

SOLUTIONS MATH ANALYSIS HONORS

$$\textcircled{1} \frac{\tan^2 x - \sec^2 x}{\cos x} = -1 \sec x$$

Note $\sec^2 x - \tan^2 x = 1$

So $\tan^2 x - \sec^2 x = -1$

because $-1(\sec^2 x - \tan^2 x) = -1 \cdot 1$

$$-\sec^2 x + \tan^2 x = -1$$

$$\tan^2 x - \sec^2 x = -1$$

$$\frac{1}{\cos x} \cdot \frac{\tan^2 x - \sec^2 x}{\cos x} = -1 \sec x$$

$$\textcircled{A} \sec x \cdot \tan^2 x - \sec^2 x = -1 \sec x$$

$$\sec x \cdot -1(\sec^2 x - \tan^2 x) = -1 \sec x$$

$$\sec x \cdot -1(1) \textcircled{B} = -1 \sec x$$

$$\boxed{-1 \sec x = -1 \sec x}$$

- Ⓐ Reciprocal Id
- Ⓑ Pyth Id

$$\textcircled{2} \tan x + \sec x = \frac{1 + \sin x}{\cos x}$$

Method (1)

$$\textcircled{A} \frac{\sin x}{\cos x} + \sec x = \frac{1 + \sin x}{\cos x}$$

$$\frac{\sin x}{\cos x} + \frac{1}{\cos x} \textcircled{B} = \frac{1 + \sin x}{\cos x}$$

$$\frac{\sin x + 1}{\cos x} = \frac{1 + \sin x}{\cos x}$$

$$\boxed{\frac{1 + \sin x}{\cos x} = \frac{1 + \sin x}{\cos x}}$$

\textcircled{A} Quotient Id

\textcircled{B} Reciprocal Id

Method (2)

$$\tan x + \sec x = \frac{1 + \sin x}{\cos x}$$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \textcircled{A} \sec x + \frac{\sin x}{\cos x}$$

$$= \sec x + \tan x \textcircled{B}$$

$$\boxed{\tan x + \sec x = \tan x + \sec x}$$

\textcircled{A} Reciprocal Id

\textcircled{B} Quotient Id

$$\textcircled{3} \frac{\sec x}{\sin^3 x} = \frac{\csc^3 x}{\cos x}$$

$$\frac{\sec x}{1} \cdot \frac{1}{\sin^3 x} = \frac{\csc^3 x}{\cos x}$$

$$\textcircled{A} \frac{1}{\cos x} \cdot \frac{1}{\sin^3 x}$$

$$\frac{1}{\cos x} \cdot \frac{\csc^3 x}{1} \textcircled{B}$$

$$\boxed{\frac{\csc^3 x}{\cos x} = \frac{\csc^3 x}{\cos x}}$$

\textcircled{A} Reciprocal
 \textcircled{B} Reciprocal

Method 2

$$\frac{\sec x}{\sin^3 x} = \frac{\csc^3 x}{\cos x}$$

$$= \frac{\csc^3 x}{1} \cdot \frac{1}{\cos x} \textcircled{A}$$

$$= \frac{\csc^3 x}{1} \cdot \sec x$$

$$= \textcircled{B} \frac{1}{\sin^3 x} \cdot \frac{\sec x}{1}$$

$$\boxed{\frac{\sec x}{\sin^3 x} = \frac{\sec x}{\sin^3 x}}$$

\textcircled{A} Reciprocal
 \textcircled{B} Reciprocal

$$(4) \cos x + \sec x = \frac{\cos^2 x + 1}{\cos x}$$

Method (1)

$$\frac{\cos x}{1} + \frac{1}{\cos x} \textcircled{A} = \frac{\cos^2 x + 1}{\cos x}$$

$$\frac{\cos x}{1} \cdot \frac{\cos x}{\cos x} + \frac{1}{\cos x} =$$

$$\frac{\cos^2 x}{\cos x} + \frac{1}{\cos x} =$$

$$\boxed{\frac{\cos^2 x + 1}{\cos x} = \frac{\cos^2 x + 1}{\cos x}}$$

(A) Reciprocal Id

Method (2)

