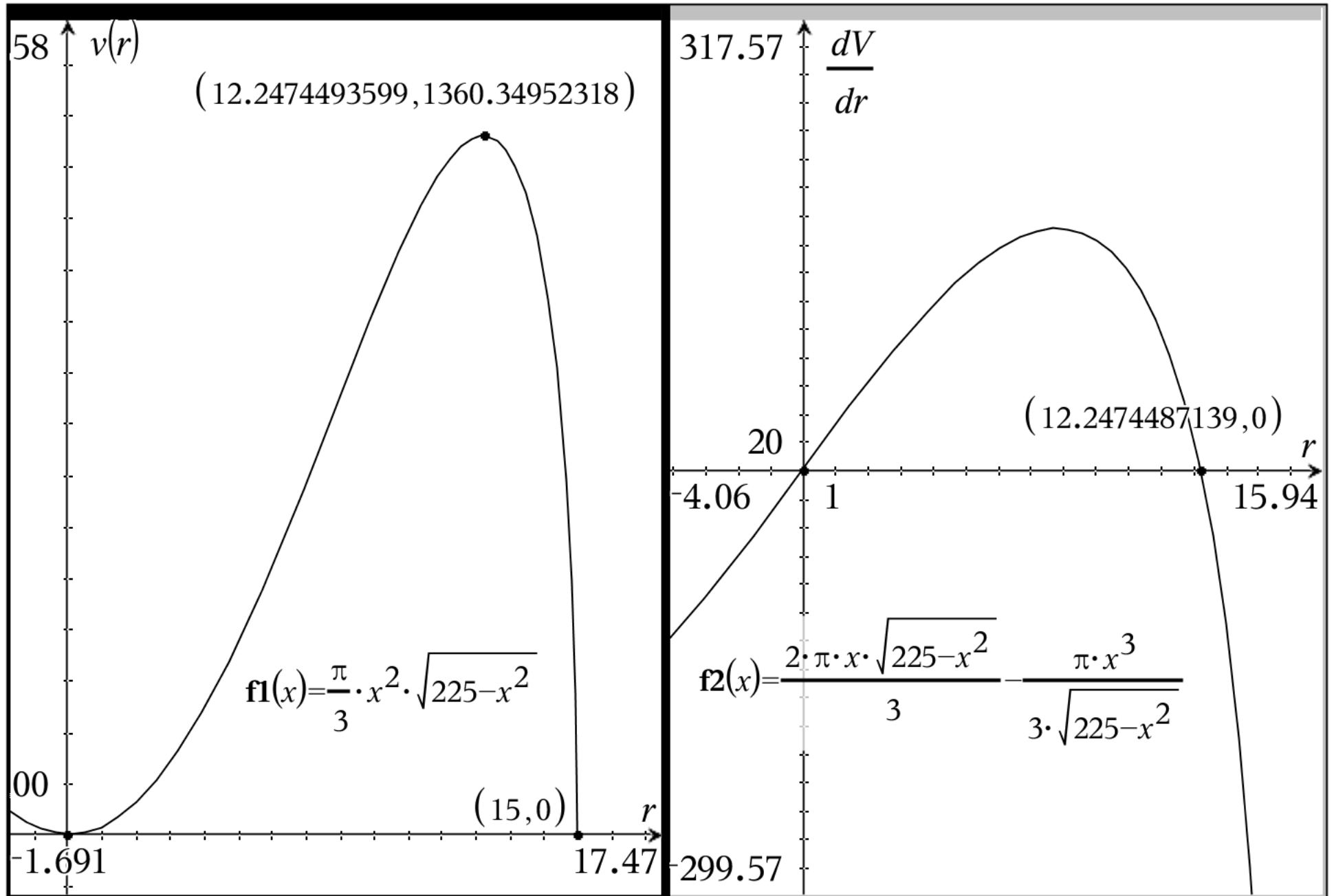
$$\frac{dV}{dh} = 32 \cdot \pi \approx 100.531$$



Volume formula in terms of radius $V(h) = 48 \cdot h \cdot \pi - \frac{h^3 \cdot \pi}{3}$

