

Trig Identities

$$(1) \sec x - \tan x \sin x = \frac{1}{\sec x}$$

$$\begin{array}{c} \text{B} \rightarrow \\ \frac{1}{\cos x} - \frac{\sin x \sin x}{\cos x \cdot 1} = \frac{1}{\sec x} \end{array}$$

$$\begin{array}{c} \text{A} \rightarrow \\ \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x} = \frac{1}{\sec x} \end{array}$$

$$\frac{1 - \sin^2 x}{\cos x} = \frac{1}{\sec x}$$

$$\text{C} \rightarrow \frac{\cos^2 x}{\cos x} = \frac{1}{\sec x}$$

$$\cos x = \frac{1}{\sec x}$$

- A Reciprocal Id
- B Quotient Id
- C Py Id
- D Reciprocal Id

$$\text{D} \rightarrow \frac{1}{\sec x} = \frac{1}{\sec x}$$

Trig Id #2

$$(2) \frac{1 + \cos X}{\sin X} = \csc X + \cot X$$

(A) → $\frac{1}{\sin X} + \frac{\cos X}{\sin X} = \csc X + \cot X$

$$\csc X + \frac{\cos X}{\sin X} = \csc X + \cot X$$

$$\csc X + \cot X = \csc X + \cot X$$

(B) →

(A) Reciprocal Id

(B) Quotient Id

$$\frac{1 + \cos X}{\sin X} = \csc X + \cot X$$

$$= \frac{1}{\sin X} + \cot X$$

(C) → $\frac{1}{\sin X} + \frac{\cos X}{\sin X}$ ← (D)

$$= \frac{1 + \cos X}{\sin X}$$

(C) Reciprocal Id

(D) Quotient Id

Trig Id #3

$$\textcircled{\#3} \quad \frac{\sec \theta \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$$

$$\textcircled{A} \rightarrow \frac{\frac{1}{\cos \theta} \cdot \sin \theta}{\tan \theta + \cot \theta} = \sin^2 \theta$$

$$\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \cot \theta} = \sin^2 \theta$$

$$\textcircled{B} \rightarrow \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \cot \theta}$$

$$\ast \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} \leftarrow \textcircled{C} = \sin^2 \theta$$

Two methods from here ① combine fractions in denominator
② clear fractions using LCD

Method ① $\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \cot \theta} = \sin^2 \theta$

$$\frac{\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta} + \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta}}$$

$$\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin^2 \theta}{\cos \theta} + \cos^2 \theta} = \sin^2 \theta$$

Trig Id #3

method

(A)

$$\frac{\frac{\sin \theta}{\cos \theta}}{\sin^2 \theta + \cos^2 \theta} = \sin^2 \theta$$

$$\frac{\sin \theta}{\cos \theta}$$

$$\frac{\sin \theta}{\cos \theta}$$

$$= \sin^2 \theta$$

(B)

$$\rightarrow \frac{1}{\sin \theta \cos \theta}$$

$$\frac{\sin \theta}{\cos \theta} \div \frac{1}{\sin \theta \cos \theta} = \sin^2 \theta$$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta \cos \theta}{1} = \sin^2 \theta$$

$$\sin^2 \theta \cdot \frac{\cos \theta}{\cos \theta} = \sin^2 \theta$$

$$\sin^2 \theta \cdot 1 = \sin^2 \theta$$

$$\sin^2 \theta = \sin^2 \theta$$

- (A) Reciprocal Id
- (B) Quotient Id
- (C) Quotient Id
- (D) Py Th Id.

Trig Id # 3 cont

Method (2)

$$* \frac{\sin \theta}{\cos \theta} = \sin^2 \theta$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}$$

$$\text{LCD} = \cos \theta \sin \theta$$

$$\frac{\sin \theta \cos \theta}{\sin \theta \cos \theta} \cdot \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = \sin^2 \theta$$

$$\frac{\sin^2 \theta \cos \theta}{\cos \theta} + \frac{\sin \theta \cos^2 \theta}{\sin \theta} = \sin^2 \theta$$

$$\frac{\sin^2 \theta \cdot \frac{\cos \theta}{\cos \theta}}{\sin^2 \theta \cdot \frac{\cos \theta}{\cos \theta} + \frac{\sin \theta \cos^2 \theta}{\sin \theta}} = \sin^2 \theta$$

$$\frac{\sin^2 \theta \cdot 1}{\sin^2 \theta \cdot 1 + 1 \cdot \cos^2 \theta} = \sin^2 \theta$$

$$\frac{\sin^2 \theta \cdot 1}{\sin^2 \theta \cdot 1 + 1 \cdot \cos^2 \theta} = \sin^2 \theta$$

$$\text{(E)} \rightarrow \frac{\sin^2 \theta}{\sin^2 \theta + \cos^2 \theta} = \frac{\sin^2 \theta}{1} = \sin^2 \theta$$

