

PRECALCULUS ADVANCED  
WORKSHEET ON DOUBLE-ANGLE IDENTITIES

Use a double-angle formula to rewrite the expression.

$$1. 6 \sin x \cos x = \boxed{3 \sin 2x}$$

$$3(2 \sin x \cos x)$$

$$2. 5 - 10 \sin^2 x = \boxed{5 \cos 2x}$$

$$5(1 - 2 \sin^2 x)$$

$$3. \cos^2(5x) - \sin^2(5x) = \boxed{\cos 10x}$$

$$5(2) = 10$$

$$4. 14 \sin(3x) \cos(3x) = \boxed{7 \sin 6x}$$

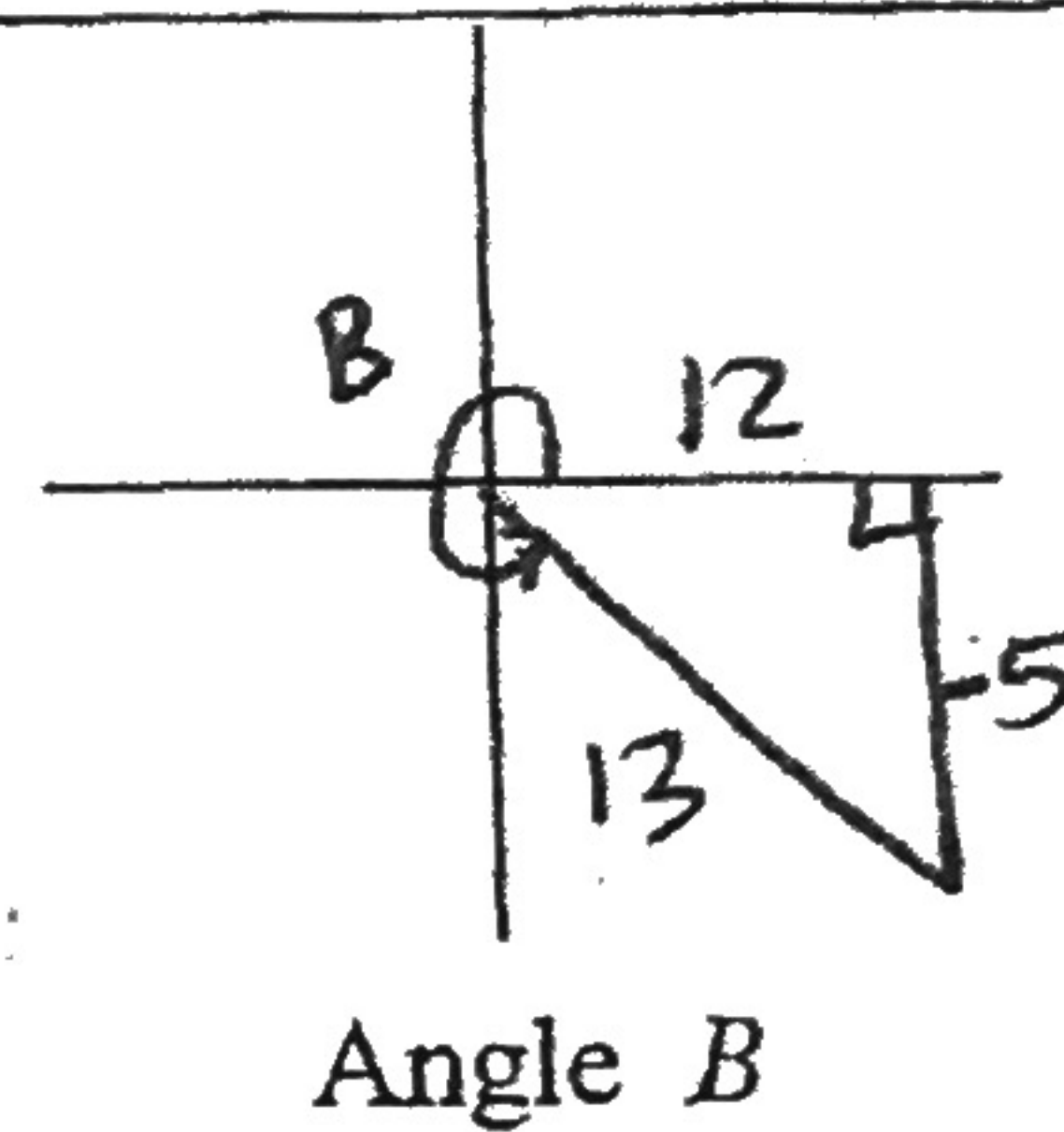
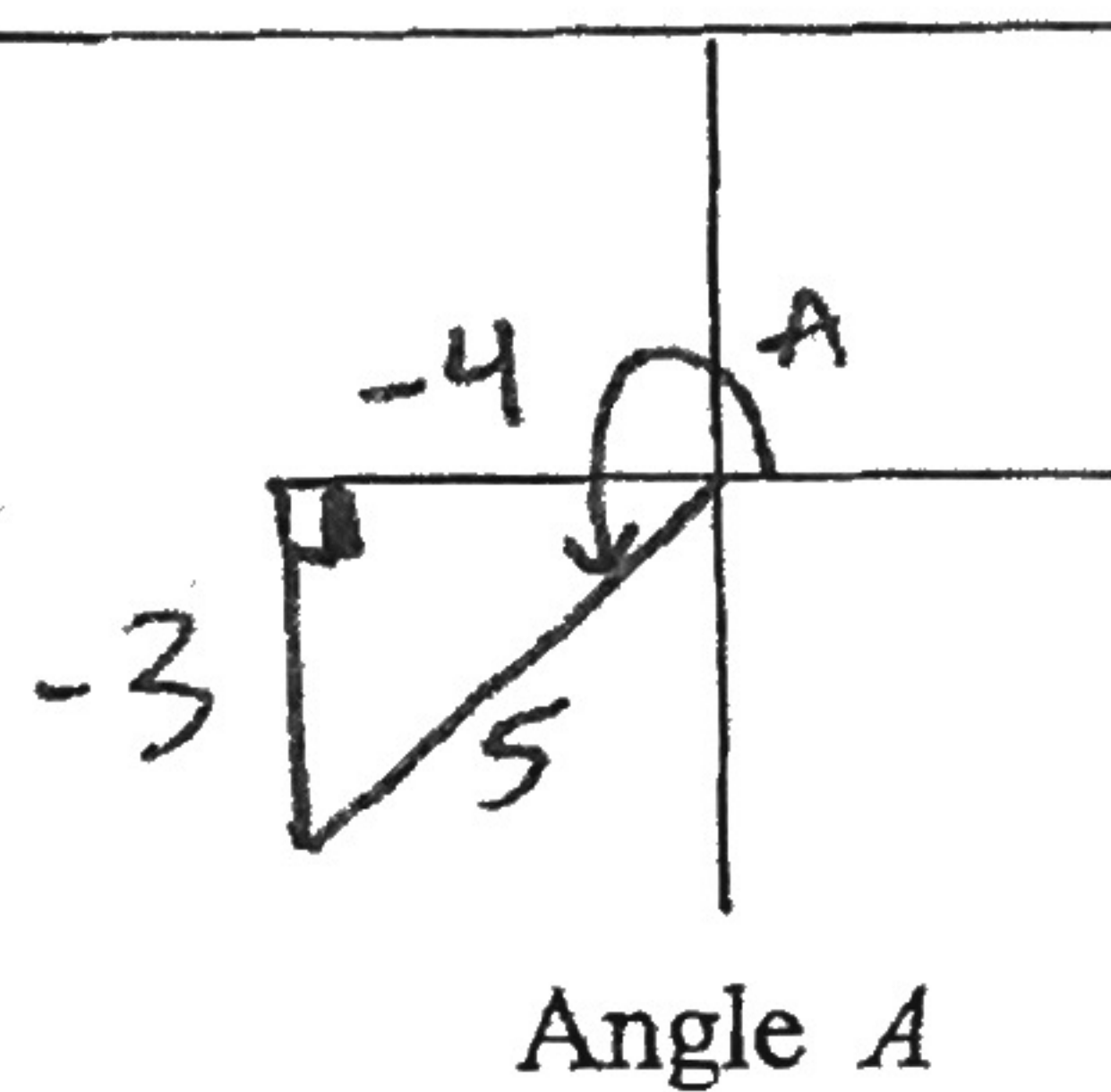
$$7(2 \sin(3x) \cos(3x))$$

