

EXAMPLE 1: Consider a right triangle which is changing shape in the following way. The horizontal leg is increasing at the rate of  $5 \text{ in. /min.}$  and the vertical leg is decreasing at the rate of  $6 \text{ in. /min.}$  At what rate is the hypotenuse changing when the horizontal leg is  $12 \text{ in.}$  and the vertical leg is  $9 \text{ in.}$  ?

In the list of Related Rates Problems which follows, most problems are average and a few are somewhat challenging.

- PROBLEM 1 : The edge of a square is increasing at the rate of  $3 \text{ cm/sec.}$  At what rate is the square's

- a.) perimeter changing
- b.) area changing

when the edge of the square is  $10 \text{ cm.}$  ?

- PROBLEM 2 : The length of a rectangle is increasing at the rate of  $4 \text{ ft/hr.}$  and the width of the rectangle is decreasing at the rate of  $3 \text{ ft/hr.}$  At what rate is the rectangle's

- a.) perimeter changing
- b.) area changing

when the length is  $8 \text{ ft.}$  and the width is  $5 \text{ ft.}$  ?

- PROBLEM 3 : Leg one of a right triangle is decreasing at the rate of  $5 \text{ in/sec.}$  and leg two of the right triangle is increasing at the rate of  $7 \text{ in/sec.}$  At what rate is the triangle's

- a.) hypotenuse changing
- b.) perimeter changing
- c.) area changing

when leg one is  $8 \text{ in.}$  and leg two is  $6 \text{ in.}$  ?

- PROBLEM 4 : The radius of a circular oil slick on the surface of a pond is increasing at the rate of  $10 \text{ meters}/\text{min}$ . At what rate is the circle's
  - a.) circumference changing
  - b.) area changing

when the radius of the oil slick is  $20 \text{ m}$ . ?

- PROBLEM 5 : A big block of ice is in the shape of a perfect cube. As it melts, each edge of the cube is decreasing at the rate of  $2 \text{ cm}/\text{min}$ . At what rate is the ice cube's
  - a.) surface area changing
  - b.) volume changing

when the edge of the ice cube is  $80 \text{ cm}$ . ?

- PROBLEM 6 : A ladder 13 feet long is leaning against a high wall. If the base of the ladder is pushed toward the wall at the rate of  $2 \text{ ft}/\text{sec}$ ., at what rate is the top of the ladder moving up the wall when the base of the ladder is
  - a.) 5 feet
  - b.) 1 foot

from the wall ?

- PROBLEM 7 : The radius of a large sphere is increasing at the rate of  $3 \text{ ft}/\text{hr}$ . At what rate is the sphere's
  - a.) surface area changing
  - b.) volume changing

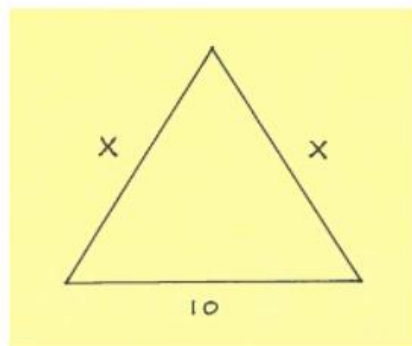
when the radius of the sphere is  $10 \text{ ft}$ . ?

- PROBLEM 8 : Consider a closed right circular cylinder of base radius  $r$  *cm.* and height  $h$  *cm.* If the radius of the cylinder is increasing at the rate of  $5$  *cm/hr.* and the height of the cylinder is decreasing at the rate of  $4$  *cm/hr.* , at what rate is the cylinder's

- a.) surface area changing
- b.) volume changing

when the radius of the cylinder is  $20$  *cm.* and the height of the cylinder is  $12$  *cm.* ?

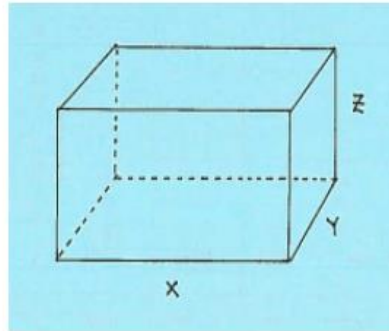
- PROBLEM 9 : Consider the given isosceles triangle of base 10 inches and side lengths  $x$  inches. If  $x$  is increasing at the rate of  $4$  *in/min.*, at what rate is the triangle's



- a.) perimeter changing
- b.) height changing
- c.) area changing

when  $x = 13$  *in.* ?