(E)
Pyth Id

Trig Id #4

$$\textcircled{\#4} \quad \frac{\sec \theta}{\cos \theta} - \frac{\tan \theta}{\cot \theta} = 1$$

$$\frac{\sec \theta}{1} \cdot \frac{1}{\cos \theta} - \frac{\tan \theta}{1} \cdot \frac{1}{\cot \theta} = 1$$

$$\sec \theta \cdot \sec \theta - \tan \theta \cdot \tan \theta = 1$$

(A) →
← (B)
(A)(B) Reciprocal Id

$\sec^2 \theta - \tan^2 \theta = 1$

this has many ways to be completed

Note $\sin^2 \theta + \cos^2 \theta = 1$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 = \sec^2 \theta - \tan^2 \theta \quad \text{Pyth Id}$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} = 1$$

© →

ⓓ →

$$\frac{1 - \sin^2 \theta}{\cos^2 \theta} = 1$$

$$\frac{\cos^2 \theta}{\cos^2 \theta} = 1$$

1 = 1

ⓔ →

- © Reciprocal Id
- ⓓ Quotient Id
- ⓔ Pyth Id

Trig Id ⑤

$$\textcircled{\#5} \quad \cos^2 y - \sin^2 y = 1 - 2\sin^2 y$$

Method ①

$$[1 - \sin^2 y] - \sin^2 y = 1 - 2\sin^2 y$$

① →

$$\textcircled{A} \quad 1 - 1\sin^2 y - 1\sin^2 y = 1 - 2\sin^2 y$$

① Py.Th Id

$$1 - 2\sin^2 y = 1 - 2\sin^2 y$$

Method ②

$$\cos^2 y - \sin^2 y = 1 - 2\sin^2 y$$

$$\textcircled{B} \rightarrow = [\cos^2 y + 1\sin^2 y] - 2\sin^2 y$$

$$= \cos^2 y + (1-2)\sin^2 y$$

$$= \cos^2 y - \sin^2 y$$

③ Py.Th Id

Trig Id #6

$$\textcircled{\#6} \csc^2 \theta \tan^2 \theta - 1 = \tan^2 \theta$$

$$\frac{1}{\sin^2 \theta} \cdot \frac{\tan^2 \theta}{1} - 1 = \tan^2 \theta$$

$$\textcircled{A} \rightarrow \frac{1}{\sin^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} - 1 = \tan^2 \theta$$

$$\frac{\sin^2 \theta}{\sin^2 \theta} \cdot \frac{1}{\cos^2 \theta} - 1 = \tan^2 \theta$$

$$\textcircled{A} \text{ Reciprocal Id} \quad 1 \cdot \frac{1}{\cos^2 \theta} - 1 = \tan^2 \theta$$

$$\boxed{\sec^2 \theta - 1 = \tan^2 \theta}$$

this is a
Pyth Id.

Note: $\cos^2 \theta + \sin^2 \theta = 1$

$$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$[1 + \tan^2 \theta] - 1 = \tan^2 \theta$$

$$\textcircled{B} \rightarrow \tan^2 \theta = \tan^2 \theta$$

\textcircled{B} Pyth. Id.

Trig Id #7

$$\textcircled{\#7} \frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$$

Method C

$$\frac{\sec^2 \theta}{1} \cdot \frac{1}{\sec^2 \theta - 1} = \csc^2 \theta$$

Method C

$$\frac{1}{\cos^2 \theta} \frac{1}{(\sec^2 \theta - 1)} = \csc^2 \theta$$

$\textcircled{A} \rightarrow$

$$\frac{1}{\cos^2 \theta \sec^2 \theta - \cos^2 \theta} = \csc^2 \theta$$

$$\frac{1}{\cos^2 \theta \frac{1}{\cos^2 \theta} - \cos^2 \theta} = \csc^2 \theta$$