Given:  $\sin A = -\frac{3}{5}$ ,  $\pi \leq A \leq \frac{3\pi}{2}$

$\cos B = \frac{12}{13}$ ,  $\frac{3\pi}{2} \leq B \leq 2\pi$

$$5. \sin(2B) = 2 \sin B \cos B$$

$$= 2 \left( \frac{-5}{13} \right) \left( \frac{12}{13} \right) = \boxed{\frac{-120}{169}}$$



$$6 \sqrt{144} = 24$$

$$7 \sqrt{25} = 35$$

$$12 \sqrt{3} = 20.78$$

$$13 \sqrt{25} = 65$$

$$14 \sqrt{25} = 70$$

$$15 \sqrt{25} = 75$$

$$16 \sqrt{25} = 80$$

$$17 \sqrt{25} = 85$$

$$18 \sqrt{25} = 90$$

$$19 \sqrt{25} = 95$$

$$20 \sqrt{25} = 100$$

$$6. \cos(2A) = 2 \cos^2 A - 1$$

$$= 2 \left( \frac{-4}{5} \right)^2 - 1$$

$$= 2 \left( \frac{16}{25} \right) - 1 = \frac{32}{25} - \frac{25}{25} = \boxed{\frac{7}{25}}$$

$$7. \tan(2B) = \frac{2 \tan B}{1 - \tan^2 B} = \frac{2 \left( \frac{-5}{12} \right)}{1 - \left( \frac{-5}{12} \right)^2} = \frac{-\frac{5}{6}}{1 - \frac{25}{144}} = \frac{-\frac{5}{6}}{\frac{119}{144}} = \frac{-5(24)}{119} = \boxed{\frac{-120}{119}}$$

$$8. \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \left( \frac{-4}{5} \right) \left( \frac{12}{13} \right) - \left( \frac{-3}{5} \right) \left( \frac{-5}{13} \right)$$

$$= \frac{-48}{65} - \frac{15}{65} = \boxed{\frac{-63}{65}}$$

$$9. \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$= \left( \frac{-3}{5} \right) \left( \frac{12}{13} \right) - \left( \frac{-4}{5} \right) \left( \frac{-5}{13} \right)$$

$$= \frac{-36}{65} - \frac{20}{65} = \boxed{\frac{-56}{65}}$$

$$10. \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{3}{4} + \frac{-5}{12}}{1 - \left( \frac{3}{4} \right) \left( \frac{-5}{12} \right)} = \frac{\frac{3}{4} - \frac{5}{12}}{1 + \frac{15}{48}} = \frac{\frac{48}{48} - \frac{50}{48}}{\frac{48+15}{48}} = \frac{-2}{63} = \boxed{\frac{-2}{63}}$$

Simplify using the sum and difference identities.

$$11. \cos(8x) \cos(2x) - \sin(8x) \sin(2x) =$$

$$= \cos(8x+2x)$$

$$= \cos(10x) = \boxed{\cos 10x}$$

$$12. \sin(140^\circ) \cos(30^\circ) - \cos(140^\circ) \sin(30^\circ) =$$

$$\sin(140-30)$$

$$\boxed{\sin 110^\circ}$$

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$$\frac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - \sqrt{3} + \sqrt{3} - 3} = \frac{-4 + 2\sqrt{3}}{-2} = \boxed{2 - \sqrt{3}}$$

Use a sum or difference identity to find the exact value.

$$13. \sin \frac{13\pi}{12} = \sin \left( \frac{4\pi}{12} + \frac{9\pi}{12} \right) = \sin \left( \frac{\pi}{3} + \frac{3\pi}{4} \right)$$

$$\sin \frac{\pi}{3} \cos \frac{3\pi}{4} + \cos \frac{\pi}{3} \sin \frac{3\pi}{4} = \left( \frac{\sqrt{3}}{2} \right) \left( -\frac{\sqrt{2}}{2} \right) + \left( \frac{1}{2} \right) \left( \frac{\sqrt{2}}{2} \right) = \boxed{\frac{-\sqrt{6} + \sqrt{2}}{4}}$$

$$14. \tan \frac{\pi}{12} = \tan \left( \frac{4\pi}{12} - \frac{3\pi}{12} \right) = \tan \left( \frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$\frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{4}} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}(1)} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$14. \cos(165^\circ) = \cos(135^\circ + 30^\circ)$$

$$\cos 135^\circ \cos 30^\circ - \sin 135^\circ \sin 30^\circ = \left( -\frac{\sqrt{2}}{2} \right) \left( \frac{\sqrt{3}}{2} \right) - \left( \frac{\sqrt{2}}{2} \right) \left( \frac{1}{2} \right) = \boxed{\frac{-\sqrt{6} - \sqrt{2}}{4}}$$

$$16. \sin(75^\circ) = \sin(30^\circ + 45^\circ)$$

$$\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ = \left( \frac{1}{2} \right) \left( \frac{\sqrt{2}}{2} \right) + \left( \frac{\sqrt{3}}{2} \right) \left( \frac{\sqrt{2}}{2} \right) = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$