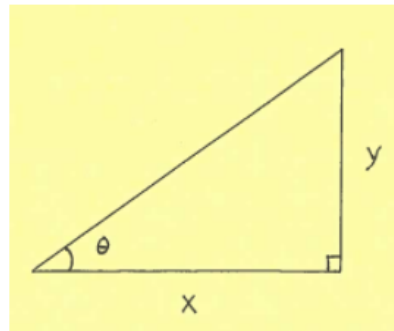
- PROBLEM 10 : Consider the given closed rectangular box with dimensions  $x$  feet by  $y$  feet by  $z$  feet. Assume that  $x$  is increasing at the rate of  $4 \text{ ft/hr.}$ ,  $y$  is decreasing at the rate of  $6 \text{ ft/hr.}$ , and  $z$  is increasing at the rate of  $3 \text{ ft/hr.}$  At what rate is the box's



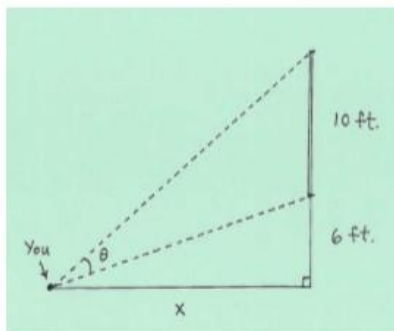
- surface area changing
- volume changing
- main diagonal changing

when  $x = 5 \text{ ft.}$ ,  $y = 4 \text{ ft.}$ , and  $z = 6 \text{ ft.}$  ?

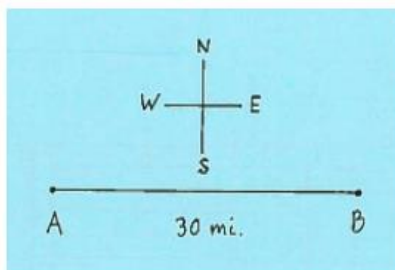
- PROBLEM 11 : The volume of a large spherical balloon is increasing at the rate of  $64\pi \text{ meters}^3/\text{hr.} \approx 201.06 \text{ meters}^3/\text{hr.}$  At what rate is the balloon's surface area changing when the radius of the balloon is  $2 \text{ m.}$  ?
- PROBLEM 12 : The surface area of a cube is increasing at the rate of  $600 \text{ in}^2/\text{hr.}$  At what rate is the cube's volume changing when the edge of the cube is  $10 \text{ in.}$  ?
- PROBLEM 13 : Consider the given right triangle with legs of length  $x \text{ cm.}$  and  $y \text{ cm.}$  and angle  $\theta$  radians. If  $x$  is decreasing at the rate of  $3 \text{ cm/min.}$  and  $y$  is increasing at the rate of  $4 \text{ cm/min.}$ , at what rate is angle  $\theta$  changing when  $x = 5 \text{ cm.}$  and  $y = 2 \text{ cm.}$  ?



- PROBLEM 14 : You are sitting  $x$  ft. from a wall and watching a movie screen which is 10 feet high and is 6 feet above the floor. Your viewing angle is  $\theta$  radians. (See the side view diagram below.)

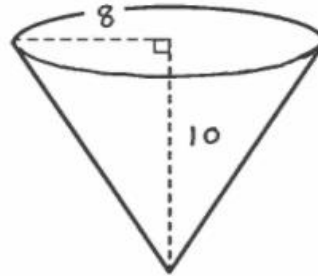


- Write your viewing angle  $\theta$  as a function of  $x$ .
  - If  $x$  is increasing at the rate of  $10 \text{ ft/min.}$ , at what rate is  $\theta$  changing when
    - $x = 8 \text{ ft.}$  ?
    - $x = 20 \text{ ft.}$  ?
- PROBLEM 15 : Car B starts 30 miles directly east of car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph. At what rate is the distance between the cars changing after
    - $t = 1/5 \text{ hr.}$  ?
    - $t = 1/3 \text{ hr.}$  ?



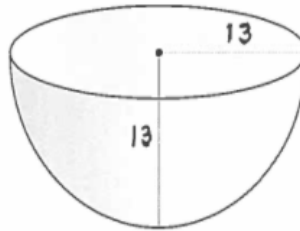
- PROBLEM 16 : An open right circular conical tank (vertex down) has height 10 meters and base radius 8 meters. Water begins flowing into the tank at the rate of  $\pi \text{ meters}^3/\text{min}$ . At what rate is the depth  $h$  of the water in the tank changing when

- a.)  $h = 1 \text{ m}$ . ?
- b.)  $h = 9 \text{ m}$ . ?