$\textcircled{B} \rightarrow$

$$\frac{1}{\frac{\cos^2 \theta}{\cos^2 \theta} - \cos^2 \theta} = \csc^2 \theta$$

$$\frac{1}{1 - \cos^2 \theta} = \csc^2 \theta$$

$$\frac{1}{\sin^2 \theta} = \csc^2 \theta$$

$\textcircled{C} \rightarrow$

$$\csc^2 \theta = \csc^2 \theta$$

\textcircled{A} Reciprocal Id

\textcircled{B} Reciprocal Id

\textcircled{C} PyTh Id

Trig Id #7 Method 2

$$\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$$

Note $\cos^2 \theta + \sin^2 \theta = 1$

$$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\tan^2 \theta = \sec^2 \theta - 1$$

$$\frac{\sec^2 \theta}{\tan^2 \theta} = \csc^2 \theta$$

(A) \rightarrow

$$\frac{\sec^2 \theta}{1} \cdot \frac{1}{\tan^2 \theta} = \csc^2 \theta$$

$$\sec^2 \theta \cdot \cot^2 \theta = \csc^2 \theta$$

(B) \rightarrow

$$\frac{1}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta} = \csc^2 \theta$$

(C) \rightarrow

$$\frac{\cos^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\sin^2 \theta} = \csc^2 \theta$$

$$1 \cdot \frac{1}{\sin^2 \theta} = \csc^2 \theta$$

$$\therefore \rightarrow \csc^2 \theta = \csc^2 \theta$$

(A) Pyth Id

(B) Reciprocal Id

(C) Reciprocal Id

(D) Reciprocal Id

Trig Id #7

$$\frac{\sec^2 \theta}{\sec^2 \theta - 1} = \csc^2 \theta$$

$$\textcircled{A} \rightarrow \frac{1}{\cos^2 \theta} = \csc^2 \theta$$

↘ $\frac{1}{\cos^2 \theta} - 1$

We can "clear" the fraction using $\frac{\cos^2 \theta}{\cos^2 \theta}$

$$\frac{\cos^2 \theta}{\cos^2 \theta} \left(\frac{1}{\cos^2 \theta} \right) = \csc^2 \theta$$
$$\frac{\cos^2 \theta}{\cos^2 \theta} \left(\frac{1}{\cos^2 \theta} - 1 \right)$$

$$\frac{\frac{\cos^2 \theta}{\cos^2 \theta}}{\frac{\cos^2 \theta}{\cos^2 \theta} - \cos^2 \theta} = \csc^2 \theta$$

$$\frac{1}{1 - \cos^2 \theta} = \csc^2 \theta$$

$$\textcircled{B} \rightarrow \frac{1}{\sin^2 \theta} = \csc^2 \theta$$

$$\textcircled{C} \rightarrow \csc^2 \theta = \csc^2 \theta$$

Ⓐ Reciprocal Id

Ⓑ Pyth Id

Ⓒ Reciprocal Id

Trig Id # 8

$$\textcircled{\#8} \quad \tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

$$\textcircled{A} \quad \tan^2 x \cdot (1 - \cos^2 x) = \tan^2 x - \sin^2 x$$

$$\tan^2 x - \tan^2 x \cos^2 x = \tan^2 x - \sin^2 x$$

$$\tan^2 x - \frac{\sin^2 x}{\cos^2 x} \frac{\cos^2 x}{1} = \tan^2 x - \sin^2 x$$

\textcircled{B} \nearrow

$$\tan^2 x - \frac{\cos^2 x}{\cos^2 x} \cdot \sin^2 x = \tan^2 x - \sin^2 x$$

$$\tan^2 x - 1 - \sin^2 x = \tan^2 x - \sin^2 x$$

$$\tan^2 x - \sin^2 x = \tan^2 x - \sin^2 x$$

\textcircled{A} Pyth Id

\textcircled{B} Quotient Id

Trig Id #8

Method 2

$$\textcircled{8} \quad \tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

Note $\cos^2 x + \sin^2 x = 1$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\boxed{\tan^2 x = \sec^2 x - 1}$$

$$\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$$

$$\textcircled{A} \rightarrow [\sec^2 x - 1] \sin^2 x = \tan^2 x - \sin^2 x$$

$$\sec^2 x \cdot \sin^2 x - \sin^2 x = \tan^2 x - \sin^2 x$$

$$\textcircled{B} \rightarrow \frac{1}{\cos^2 x} \sin^2 x - \sin^2 x = \tan^2 x - \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} - \sin^2 x = \tan^2 x - \sin^2 x$$

