

# VERSION 24

Given  $\frac{dV}{dt} = -24 \frac{\text{cm}^3}{\text{min}}$

$\frac{dr}{dt} = 0$

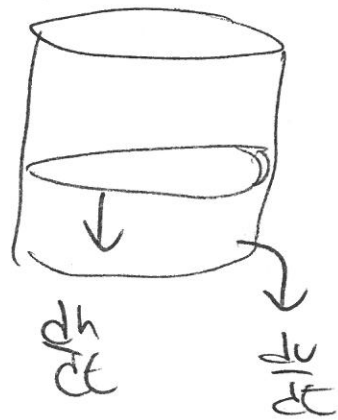
$r = ?$   $r = 20 \text{ cm}$

$d = 40 \text{ cm}$

$\frac{dh}{dt} = ?$

$$V = \pi r^2 h$$

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$$



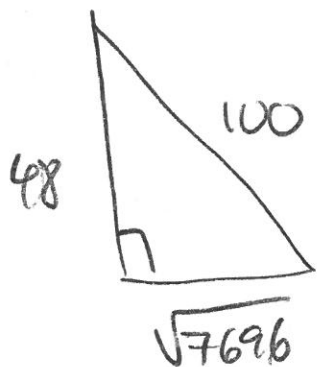
$$-24 = 2\pi(40)h(0) + \pi(40)^2 \frac{dh}{dt}$$

$$-24 = 0 + 20^2 \pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-24 \text{ cm}}{20^2 \pi \text{ Sec}} = \frac{-24 \text{ cm}}{400\pi \text{ Sec}} = \frac{-3}{50\pi} \frac{\text{cm}}{\text{Sec}}$$

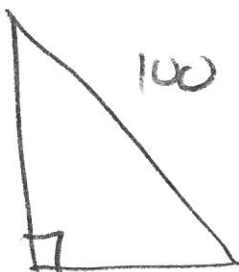
$$\approx -0.0191 \frac{\text{cm}}{\text{Sec}}$$

VERSION 24 #2 cont



$t=0$

$\frac{dy}{dt}?$   
↓  
 $y?$



$t > 0$



$$\frac{dz}{dt} = 0$$

$$\frac{dx}{dt} = 3$$

$$x(t) = \sqrt{7696} + 3t$$

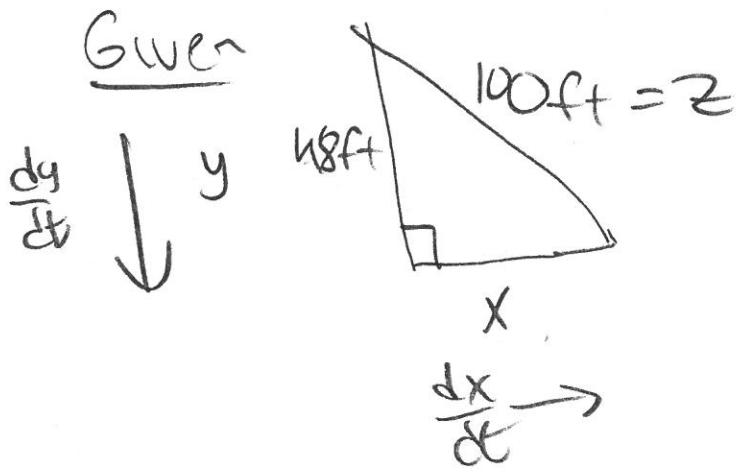
$$y(t) = \sqrt{100^2 - (\sqrt{7696} + 3t)^2}$$

Recall  $\frac{dy}{dt} = \frac{-3(x(t))}{y(t)} = \frac{-3(\sqrt{7696} + 3t)}{\sqrt{100^2 - (\sqrt{7696} + 3t)^2}}$

$$\left. \frac{dy}{dt} \right|_{t=4} = \frac{-3(x(4))}{y(4)} = \frac{-3(\sqrt{7696} + 12)}{\sqrt{100^2 - (\sqrt{7696} + 12)^2}}$$

$$\approx -40.505 \frac{\text{ft}}{\text{Sec}}$$

24 Version #2



$x = ?$   
 $y = 48ft$   
 $z = 100$

$\frac{dx}{dt} = +3 \frac{ft}{sec}$   
 $\frac{dy}{dt} = ?$   
 $\frac{dz}{dt} = 0$