Feasible Domain of h : $0 \leq h \leq 12$ $\frac{dV}{dh} = 48 \cdot \pi - h^2 \cdot \pi$

cone slant height 15



$$f1(x) = \frac{\pi \cdot x^2 \cdot \sqrt{225 - x^2}}{3}$$

$$f2(x) := \frac{2 \cdot \pi \cdot x \cdot \sqrt{225 - x^2}}{3} - \frac{\pi \cdot x^3}{3 \cdot \sqrt{225 - x^2}}$$

Done

$$\text{solve}(f2(x)=0, x) \quad x = -5\sqrt{6} \text{ or } x = 0 \text{ or } x = 5\sqrt{6}$$

$$\text{solve}(f2(x)=0., x) \quad x = -12.2474 \text{ or } x = 0. \text{ or } x = 12.2474$$

$$f1(12.247) = 1360.35$$

$$f3(x) := \sqrt{225 - x^2}$$

Done

$$f3(12.247) = 8.66089$$

□

$$V(r) = \frac{\pi \cdot r^2 \cdot \sqrt{225-r^2}}{3}$$

Feasible Domain of r: $0 \leq r \leq 15$

$$\frac{dV}{dr} = \frac{2 \cdot \pi \cdot r \cdot \sqrt{225-r^2}}{3} - \frac{\pi \cdot r^3}{3 \cdot \sqrt{225-r^2}}$$

If $V(r)$ has a maximum over its feasible domain then it occurs at one of the following:

$V(0)$, $V(15)$ these are values of volume at the extremes

Or at the solutions of $\frac{dV}{dr} = 0$

$$V(0) = 0$$

$$V(15) = 0$$

$$\text{or at } x = \sqrt{150} = 5 \cdot \sqrt{6} = 12.2474$$

$$V(\sqrt{150}) = 250 \cdot \pi \cdot \sqrt{3} = 1360.35$$

Volume formula in terms of radius $V(r) = \frac{\pi \cdot r^2 \cdot \sqrt{225-r^2}}{3}$

Feasible Domain of r : $0 \leq r \leq 15$

$$\frac{dV}{dr} = \frac{2 \cdot \pi \cdot r \cdot \sqrt{225-r^2}}{3} - \frac{\pi \cdot r^3}{3 \cdot \sqrt{225-r^2}}$$

The maximum volume for this scenario occurs at $r = \sqrt{150} \approx 12.2474$

The height that maximizes the volume is $h = \sqrt{75} = 5 \cdot \sqrt{3} \approx 961.912$

The maximum volume is $V(\sqrt{150}) = 250 \cdot \pi \cdot \sqrt{3} = 1360.35$

$$f3(x) := \frac{\pi}{3} \cdot (225 - x^2) \cdot h$$

Done

$$f3(x) := \frac{\pi}{3} \cdot (225 - x^2) \cdot x$$

Done

$$\frac{d}{dx}(f3(x))$$

$$-\pi \cdot (x^2 - 75)$$

$$\text{expand}(f3(x))$$

$$75 \cdot \pi \cdot x - \frac{\pi \cdot x^3}{3}$$

$$f4(x) := \frac{d}{dx}(f3(x))$$

Done

$$\text{expand}(f4(x))$$

$$75 \cdot \pi - \pi \cdot x^2$$

$$\text{solve}(f4(x)=0, x)$$

$$x = -5\sqrt{3} \text{ or } x = 5\sqrt{3}$$

$$\sqrt{75}$$

$$5 \cdot \sqrt{3}$$

□

Volume formula in terms of radius $V(h) = 75 \cdot h \cdot \pi - \frac{h^3 \cdot \pi}{3}$