$$\cos x + \sec x = \frac{\cos^2 x}{\cos x} + \frac{1}{\cos x}$$

$$= \frac{\cancel{\cos x} \cos x}{\cancel{\cos x}} + \frac{1}{\cos x}$$

$$= \cos x + \frac{1}{\cos x}$$

$$\boxed{\cos x + \sec x = \cos x + \sec x} \textcircled{A}$$

(A) Reciprocal Id

$$\textcircled{5} \cot^2 x + 1 = \frac{1}{\sin^2 x}$$

Method 1

$$\text{csc}^2 x \textcircled{A} = \frac{1}{\sin^2 x}$$

$$\frac{1}{\sin^2 x} \textcircled{B} = \frac{1}{\sin^2 x}$$

Ⓐ Pyth Id

Ⓑ Reciprocal Id

Method 2

$$\cot^2 x + 1 = \text{csc}^2 x \textcircled{A}$$

$$\cot^2 x + 1 = \cot^2 x + 1 \textcircled{B}$$

Ⓐ Reciprocal Id

Ⓑ Pyth Id

$$\textcircled{6} \tan x \csc x \sec x = \tan^2 x + 1$$

$$\frac{\tan x}{1} \cdot \frac{1}{\sin x} \cdot \frac{\sec x}{1} \textcircled{A}$$

$$\textcircled{B} \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \frac{\sec x}{1}$$

$$\frac{\sin x}{\sin x} \cdot \frac{1}{\cos x} \cdot \frac{\sec x}{1}$$

$$1 \cdot \frac{1}{\cos x} \cdot \frac{\sec x}{1}$$

$$\textcircled{C} \sec x \cdot \sec x$$

$$\sec^2 x$$

$$\textcircled{D} \tan^2 x + 1 = \tan^2 x + 1$$

Ⓐ Reciprocal

Ⓑ Quotient

Ⓒ Reciprocal

Ⓓ Pyth Id

⑥ Method 2

$$\begin{aligned}\tan x \csc x \sec x &= \tan^2 x + 1 \\ &= \sec^2 x \text{ (A)} \\ &= \sec x \sec x\end{aligned}$$

$$\text{(B)} \quad = \frac{1}{\cos x} \cdot \frac{\sec x}{1}$$

$$= \frac{\sin x}{\sin x} \cdot \frac{1}{\cos x} \cdot \frac{\sec x}{1}$$

$$= \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \frac{\sec x}{1}$$

$$\text{(C)} \quad = \tan x \cdot \frac{1}{\sin x} \cdot \sec x$$

$$= \tan x \cdot \csc x \text{ (D)} \cdot \sec x$$

④ Pyth Id

③ Recip-ocal Id

② Quotient Id

① Recip-ocal Id

$$\textcircled{7} \sin^2 x (\tan^2 x + 1) = \frac{\tan x}{\cot x}$$

$$\sin^2 x \cdot \textcircled{A} \sec^2 x =$$

$$\frac{\sin^2 x}{1} \cdot \frac{1}{\cos^2 x} \textcircled{B} =$$

$$\frac{\sin^2 x}{\cos^2 x} =$$

$$\tan^2 x \textcircled{C} =$$

$$\frac{\tan x}{1} \cdot \frac{\tan x}{1} =$$

$$\frac{\tan x}{1} \cdot \frac{1}{\cot x} \textcircled{D} =$$

$\frac{\tan x}{\cot x}$	=	$\frac{\tan x}{\cot x}$
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Ⓐ Pyth Id

Ⓑ Reciprocal Id

Ⓒ Quotient Id

Ⓓ Reciprocal Id

⑦ Method ②

$$\sin^2 x (\tan^2 x + 1) = \frac{\tan x}{1} \cdot \frac{1}{\cot x}$$
$$= \tan x \cdot \tan x \text{ (A)}$$

$$= \tan^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} \text{ (B)}$$

$$= \frac{\sin^2 x}{1} \cdot \frac{1}{\cos^2 x}$$

$$= \frac{\sin^2 x}{1} \cdot \frac{\sec^2 x}{1} \text{ (C)}$$

$$= \sin^2 x (\tan^2 x + 1) \text{ (D)}$$

(A) Reciprocal Id

(B) Quotient Id

(C) Reciprocal Id

(D) Pyth Id