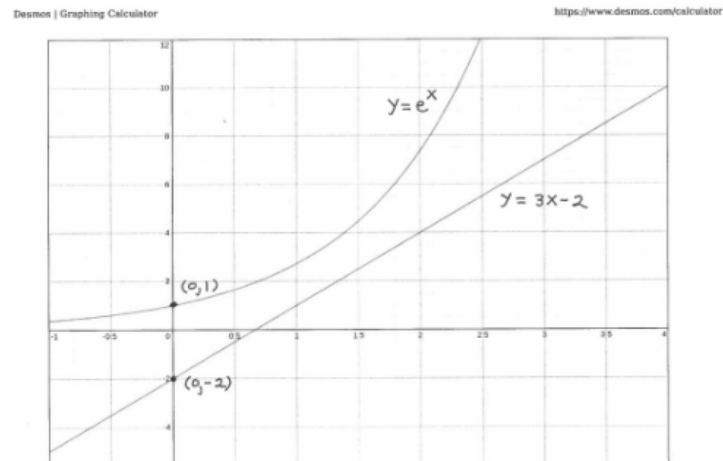


- PROBLEM 17 : An open hemispherical tank has radius 13 feet. Oil begins flowing into the tank in such a way that the depth  $h$  of the oil in the tank changes at the rate of  $3 \text{ ft/hr}$ . At what rate is the top circular surface area of the oil changing when the depth of oil is

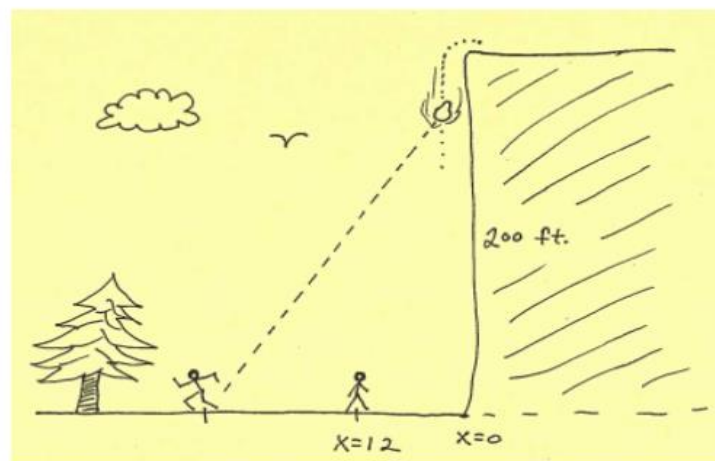
- a.)  $h = 1 \text{ ft}$ . ?
- b.)  $h = 8 \text{ ft}$ . ?



- PROBLEM 18 : Car 1 starts on the graph of  $y = e^x$  at the point  $(0, 1)$ , and car 2 starts on the graph of  $y = 3x - 2$  at the point  $(0, -2)$  and distance is measured in miles. If both cars start moving to the right at the same time in such a way that  $\frac{dx}{dt} = 1 \text{ mile/min.}$ , at what rate is the distance between the cars changing when
  - $t = 1 \text{ min.}$  ?
  - $t = 3 \text{ min.}$  ?



- PROBLEM 19 : You are standing 12 feet from the base of a 200-ft. cliff. As a boulder rolls off the cliff, you begin running away at  $10 \text{ ft/sec.}$  At what rate is the distance between you and the boulder changing after
  - $t = 1 \text{ sec.}$  ?
  - $t = 3 \text{ sec.}$  ?



I would do each problem set on a separate piece of paper and follow the suggestions below

- 1.) Read the problem slowly and carefully.
- 2.) Draw an appropriate sketch.
- 3.) Introduce and define appropriate variables. Use variables if quantities are changing. Use constants if quantities are not changing.
- 4.) Read the problem again.
- 5.) Clearly label the sketch using your variables.
- 6.) State what information is given in the problem.
- 7.) State what information is to be determined or found.
- 8.) Use a given equation or create an appropriate equation relating the given variables.
- 9.) Differentiate this equation with respect to the time variable  $t$ .
- 10.) Plug in the given rates and numbers to the differentiated equation.
- 11.) Solve for the unknown rate.
- 12.) Put proper units on your final answer.

I chose this practice set because it has many of the popular related rate problems Calculus 1 has to offer when considering geometric applications and is fortunate that the website that I found these problems on has links to detailed solutions. I have not as of yet verified those solutions, but you are welcome to the link <https://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/relatedratesdirectory/RelatedRates.html>