Feasible Domain of h: $0 \leq h \leq 15$

$$\frac{dV}{dh} = 75 \cdot \pi - h^2 \cdot \pi$$

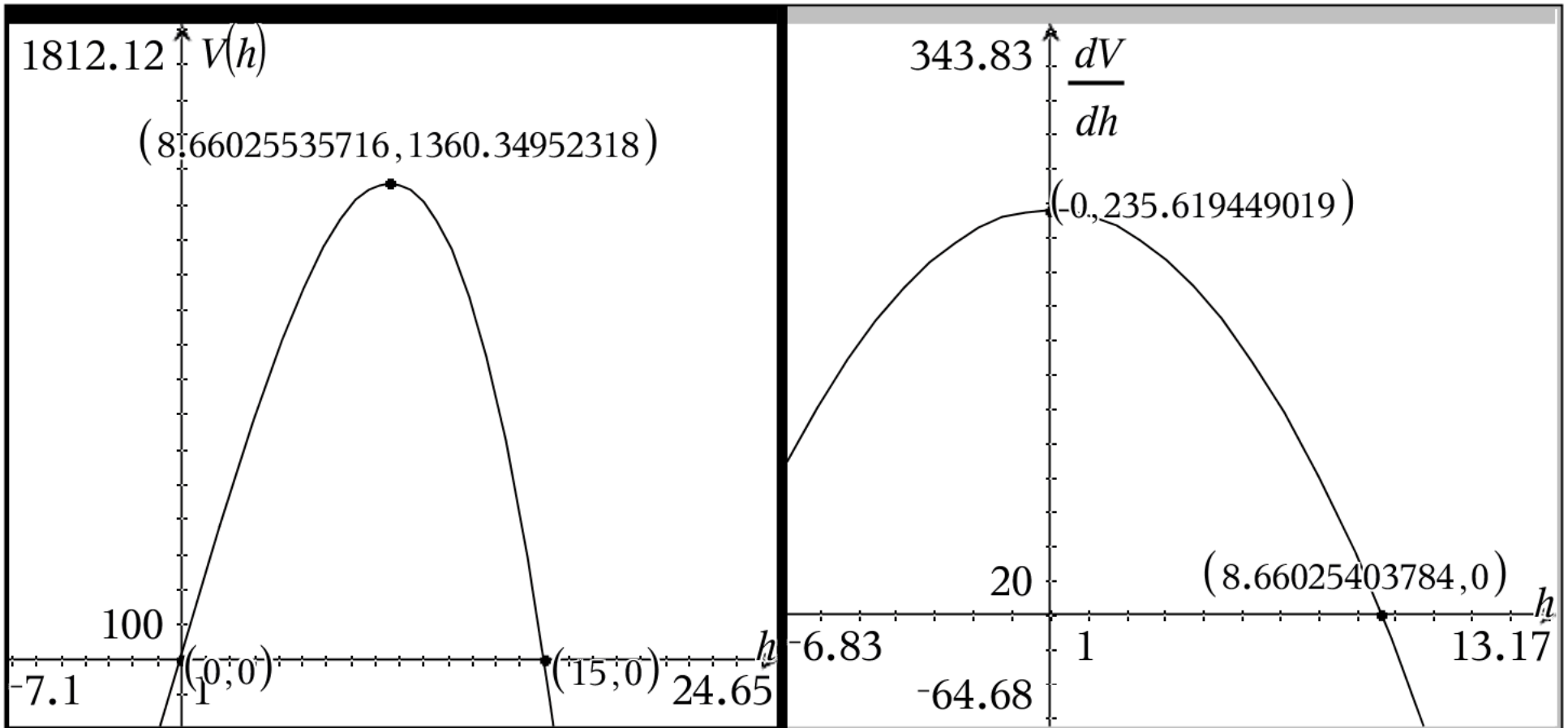
The maximum volume for this scenario occurs at $h = \sqrt{75} \approx 8.66025$

The radius that maximizes the volume is $r = \sqrt{150} = 5 \cdot \sqrt{6} \approx 12.2474$

The maximum volume is $V(\sqrt{75}) = 250 \cdot \pi \cdot \sqrt{3} \approx 1360.35$

$$\frac{dV}{dh} \text{ at } r = 5 \quad h = 10 \cdot \sqrt{2}$$

$$\frac{dV}{dh} = -125 \cdot \pi \approx -392.699$$



Volume formula in terms of radius $V(h) = 75 \cdot h \cdot \pi - \frac{h^3 \cdot \pi}{3}$

Feasible Domain of h : $0 \leq h \leq 15$ $\frac{dV}{dh} = 75 \cdot \pi - h^2 \cdot \pi$