$$\textcircled{C} \rightarrow \tan^2 x - \sin^2 x = \tan^2 x - \sin^2 x$$

Ⓐ Pyth Id

Ⓑ Reciprocal Id

Ⓒ Quotient Id

$$(9) \quad \underbrace{(\sin\theta + \cos\theta)^2}_{\textcircled{I}} + \underbrace{(\sin\theta - \cos\theta)^2}_{\textcircled{II}} = 2$$

$$\begin{aligned} \textcircled{I} \quad (\sin\theta + \cos\theta)^2 &= (\sin\theta + \cos\theta)(\sin\theta + \cos\theta) \\ &= \sin^2\theta + \cos\theta \sin\theta + \cos\theta \sin\theta + \cos^2\theta \\ &= \sin^2\theta + \cos^2\theta + 2\cos\theta \sin\theta \end{aligned}$$

$$\begin{aligned} \textcircled{II} \quad (\sin\theta - \cos\theta)^2 &= (\sin\theta - \cos\theta)(\sin\theta - \cos\theta) \\ &= \sin^2\theta - \cos\theta \sin\theta - \cos\theta \sin\theta + \cos^2\theta \\ &= \sin^2\theta + \cos^2\theta - 2\cos\theta \sin\theta \end{aligned}$$

$$\textcircled{I} + \textcircled{II} = 2$$

$$\underbrace{\boxed{\sin^2\theta + \cos^2\theta}}_{\textcircled{A}} + 2\sin\theta\cos\theta + \underbrace{\boxed{\sin^2\theta + \cos^2\theta}}_{\textcircled{B}} - 2\sin\theta\cos\theta = 2$$

$$1 + 2\sin\theta\cos\theta + 1 - 2\sin\theta\cos\theta = 2$$

$$1 + 1 + 0 = 2$$

$$2 = 2$$

\textcircled{A} Pyth I
 \textcircled{B} Pyth II

$$(10) (\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \csc\theta$$

$$\sin\theta \overset{\text{I}}{\tan\theta} + \cos\theta \overset{\text{II}}{\tan\theta} + \sin\theta \overset{\text{III}}{\cot\theta} + \cos\theta \overset{\text{IV}}{\cot\theta} = \sec\theta + \csc\theta$$

$$\text{I} \quad \sin\theta \cdot \frac{\sin\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

$$\text{II} \quad \cos\theta \cdot \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{1} \cdot \frac{\cos\theta}{\cos\theta} = \sin\theta \cdot 1 = \sin\theta$$

$$\text{III} \quad \sin\theta \cdot \frac{\cos\theta}{\sin\theta} = \frac{\sin\theta}{\sin\theta} \cdot \frac{\cos\theta}{1} = \cos\theta \cdot 1$$

$$\text{IV} \quad \cos\theta \cdot \frac{\cos\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$$

$$\frac{\sin^2\theta}{\cos\theta} + \sin\theta + \cos\theta + \frac{\cos^2\theta}{\sin\theta} = \sec\theta + \csc\theta$$

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1} + \frac{\cos^2\theta}{\sin\theta} + \frac{\sin\theta}{1} = \sec\theta + \csc\theta$$

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta} + \frac{\sin\theta}{1} \cdot \frac{\sin\theta}{\sin\theta} = \sec\theta + \csc\theta$$

$$\frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta} = \sec\theta + \csc\theta$$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \sec\theta + \csc\theta$$

(1b) cont

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \sec\theta + \csc\theta$$

$$\textcircled{B} \frac{1}{\cos\theta} + \textcircled{B} \frac{1}{\sin\theta} = \sec\theta + \csc\theta$$

$$\textcircled{C} \sec\theta + \frac{1}{\sin\theta} = \sec\theta + \csc\theta$$

$$\boxed{\sec\theta + \csc\theta \textcircled{D} = \sec\theta + \csc\theta}$$

(A) Quotient Id

(B) Pyth Id

(C) Reciprocal Id

(D) Reciprocal Id

⑪ Method ①

$$\frac{\tan \theta - 1}{\tan \theta + 1} = \frac{1 - \cot \theta}{1 + \cot \theta}$$

$$\frac{(\tan \theta - 1) \cdot \frac{\cot \theta}{\cot \theta}}{(\tan \theta + 1) \cdot \frac{\cot \theta}{\cot \theta}} = \frac{1 - \cot \theta}{1 + \cot \theta}$$

$$\frac{\tan \theta \cot \theta - \cot \theta}{\tan \theta \cot \theta + \cot \theta} = \frac{1 - \cot \theta}{1 + \cot \theta}$$

$$\frac{\tan \theta \cdot \frac{1}{\tan \theta} - \cot \theta}{\tan \theta \cdot \frac{1}{\tan \theta} + \cot \theta} = \frac{1 - \cot \theta}{1 + \cot \theta}$$

$$\frac{\frac{\tan \theta}{\tan \theta} - \cot \theta}{\frac{\tan \theta}{\tan \theta} + \cot \theta} = \frac{1 - \cot \theta}{1 + \cot \theta}$$

$$\boxed{\frac{1 - \cot \theta}{1 + \cot \theta} = \frac{1 - \cot \theta}{1 + \cot \theta}}$$