Solve for  $x$ ,  $0 \leq x < 2\pi$ .

$$17. \sin(2x) - \sin x = 0$$

$$2 \sin x \cos x - \sin x = 0$$

$$\sin x (2 \cos x - 1) = 0$$

$$\sin x = 0 \quad 2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$\boxed{x = 0, \pi}$$

$$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$$

$$18. 4 \sin x \cos x = -1$$

$$2(2 \sin x \cos x) = -1 \quad x = \frac{7\pi}{12} + 2\pi n \quad 2x = \frac{11\pi}{6} + 2\pi n$$

$$2 \sin 2x = -1$$

$$\sin 2x = -\frac{1}{2}$$

$$\boxed{x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}}$$

$$19. \cos(2x) - \cos x = 0$$

$$2 \cos^2 x - 1 - \cos x = 0$$

$$2 \cos^2 x - \cos x - 1 = 0$$

$$(2 \cos x + 1)(\cos x - 1) = 0$$

$$2 \cos x + 1 = 0 \quad \cos x = 1$$

$$\cos x = -\frac{1}{2} \quad \boxed{x = 0}$$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

$$21. \sec^2 x + \sec x - 2 = 0$$

$$(\sec x + 2)(\sec x - 1) = 0$$

$$\sec x = -2 \quad \sec x = 1$$

$$\cos x = -\frac{1}{2} \quad \cos x = 1$$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

$$\boxed{x = 0}$$

$$20. \cos 2x - \sin x = 0$$

$$1 - 2 \sin^2 x - \sin x = 0$$

$$0 = 2 \sin^2 x + \sin x - 1$$

$$0 = (2 \sin x - 1)(\sin x + 1)$$

$$2 \sin x - 1 = 0 \quad \sin x = -1$$

$$\sin x = \frac{1}{2}$$

$$\boxed{x = \frac{3\pi}{2}}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$$

$$22. 2 \cos(3x) - \sqrt{3} = 0$$

$$\cos(3x) = \frac{\sqrt{3}}{2}$$

$$\cos x = \frac{\sqrt{3}}{2} \quad x = \frac{\pi}{6}, \frac{11\pi}{6}$$

$$3x = \frac{\pi}{6} + 2\pi n$$

$$3x = \frac{11\pi}{6} + 2\pi n$$

$$x = \frac{\pi}{18} + \frac{2\pi n}{3}$$

$$x = \frac{11\pi}{18} + \frac{2\pi n}{3}$$

$$x = \frac{\pi}{18} + \frac{12\pi n}{18}$$

$$x = \frac{11\pi}{18} + \frac{12\pi n}{18}$$

$$x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$$

$$\frac{36\pi}{18} = 2\pi$$

Solve for all values of  $x$  (general solution).

$$23. 2 \cos^2 x + 5 \cos x - 3 = 0$$

$$(2 \cos x - 1)(\cos x + 3) = 0$$

$$\cos x = \frac{1}{2}$$

$$\cos x = -3$$

$$x = \emptyset$$

$$\boxed{x = \left\{ \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n \right\}}$$

$$24. 2 \sin^2 x + \sin x = 0$$

$$\sin x (2 \sin x + 1) = 0$$

$$\sin x = 0 \quad \sin x = -\frac{1}{2}$$

$$x = 0, \pi$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\boxed{x = \left\{ 0 + 2\pi n, \pi + 2\pi n, \frac{7\pi}{6} + 2\pi n, \frac{11\pi}{6} + 2\pi n \right\}}$$

$$25. \tan^2 x - 3 = 0$$

$$\tan^2 x = 3$$

$$\tan x = \pm \sqrt{3}$$

$$\boxed{x = \left\{ \frac{\pi}{3} + 2\pi n, \frac{2\pi}{3} + 2\pi n, \frac{4\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n \right\}}$$