$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = z \frac{dz}{dt}$$

$$x(t)(3) + y(t) \frac{dy}{dt} = 100(0)$$

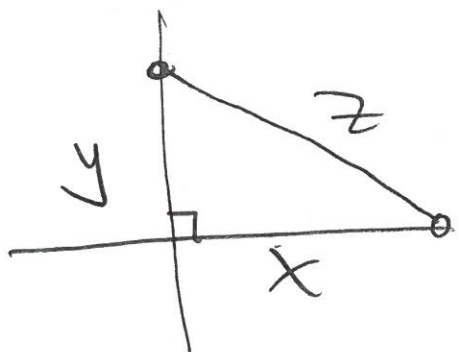
$$3x(t) + \frac{dy}{dt} y(t) = 0$$

$$\frac{dy}{dt} [y(t)] = -3x(t)$$

$$\boxed{\frac{dy}{dt} = \frac{-3x(t)}{y(t)}}$$

VERSION 24

#3



Given

$$x(t) = 45t$$

$$y(t) = 56t$$

$$z(t) = \sqrt{(45t)^2 + (56t)^2}$$

$$\frac{dx}{dt} = 45 \text{ mph}$$

$$\frac{dy}{dt} = 56 \text{ mph}$$

$$x^2 + y^2 = z^2 \longrightarrow x \frac{dx}{dt} + y \frac{dy}{dt} = z \frac{dz}{dt}$$

$$\longrightarrow (45t)45 + (56t)56 = \sqrt{(45t)^2 + (56t)^2} \frac{dz}{dt}$$

$$\longrightarrow \boxed{\frac{45^2 t + 56^2 t}{\sqrt{45^2 t^2 + 56^2 t^2}} = \frac{dz}{dt}}$$

$$\begin{aligned} \left. \frac{dz}{dt} \right|_{t=1.5} &= \frac{45^2(1.5) + 56^2(1.5)}{\sqrt{45^2(1.5)^2 + 56^2(1.5)^2}} \\ &= \frac{7741.5}{\sqrt{11612.25}} \end{aligned}$$

$$\boxed{\frac{dz}{dt} \approx 71.840 \text{ mph}}$$

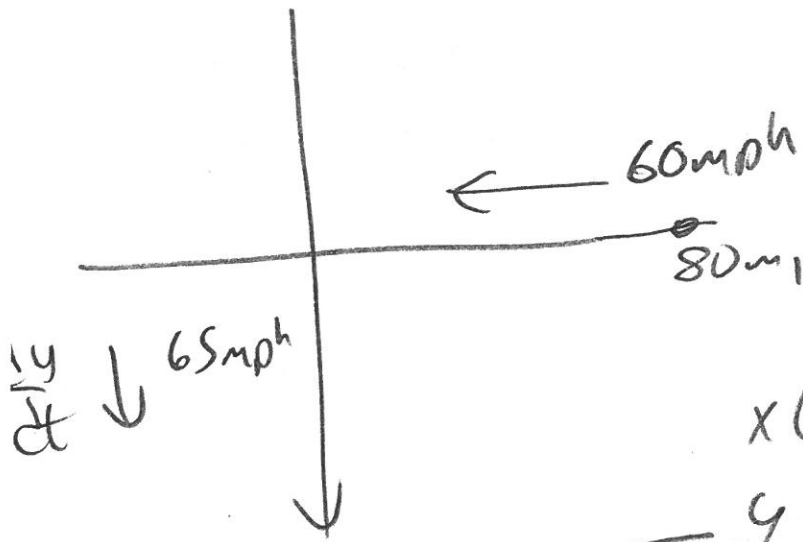
Question 24 #4 cont

$$\frac{dz}{dt} \Big|_{t=\frac{4}{3}} = \frac{60^2 \left(\frac{4}{3} - \frac{4}{3}\right) + 65^2 \left(\frac{4}{3} + \frac{1}{2}\right)}{\sqrt{60^2 \left(\frac{4}{3} - \frac{4}{3}\right)^2 + 65^2 \left(\frac{4}{3} + \frac{1}{2}\right)^2}}$$

$$= \frac{0 + 65^2 \left(\frac{4}{3}\right)}{\sqrt{0 + 65^2 \left(\frac{4}{3}\right)^2}}$$

$$= \boxed{65 \text{ mph}}$$

VERSION 24 (#4)



$$t_{\text{semi}} = t$$

$$t_{\text{car}} = t + \frac{1}{2}$$

when  $t = \frac{4}{3}$  hr  
semi at restaurant

$$x(t) = 80 - 60t$$

$$y(t) = 65(t + \frac{1}{2})$$

$$z(t) = \sqrt{60^2(\frac{4}{3} - t)^2 + 65^2(t + \frac{1}{2})^2}$$

$$z(t) = \sqrt{x(t)^2 + y(t)^2}$$

$$= \sqrt{(80 - 60t)^2 + (65(t + \frac{1}{2}))^2}$$

$$\frac{dx}{dt} = -60 \text{ mph}$$

$$\frac{dy}{dt} = 65 \text{ mph}$$

$$z^2 = x^2 + y^2$$

$$\frac{dz}{dt} z = x \frac{dx}{dt} + y \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{(80 - 60t)60 + 65(t + \frac{1}{2})65}{\sqrt{60^2(\frac{4}{3} - t)^2 + 65^2(t + \frac{1}{2})^2}}$$

$$= \frac{60^2(\frac{4}{3} - t) + 65^2(t + \frac{1}{2})}{\sqrt{60^2(\frac{4}{3} - t)^2 + 65^2(t + \frac{1}{2})^2}}$$