Ⓐ Reciprocal Id

Ⓑ Reciprocal Id

(11) Method 2

$$\frac{\tan \theta - 1}{\tan \theta + 1} = \frac{(1 - \cot \theta)}{(1 + \cot \theta)} \cdot \frac{\tan \theta}{\tan \theta}$$

$$= \frac{\tan \theta - \cot \theta \tan \theta}{\tan \theta + \cot \theta \tan \theta}$$

$$= \frac{\tan \theta - \cot \theta \frac{1}{\cot \theta}}{\tan \theta + \cot \theta \frac{1}{\cot \theta}} \quad \text{(A)}$$

$$\frac{\tan \theta - 1}{\tan \theta + 1} \quad \text{(B)}$$

$$= \frac{\tan \theta - \frac{\cot \theta}{\cot \theta}}{\tan \theta + \frac{\cot \theta}{\cot \theta}}$$

$$= \frac{\tan \theta - 1}{\tan \theta + 1}$$

(A) Reciprocal Id

(B) Reciprocal Id

$$\textcircled{12} \quad \frac{1 - \tan^2 x}{1 + \tan^2 x} = 1 - 2\sin^2 x$$

$$\frac{1 - \tan^2 x}{1} \cdot \frac{1}{1 + \tan^2 x} = 1 - 2\sin^2 x$$

$$\frac{1 - \tan^2 x}{1} \cdot \frac{1}{\sec^2 x} \textcircled{A} = 1 - 2\sin^2 x$$

$$(1 - \tan^2 x) (\cos^2 x) \textcircled{B} = 1 - 2\sin^2 x$$

$$\cos^2 x - \tan^2 x \cos^2 x = 1 - 2\sin^2 x$$

$$\cos^2 x - \frac{\sin^2 x}{\cos^2 x} \frac{\cos^2 x}{1} = 1 - 2\sin^2 x$$

$$\cos^2 x - \frac{\sin^2 x}{1} \frac{\cos^2 x}{\cos^2 x} = 1 - 2\sin^2 x$$

$$\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$\textcircled{D} \quad 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$$

$$\boxed{1 - 2\sin^2 x = 1 - 2\sin^2 x}$$

Ⓐ Pythid

Ⓑ Reciprocal Id

Ⓒ Quotient Id

$$\textcircled{13} \quad \frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{1 + \cos x}{\sin x (\sin^2 x)} = \frac{\csc x}{1 - \cos x}$$

$$\frac{1}{\sin x} \cdot \frac{1 + \cos x}{\sin^2 x} = \frac{\csc x}{1 - \cos x}$$

$$\textcircled{A} \quad \frac{\csc x}{1} \cdot \frac{1 + \cos x}{\sin^2 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\csc x}{1} \cdot \frac{1 + \cos x}{1 - \cos^2 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\csc x}{1} \cdot \frac{1 + \cos x}{(1 - \cos x)(1 + \cos x)} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\csc x}{1} \cdot \frac{1}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{\csc x}{1 - \cos x} \cdot 1 = \frac{\csc x}{1 - \cos x}$$

$$\frac{\csc x}{1 - \cos x} = \frac{\csc x}{1 - \cos x}$$

- Ⓐ Reciprocal
- Ⓑ Pythagorean
- Ⓒ DOYS

⑬ Method ②

$$\frac{\cos x + 1}{\sin^3 x} = \frac{\csc x}{1 - \cos x}$$

$$\frac{1 + \cos x}{\sin^3 x} = \frac{\csc x}{1} \cdot \frac{1}{1 - \cos x}$$

$$\frac{1 + \cos x}{\sin^3 x} = \frac{\csc x}{1} \cdot \frac{1}{1 - \cos x} \cdot \frac{(1 + \cos x)}{(1 + \cos x)}$$

$$= \frac{\csc x}{1} \cdot \frac{1 + \cos x}{1 - \cos^2 x} \text{ (A)}$$

$$= \frac{\csc x}{1} \cdot \frac{1 + \cos x}{\sin^2 x} \text{ (B)}$$

$$= \frac{1}{\sin x} \cdot \frac{1 + \cos x}{\sin^2 x}$$

$$= \text{④} \frac{1 + \cos x}{\sin^3 x}$$

① DOTS

② Pyth Id

③ Reciprocal

$$\textcircled{14} \csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$$

$$\textcircled{A} (\csc^2 x - \cot^2 x)(\csc^2 x + \cot^2 x) = \csc^2 x + \cot^2 x$$

$$\textcircled{B} 1 \cdot (\csc^2 x + \cot^2 x) = \csc^2 x + \cot^2 x$$

$$\csc^2 x + \cot^2 x = \csc^2 x + \cot^2 x$$

\textcircled{A} DOTs

\textcircled{B} Pyth Id

$$(15) \frac{\tan \theta}{\sec \theta} + \frac{\cot \theta}{\csc \theta} = \sin \theta + \cos \theta$$

$$\frac{\tan \theta}{1} \cdot \frac{1}{\sec \theta} + \frac{\cot \theta}{1} \cdot \frac{1}{\csc \theta} = \sin \theta + \cos \theta$$

$$\frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sec \theta} + \frac{\cot \theta}{1} \cdot \frac{1}{\csc \theta} = \sin \theta + \cos \theta$$

$$(A) \frac{\sin \theta}{\cos \theta} \cdot \frac{\cos \theta}{1} (B) + \frac{\cot \theta}{1} \cdot \frac{1}{\csc \theta} = \sin \theta + \cos \theta$$

$$\sin \theta \cdot \frac{\cos \theta}{\cos \theta} + \frac{\cot \theta}{1} \cdot \frac{1}{\csc \theta} = \sin \theta + \cos \theta$$

$$\sin \theta \cdot 1 + \frac{\cot \theta}{1} \cdot \frac{1}{\csc \theta} = \sin \theta + \cos \theta$$

$$\sin \theta + \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\csc \theta} = \sin \theta + \cos \theta$$

$$\sin \theta + \frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{1} (D) = \sin \theta + \cos \theta$$

$$\sin \theta + \frac{\cos \theta}{1} \cdot \frac{\sin \theta}{\sin \theta} = \sin \theta + \cos \theta$$

$$\sin \theta + \cos \theta \cdot 1 = \sin \theta + \cos \theta$$

$$\boxed{\sin \theta + \cos \theta = \sin \theta + \cos \theta}$$

(A) Quotient

(B) Reciprocal

(C) Quotient

(D) Reciprocal

$$\textcircled{16} \frac{\sin y + \tan y}{1 + \sec y} = \sin y$$

$$\frac{\frac{\sin y}{1} + \frac{\sin y}{\cos y} \textcircled{A}}{1 + \sec y} = \sin y$$

$$\frac{\frac{\sin y}{1} \frac{\cos y}{\cos y} + \frac{\sin y}{\cos y}}{1 + \sec y} = \sin y$$

$$\frac{\sin y \cos y + \sin y}{\cos y}$$

$$\frac{\sin y \cos y + \sin y}{\cos y} = \sin y$$

$$\frac{\sin y \cos y + \sin y}{1 + \sec y}$$

$$\left[\frac{\sin y (\cos y + 1)}{\cos y} \right] = \sin y$$

$$1 + \sec y$$

$$\left[\frac{\sin y (\cos y + 1)}{\cos y} \right] = \sin y$$
$$1 + \frac{1}{\cos y} \textcircled{B}$$

Ⓐ Quotient
Ⓑ Reciprocal

⑩ cont

$$\left[\frac{\sin y (\cos y + 1)}{\cos y} \right] = \sin y$$

$$\frac{1}{1} + \frac{1}{\cos y}$$

$$\left[\frac{\sin y (\cos y + 1)}{\cos y} \right] = \sin y$$

$$\left[\frac{\cos y}{\cos y} + \frac{1}{\cos y} \right]$$

$$\frac{\sin y (\cos y + 1)}{\cos y} = \sin y$$

$$\frac{\cos y + 1}{\cos y}$$

$$\sin y \cdot \left[\frac{\cos y + 1}{\cos y} \right] = \sin y$$

$$1 \cdot \left[\frac{\cos y + 1}{\cos y} \right]$$

$$\sin y \cdot 1 = \sin y$$

$$\boxed{\sin y = \sin y}$$